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An analysis of BCS-BEC crossover under three different approaches and evaluation of critical temperature T_c

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Abstract
BCS-BEC crossover regimes can be studied using three approaches (i) from 1D and 2D Hubbard model (ii) from two-channel model for system of ultra-cold alkali atoms (iii) via Feshbach resonance method. It has been observed that in ^{40}K, the appearance of the condensate in BCS regime is accompanied by a significant decrease in the width of the non-condensate fraction. The origin of this phenomenon is yet unknown and requires further study. Another important aspect of the BCS-BEC transition is that one studies the nature of the condensate formed when the system is taken across the resonance. It is not known if the condensate is in the intermediate region of resonance then what will happen whether one will take the linear combination of Cooper pairs and tightly bound molecules or some complex state.

Keywords: Feshbach resonance, Hyperfine states, Bathe ansatz, GSCF, Zeeman level, Riemanian Zeta function, Coordination number, Detuning parameter

1. Introduction

BCS theory or Bardeen–Cooper–Schrieffer theory is a microscopic theory of superconductivity. The theory describes superconductivity as a microscopic effect caused by a condensation of Cooper pair. The theory is used in nuclear physics to describe the pairing interaction between nucleons in an atomic nucleus. It was proposed by Bardeen, Cooper, and Schrieffer in 1957; they received the Nobel Prize in Physics for this theory in 1972.

In condensed matter physics, a Bose–Einstein condensate (BEC) is a state of matter (also called the fifth state of matter) which is typically formed when a gas of bosons at low densities is cooled to temperatures very close to absolute zero (-273.15 °C). Under such conditions, a large fraction of bosons occupy the lowest quantum state, at which point microscopic quantum mechanical phenomena, particularly wave function interference, become apparent macroscopically. A BEC is formed by cooling a gas of extremely low density (about one-hundred-thousandth (1/100,000) the density of normal air) to ultra-low temperatures. This state was first predicted, generally, in 1924–1925 by Albert Einstein following and crediting a pioneering paper by Satyendra Nath Bose on the new field now known as quantum statistics.

In the context of ultra-cold Fermi gases, a BEC-BCS crossover means that by tuning the interaction strength (the s-wave scattering length), one goes from a BEC state to a BCS state without encountering a phase transition (thus the word "crossover"). BCS-BEC crossover physics now a days are considered challenging problem in condensed matter physics, This is because, there exists a three distinct limit which contains rich varieties of physics. BCS is a weak coupling limit of loosely bound Cooper pairs whereas BEC is a strong coupling limit of tightly bound molecules (composite bosons). In between these, there exists an unitarily limit which itself contains rich amount of physics, because in this case an important parameter s-wave scattering length goes to zero and the system becomes independent of various constraints. BCS-BEC crossover has been studied with different formalisms like using BCS theory, and various models used for the study of ultracold physics.

As one knows that unlike conventional superconductors in which the phase coherence and pairing occur simultaneously at the same temperature, under-doped cuprates, there exists a separation between the pair binding regime (energy gap) and the superconductivity regime (coherency between pairs) as the doping exceeds the optimal value. The smooth crossover occurs from BCS regime (weak) to localized Bose condensation (BEC) strong coupling superconductivity. This has been investigated in low dimensional Hubbard lattices due to possible relevance in high temperature superconductivity. In these studies, one applies the generalized self-consistent field (GSCF) in one and two-dimensional (1D and 2D) lattice by introducing an order (correlation) parameter $\Delta^{\rightarrow(-)}{}_q$ with wave vector $0\leq|q\rightarrow|\leq\pi$ and the renormalized chemical potential μ to study the crossover. This also studies the ground state properties within the attractive Hubbard model in wide range of coupling strength U and electron concentration n.

Ultra cold fermion atoms such as ^{40}K and ^{6}Li are used to achieve a new kind of superfluidity where effective interactions are tuned via the magnetic field. In two component system made of the fermion atoms in two or different hyperfine states, it is possible to produce the weakly bound molecules with binding energy in three order of magnitude smaller than helium dimmers (the weakest bound pairs in the solid state physics) . Such small size molecules get formed in the presence of magnetic field $B < B_0$

where the characteristic Feshbach resonance B_0depends on specific atoms and their Zeeman configuration. At ultralow low temperature $T \sim T_f$ (for 10^5 trapped atoms with concentration $10^{13}cm^{-3}$ the typical Fermi temperature T_F=100nK) these weakly bound molecules have been observed to undergo Bose-Einstein condensation (BEC). One investigates the fluctuation model by time dependent interchange between BCS and BEC regimes in the ultracold gas of fermions atoms. Experimentally switching between the BCS and BEC regimes is often done in time interval much shorter than intrinsic scale set by the Fermi velocity v_F. The fast time dynamic has been used to study this effect analytically and numerically. One shows that for sufficiently large frequency there appears oscillations of the order parameter.

As one knows, degenerate Fermi gas and Bose-Einstein Condensation possess vastly different properties because of their respective statistics. Now it has become possible to connect these two systems and convert directly between them by using a Feshbach resonance. This effect has first observed in condensed matter systems by Shin Enoye et al. MIT. This method uses an applied dc magnetic field to manipulate atoms in different hyperfine states and exploit quasi-bound molecular states in order to vary the s-wave scattering length. This works because variable magnetic field allows for vanish tuning of the molecular state energy is equal to the energy of the two (free) scattering atoms, a possibility due to different magnetic moments ($\frac{\partial E}{\partial B}$) of the different states.

2. Approaches to study of BCS-BEC crossover

From the theoretical formalism of A N Kocharian et al., T. Domanski and A Donabidowicz and Phillip Powell, we have studied BCS-BEC crossover physics with three different approaches. In the first approach, we have used 1D and 2D Hubbard model. In the second approach, we have used two-channel model for system of ultracold alkali atoms. In the third approach, we have used Feshbach resonances.

In the first approach, our evaluated results of electron concentration n are obtained as a function of (U/zt). Here U is the coupling strength, z is the coordination number and for 1D, z=2 and for 2D, z=4, t is hopping sites. We have shown the results of n as a function of (U/zt) for both BCS and BEC regimes. In both regimes and for both 1D and 2D, n decrease as a function of (U/zt) and there is no BEC of electrons both in 1D and 2D for n=l no matter how strong is |U|. The BCS-BEC crossover in one dimension at $n \neq 1$ is overall in agreement with the continuum model.

Within GSCF approach it is given by simple expression

$$D_0=\Delta^{(-)^2}{}_q/4U^2 \quad (1)$$

is valid for arbitrary dimension $U < 0$, $h > 0$ and $0 < n < l$. D_o is given by

$$D_0 = n^2/(4-\Delta^2) \quad (2)$$

D_0 is the concentration of unbound electron pairs at U=0 and in the presence[20] of magnetic field h. The concentration of bound electron pairs in given by the expression

$$n_{pair}=D-D_0 \quad (3)$$

In GSCF approach it reduces to

$$n_{pair}=\Delta^{(-)^2}{}_q/4U^2 \quad (4)$$

The position K_0 of the minimum of the excitation spectrum E_k distinguishes the BCS regime ($K_0 \neq 0$) from the BEC regime $K_0 = 0$, h=0. Similarly, for BCS regime $E^{(-)}_{gap} = \Delta^{(-)}_0$ and for BEC regime $E^{(-)}_{gap} > \Delta^{(-)}_0$. In this calculation, it appears that GSCF (general self-consistent field) theory gives a simple relationship between the inhomogeneous BCS order parameter Δ^-_q with $q \neq 0$ and the corresponding density of bound pairs n_{pair} in the presence of magnetic field.

In the second approach, we have studied BCS-BEC crossover regime by evaluating two parameters (i) critical temperature T_c as a function of detuning parameter using boson-Fermion model and (ii) time dependence of the order parameter $|\chi(t)|$ for different values of v(t).

Table 1: T1An evaluated result of critical temperature T_c as a function of detuning parameter 2v for the version of the boson-Fermion model with total concentration n_{tot} =1, D is the Fermion band width taken as unit of energy

2v/D	$k_\beta T_c/D$
-2.0	0.0052
-1.0	0.0095
-0.60	0.0125
-0.50	0.0146
-0.40	0.0168
-0.30	0.0267
-0.20	0.0143
-0.10	0.0087
0.00	0.0054
0.10	0.0048
0.20	0.0042
0.50	0.0036
0.60	0.00040
0.80	0.00050

Our evaluated results of $(K_\beta T_c/D)$ as a function of (2v/D) in the BCS-BEC regime initially increase with (2v/D) and become flat between -0.85 to -0.3 and then decrease and finally become zero. Here, v is detuning parameter and D is Fermion band width taken in the unit of energy.

In third approach, we have evaluated three parameters (i) ($\delta U/U_{cl}$) as a function of (T/T_F) (ii) normalized scattering length as a function of magnetic field (iii) Condensate fraction (N_0/N) as a function of (T/T_F).

The identification of the Feshbach resonance was made by directly measuring the scattering length 'a' as a function of the applied magnetic field. The scattering length near resonance is approximated [21] as

$$a = a_0 \left[1-\frac{C}{B-B_0}\right] \quad (5)$$

where a_0 is the scattering length far away from resonance and C is roughly independent of B. In the case of Feshbach resonance in which BCS-BEC transition is achieved by forming composite bosons from pair of fermions $m = 2m_F$ and $N_B=N_F/2$

$$\frac{T_F}{T_C} = [\frac{2}{9\pi(\xi 3/2)}]^{1/3} = 0.30 \quad (6)$$

This shows that the BEC temperature is on the same order as the Fermi temperature so it appears that if one begins with degenerate Fermi gas at low enough it may be possible to convert it into a gas of composite bosons cold enough to Bose condensate.
In the first result after comparing theoretical and experimental value show that Fermi gas energy deviates as $T \rightarrow 0$ and it is in accordance with the prediction of non-interacting Fermi gas. In the second evaluation our obtained results show quite satisfactory agreement with the experimental data.

In the third evaluation our obtained results after comparing with experiment show that due to Feshbach resonance there is a formation of bosonic molecules from the fermionic atoms. It has been observed that BEC formation from fermionic atoms ^{40}K and ^{6}Li started with initial temperature $(T/T_F) < 0.17$ and > 0.20 respectively. Our theoretical analysis of (N/N_o) as a function of (T/T_F) shows that BEC formation begins at $T=0.17T_F$. Our theoretically obtained all results are in good agreement with other theoretical workers.

3. Results

From the above theoretical investigations and analysis, we have come across the following results:

1. Our theoretically obtained results of BCS-BEC crossover using 1D and 2D Hubbard model in which electron concentration n are obtained as a function of (U/zt)[Here U is the coupling strength, z is the coordination number and for 1D, z=2 and for 2D, z=4, t is hopping sites] show that in both regimes and for both 1D and 2D, n decrease as a function of (U/zt) and there is no BEC of electrons both in 1D and 2D for n=l no matter how strong is |U|. The BCS-BEC crossover in one dimension at $n \neq l$ is overall in agreement with the continuum model. In this calculation, it appears that GSCP theory gives a simple relationship between the inhomogeneous BCS order parameter $\Delta^{-}_{q}$with $q \neq 0$ and the corresponding density of bound pairs n_{pair} in the presence of magnetic field.
2. Our theoretical results obtained for the critical temperature T_c as a function of detuning parameter for the boson-Fermion model [with the total concentration $n_{tot}= l / (K_\beta T_c / D)$ obtained as function of (2v / *D)*] show that $(K_\beta T_c/D)$ initially increase with (2v/D) and becomes flat between -0.85 to -3.0 and then decrease and becomes zero. One analyses the fluctuation for the detuning parameter oscillating between -0.20 to 0.05
3. Our theoretical results of the time dependence of the order parameter $|\chi(t)|$ for the detuning parameter $v(t) = v_0 \sin 2\pi t/T$ having $v_0 = 0.1D$ and t is expanded as a function of $\hbar/D$ and for v(t)=5, 10, 20 and 40 oscillations, appear that $|\chi(t)|$ fluctuate between 0 and 500. For small frequencies, the order parameter behaves in a non-regular way. Due to damping effects, one observes that the order parameter do not follow the modulation of v(t). With the gradual increase of the frequency the order parameter start to vary in kind of oscillatory way. Oscillation clearly shown up for frequency $\omega \geq 2\Delta_0$ with $\Delta_0 = g\langle b(0)\rangle$ which is the gap of fermion spectrum in the stationary solution. Further increase of the frequency smoothes the envelope of oscillations. Thus the dynamical variation of ground state wave function of the ultra-cold fermion atoms in the response of time dependent detuning across Feshbach resonance gives two important facts, (i) the order parameters have no regular behavior for small ω (ii) it gives quantum oscillations for large frequencies $\omega \geq 2\Delta_0$. In these calculations, one observes a sinusoidal sweep between the BCS and BEC regimes.
4. Our theoretically evaluated results of degenerate Fermi gas energy $(\delta U/U_{cl})$ as a function of (T/T_F). [Here $\delta U = U - U_{cl}$, U is the internal energy of the degenerate Fermi gas and U_{cl} is the classical values $U_{cl} = 3NK_\beta T$].Show that the gas energy deviates significantly as $T \rightarrow 0$ and it is in accordance with the prediction of non-interacting Fermi gas. Our theoretical evaluated results of normalized scattering length as a function of magnetic field using equation (5) and after comparing with the experimental data show that the agreement is quite satisfactory. Similarly our theoretically evaluated results of condensate fraction (N/N_0) as a function of (T/T_F) of Fermi gas ^{40}K and after comparing with the experimental data show that due to Feshbach resonance there is a formation of bosonic molecules from the fermionic atoms. It has been observed that BEC formation from fermionic atoms ^{40}K and ^{6}Li started with initial temperature $(T/T_F) < 0.17$ and > 0.20 respectively. Our theoretical analysis of (N/N_o) as a function of (T/T_F) shows that BEC formation begins at $T=0.17T_F$

4. Conclusions

From the above results shown in this paper, one can conclude that BCS-BEC crossover studies have not been completed yet. It is technically very challenging. It has been observed that in ^{40}K, the appearance of the condensate in BCS regime is accompanied by a significant decrease in the width of the non-condensate fraction. The origin of this phenomenon is yet unknown and requires further study. Another important aspect of the BCS-BEC transition is that one studies the nature of the condensate formed when the system is taken across the resonance. It is not known if the condensate is in the intermediate region of resonance then what will happen whether one will take the linear combination of Cooper pairs and tightly bound molecules or some complex state. This thing is unresolved yet.

5. References

1. AJ Leggett, J Physics. (Paris) C7, Collq, 1980, 41.
2. P Nozieres *et al*., J Low. Temp. Phys. (JLTP), 1985, 59, 195.
3. M Casas. *et al*., Phys. Rev. A, 1991, 44, 4915.
4. AN Kocharian, *et al*., Phys. Rev. B, 1999, 59, 7458.
5. AN Kocharian, *et al*., Phys, Rev. B, 1999, 29-31, 3538.
6. K Tanaka. *et al*., Phys. Rev. B, 1999, 60, 3508.
7. VJ Emery, *et al*., Physica C, 1993, 209, 597.
8. VJ Emery. *et al*., J phys, chem., solids, 1998, 59, 1705.
9. RA Duine. *et al*., Phys. Rep, 2004, 396, 115.
10. M Reiner. *et al*., Nature, 2003, 426, 537.
11. J Jochin. *et al*., Science, 2003, 302, 2101.
12. AV Andreev. *et al*., Phys. Rev. Lett. (PRL), 2004, 93, 130402.
13. M Haque. *et al*., Phys. Rev. A, 2005, 71, 063603.
14. S Inouye. *et al*., Nature (London), 1998, 392, 151.

15. AJ Moerdijk. *et al*., Phys. Rev. A, 1995, 51, 4852.
16. T Kholer. *et al*., Rev. Mod. Phys, 2006, 78, 1311.
17. AN Kocharian. *et al*., Physica C, 2001, 364-365, 131.
18. T Domanski, A Donabidowicz, Acta Physica Polonica, 2006, 109, 471.
19. Philip Powell, Physics, 2006, 569, 172.
20. M Guerrero. *et al*., Phys. Rev. B, 1999, 59, 1706.
21. MH Anderson. *et al*., Science, 1995, 269, 198.