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Influence of Hall current and rotation on the radiative MHD flow past an impulsively started infinite vertical porous plate

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Abstract
The present investigation deals with the effects of hall current, and Soret number on the non linear, radiative, unsteady, hydromagnetic natural convection flow of an optically thick rotating fluid past an impulsively started vertical porous plate. Exact analytical solution of the non-dimensional governing equations is obtained in closed form by using Laplace transform technique. The numerical values of primary and secondary velocity profiles are displayed graphically for different values of pertinent flow parameters.

Keywords: hall current, rotation, radiation, hydromagnetic, soret effect

1. Introduction

The Hall current aspect has an important role in many engineering applications. Several scientific and engineering processes involve the flow and mechanism of fluid-particle mixtures. Particular examples of such processes are the blood flow in arteries, combustion, wastewater treatment, cement production, flows in rocket tubes, steel manufacturing industry, dust in gas cooling systems, movement of inert solid particles in the atmosphere etc. Seth et al. [1] studied the effects of hall current and rotation on unsteady MHD natural convection flow with heat and mass transfer past an impulsively moving vertical plate in the presence of radiation and chemical reaction. Venkateswarlu et al. [2] studied the Dufour and heat source effects on radiative MHD slip flow of a viscous fluid in a parallel porous plate channel in presence of chemical reaction. Reddy et al. [3] presented the influence of thermal radiation, viscous dissipation and hall current on MHD convection flow over a stretched vertical flat plate. Venkateswarlu et al. [4] studied the influence of thermal radiation and heat generation on steady hydromagnetic flow in a vertical micro – porous – channel in presence of suction/injection. Vieru et al. [5] reported the magnetohydrodynamic natural convection flow with Newtonian heating and mass diffusion over an infinite plate that applies shear stress to a viscous fluid. Venkateswarlu et al. [6] discussed the influence of Hall current and heat source on MHD flow of a rotating fluid in a parallel porous plate channel. Uddin et al. [7] presented the influence of thermal radiation and heat generation or absorption on MHD heat transfer flow of a micropolar fluid past a wedge considering hall and ion slip currents. Venkateswarlu and Phani Kumar [8] studied the Soret and heat source effects on MHD flow of a viscous fluid in a parallel porous plate channel in presence of slip condition. Takhar et al. [9] presented the unsteady free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and Hall currents. Malapati and Dasari [10] investigated Soret and chemical reaction effects on the radiative MHD flow from an infinite vertical plate. Seth et al [11] discussed the effects of Hall current and rotation on MHD natural convection flow past an impulsively moving vertical plate with ramped temperature in the presence of thermal diffusion with heat absorption. Venkateswarlu and Makinde [12] presented the unsteady MHD slip flow with radiative heat and mass transfer over an inclined plate embedded in a porous medium. Sattar [13] reported the unsteady hydromagnetic free convection flow with Hall current mass transfer and variable suction through a porous medium near an infinite vertical porous plate with constant heat flux. Venkateswarlu et al. [14] considered the thermodynamic analysis of Hall current and Soret number on hydromagnetic couette flow in a rotating system with a convective boundary condition. Seth et al. [15] considered the unsteady hydromagnetic natural convection flow of a heat absorbing fluid within a rotating vertical channel in porous medium with Hall effects. Venkateswarlu et al. [16] presented the influence of heat generation and viscous dissipation on hydromagnetic fully developed natural convection flow in a vertical micro–channel. In another study, Venkateswarlu et al. [17] discussed the Soret and Dufour effects on radiative hydromagnetic flow of a chemically reacting fluid over an exponentially accelerated inclined porous plate in presence of heat absorption and viscous dissipation. The following strategy is pursued in the rest of the paper. Section two presents the formation of the problem. The analytical solutions are presented in section three and finally results are discussed in section four.

2. Formation of the problem

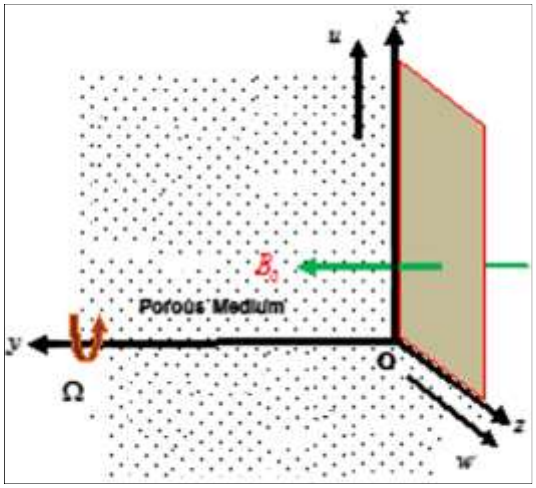

Fig 1: Geometry of the problem.

Consider the unsteady, non linear, radiative, hydromagnetic natural convection flow of an incompressible, viscous and electricity conducting fluid past a suddenly started infinite vertical porous plate in presence of a uniform transverse magnetic field of strength B_o . In view of the assumptions made by Ahmed [18], as well as of the usual Boussinesq's approximation, the dimensional governing equations can be written as

Continuity equation:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum conservation equations:

$$\frac{\partial u}{\partial t} + 2\Omega w = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left[\frac{u + mw}{1 + m^2}\right] + g\beta_T\left(T - T_\infty\right) + g\beta_C\left(C - C_\infty\right) - \nu\frac{u}{K_1} \tag{2}$$

$$\frac{\partial w}{\partial t} - 2\Omega u = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho}\left[\frac{mu - w}{1 + m^2}\right] - \nu\frac{w}{K_1} \tag{3}$$

Energy conservation equation:

$$\frac{\partial T}{\partial t} = \frac{k_T}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{4}$$

Mass diffusion equation:

$$\frac{\partial C}{\partial t} = D_M \frac{\partial^2 C}{\partial y^2} + \frac{D_M k_T}{T_M}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

We should in prior warn the reader that our model is not the same as that by Ahmed [18], in which the Hall current, rotation and permeability parameters were not taken into account. Interpretation radiation term is totally different in both papers. The initial and boundary conditions for the fluid flow problem are given below

$$\left.\begin{array}{l} t \le 0: u = 0, w = 0,\ T = T_\infty,\ C = C_\infty\ \forall\ \ y \ge 0 \\ t > 0: u = u_o, w = 0, T = T_w, C = C_w\ \text{ at }\ y = 0 \\ u \to 0, w \to 0, T \to T_\infty, C \to C_\infty\ \text{ as }\ y \to \infty \end{array}\right\} \quad (6)$$

In the case of an optically thick gray fluid, the local radiative heat flux term is simplified by using the Roseland approximation (Ahmed [18])

$$q_r = -\frac{4\sigma^*}{3a^*}\frac{\partial T^4}{\partial y} \quad (7)$$

It is assumed that the temperature difference within the fluid flow is sufficiently small such that the fluid temperature T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor's series about free stream temperature T_∞. By neglecting second and higher order terms, T^4 is expressed as,

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad (8)$$

Using the equations (7) and (8) in equation (4) we obtain

$$\frac{\partial T}{\partial t} = \frac{k_T}{\rho c_p}\left[1 + \frac{16\sigma^* T_\infty^3}{3a^* k_T}\right]\frac{\partial^2 T}{\partial y^2} \quad (9)$$

In order to write the governing equations and the boundary conditions in non dimensional form, the following non dimensional quantities are introduced.

$$\tau = \frac{u_o^2}{\nu}t, Y = \frac{u_o}{\nu}y, U = \frac{u}{u_o}, W = \frac{w}{u_o}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, u_o = \left[g\beta_T\left(T_w - T_\infty\right)\nu\right]^{1/3} \quad (10)$$

Equations (2), (3), (5) and (9) reduce to the following non dimensional form.

$$\frac{\partial U}{\partial \tau} + 2K^2 W = \frac{\partial^2 U}{\partial Y^2} - \frac{M\left(mW + U\right)}{1 + m^2} + \theta + N\phi - \frac{U}{K} \quad (11)$$

$$\frac{\partial W}{\partial \tau} - 2K^2 U = \frac{\partial^2 W}{\partial Y^2} + \frac{M(mU - W)}{1 + m^2} - \frac{W}{K} \quad (12)$$

$$\frac{\partial \theta}{\partial \tau} = \left[\frac{1 + R}{Pr}\right]\frac{\partial^2 T}{\partial Y^2} \quad (13)$$

$$\frac{\partial \phi}{\partial \tau} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial Y^2} + Sr\frac{\partial^2 \theta}{\partial Y^2} \quad (14)$$

Here $K^2 = \frac{\Omega\nu}{u_o^2}$ is the rotation parameter, $M = \frac{\sigma B_0^2\nu}{\rho u_0^2}$ is the magnetic parameter, $K = \frac{K_1 u_0^2}{\nu^2}$ is the permeability parameter, $N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}$ is the ratio of the buoyancy forces due to the temperature and concentration, $Pr = \frac{\nu\rho c_p}{k_T}$ is the Prandtl number, $R = \frac{16\sigma^* T_\infty^3}{3a^* K_T}$ is the radiation parameter, $Sr = \frac{D_M k_T (T_w - T_\infty)}{T_M \nu (C_w - C_\infty)}$ is the Soret number and $Sc = \frac{\nu}{D_M}$ is the Schmidt number respectively.

The initial and boundary conditions, in equation (6) reduced to the following non-dimensional form

$$\left.\begin{array}{l} \tau \le 0 : U = 0, W = 0, \ \theta = 0, \phi = 0 \ \forall \ Y \ge 0 \\ \tau > 0 : U = 1, W = 0, \theta = 1, \phi = 1 \text{ at } Y = 0 \\ U \to 0, W \to 0, \theta \to 0, \phi \to 0 \text{ as } Y \to \infty \end{array}\right\} \tag{15}$$

By combining the equations (11) and (12), we obtain

$$\frac{\partial F}{\partial \tau} - 2iK^2 F = \frac{\partial^2 F}{\partial Y^2} - \left[\frac{M}{1+im} + \frac{1}{K}\right] F + \theta + N\phi \tag{16}$$

Here $F(\eta, \tau) = U(\eta, \tau) + iW(\eta, \tau)$.

Initial and boundary conditions, in equation (15), in compact form, are given by

$$\left.\begin{array}{l} \tau \le 0 : F = 0, \ \theta = 0, \phi = 0 \ \forall \ Y \ge 0 \\ \tau > 0 : F = 1, \theta = 1, \phi = 1 \text{ at } Y = 0 \\ F \to 0, \theta \to 0, \phi \to 0 \text{ as } Y \to \infty \end{array}\right\} \tag{17}$$

3. Solution of the problem

Now we solve the equations (13), (14) and (16) subject to the initial and boundary conditions in equation (17) by using Laplace transform technique.

Define $\eta = \frac{Y}{2\sqrt{\tau}}$ (18)

Then the equations (13), (14) and (16) are transformed into the following form

$$\frac{\partial F}{\partial \tau} - \frac{\eta}{2\tau}\frac{\partial F}{\partial \eta} - \frac{1}{4\tau}\frac{\partial^2 F}{\partial \eta^2} + \left[\frac{M}{1+im} + \frac{1}{K} - 2iK^2\right] F = \theta + N\phi \tag{19}$$

$$\frac{\partial \theta}{\partial \tau} - \frac{\eta}{2\tau}\frac{\partial \theta}{\partial \eta} - \left[\frac{1+R}{4 Pr\,\tau}\right]\frac{\partial^2 \theta}{\partial \eta^2} = 0 \tag{20}$$

$$\frac{\partial \phi}{\partial \tau} - \frac{\eta}{2\tau}\frac{\partial \phi}{\partial \eta} - \frac{1}{4\tau Sc}\frac{\partial^2 \phi}{\partial \eta^2} = \frac{Sr}{4\tau}\frac{\partial^2 \theta}{\partial \eta^2} \tag{21}$$

The boundary conditions in equation (17) can be written as

$$\left.\begin{array}{l} F(0,\tau)=1, \quad \theta(0,\tau)=1, \quad \phi(0,\tau)=1 \\ F(\infty,\tau)=0, \; \theta(\infty,\tau)=0, \; \phi(\infty,\tau)=0 \end{array}\right\} \tag{22}$$

The exact solutions for the fluid velocity $F(\eta,\tau)$, fluid temperature $\theta(\eta,\tau)$ and species concentration $\phi(\eta,\tau)$ are obtained and are presented in the following form after simplification

$$\begin{aligned} F(\eta,\tau) = {} & a_{22}\psi_o(0,\, a_1,\, \eta,\, \tau) + Na_{17}\exp(-a_3\tau)\psi_o(-a_3,\, a_1,\, \eta,\, \tau) + \\ & Na_{16}\exp(-a_3\tau)\left[\psi_o(-a_3,\, Sc,\, \eta,\, \tau) - \psi_o(a_7 - a_3,\, 1,\, \eta,\, \tau)\right] + \\ & a_{20}\exp(-a_{10}\tau)\left[\psi_o(a_7 - a_{10},\, 1,\, \eta,\, \tau) - \psi_o(-a_{10},\, a_1,\, \eta,\, \tau)\right] + \\ & Na_{11}\psi_o(0,\, Sc,\, \eta,\, \tau) + a_{18}\psi_o(a_7,\, 1,\, \eta,\, \tau) \\ & a_{21}\exp(a_{12}\tau)\left[\psi_o(a_7 + a_{12},\, 1,\, \eta,\, \tau) - \psi_o(a_{12},\, Sc,\, \eta,\, \tau)\right] + \end{aligned} \tag{23}$$

$$\theta(\eta,\tau) = \psi_o(0,\, a_1,\, \eta,\, \tau) \tag{24}$$

$$\begin{aligned} \phi(\eta,\tau) = {} & a_6\exp(-a_3\tau)\left[\psi_o(-a_3,\, Sc,\, \eta,\, \tau) - \psi_o(-a_3,\, a_1,\, \eta,\, \tau)\right] + \\ & a_5\psi_o(0,\, Sc,\, \eta,\, \tau) - a_4\psi_o(0,\, a_1,\, \eta,\, \tau) \end{aligned} \tag{25}$$

Here the constants are not given under brevity.

4. Results and discussion

The numerical results are presented in Figs. 2 to 19. In the present study, the following parameter values are utilized for numerical computations: $K^2 = 2$, $M = 1$, $m = 0.5$, $K = 0.2$, $N = 1$, $\tau = 1$, $Pr = 0.71$, $R = 1$, $Sr = 1$ and $Sc = 0.22$. Figs. 2 to 19 depict the graphs of fluid primary velocity U, secondary velocity W, temperature θ and concentration ϕ under the influence of Soret number Sr, Buoyancy parameter N, Prandtl number Pr, Schmidt number Sc, magnetic parameter M, radiation parameter R, rotation parameter K^2, Hall current parameter m and time τ.

Figs. 2 to 5 depict the influence of rotation parameter and hall current parameter on the primary velocity and secondary velocity. It is observed that, the primary velocity decreases and secondary velocity increases on increasing the rotation parameter and hall current parameter. This implies that, rotation and hall current tends to accelerate the secondary velocity which is consistent with the fact that rotation and hall current induces secondary flow in the flow field.

Figs. 6 and 7 depict the influence of buoyancy parameter on the primary velocity and secondary velocity of the flow field. The buoyancy parameter defines the ratio of Solutal Grashof number to thermal Grashof number. It is noticed that, the primary and secondary velocities are increases on increasing the buoyancy parameter.

It is noticed that from Figs. 8 and 9, the fluid primary velocity and secondary velocity increases on increasing the radiation parameter. Figs. 10 and 11, depict the influence of Soret number on the fluid primary velocity and secondary velocity. The Soret number defines the influence of the temperature gradient inducing significant mass diffusion effects. The primary velocity and secondary velocity are found to increases on increasing the Soret number.

Figs. 12 and 13 depict the influence of magnetic parameter on the primary velocity and secondary velocity of the flow field. The transversely imposed magnetic field on the conducting dusty fluid produced a Lorentz force which acts as a resistance to the flow, consequently, the velocities decreases with an increase in magnetic parameter.

Figs. 14 and 15, shows the plot of primary velocity and secondary velocity of the flow field against different values of Prandtl number taking other parameters are constant. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is observed that, the primary velocity and secondary velocity decreases with an increase in the Prandtl number.

The influence of fluid primary velocity and secondary velocity in presence of foreign species is shown in Figs. 16 and 17. Schmidt number signifies the relative strength of viscosity to chemical molecular diffusivity. It is observed that the primary velocity and secondary velocity decreases on increasing the Schmidt number. The flow field decelerate the velocity in presence of heavier diffusing species. It is noticed that from Figs. 18 and 19, the fluid primary velocity decreases and secondary velocity increases with the progress of time.

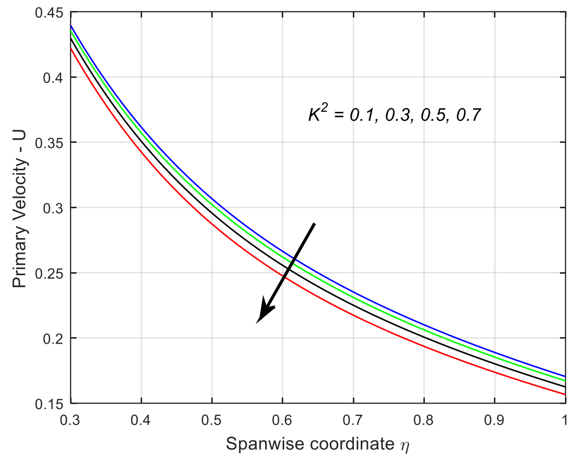

Fig 2: Influence of rotation parameter on the primary velocity.

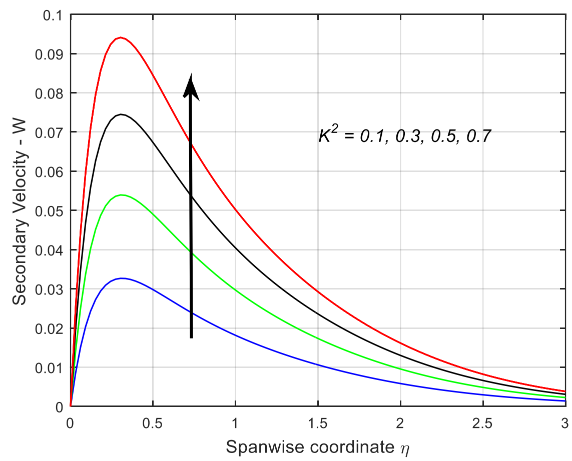

Fig. 3: Influence of rotation parameter on the secondary velocity.

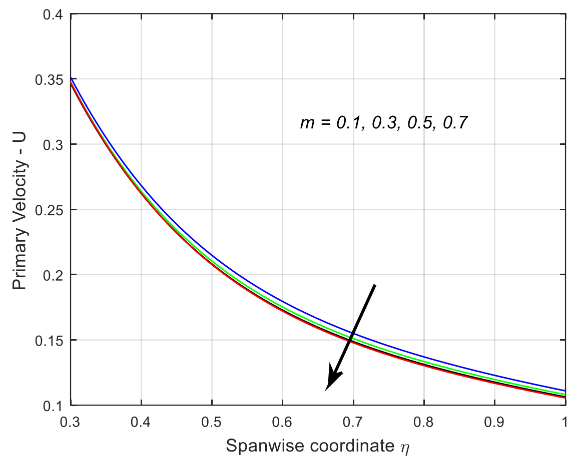

Fig 4: Influence of Hall current parameter on the primary velocity.

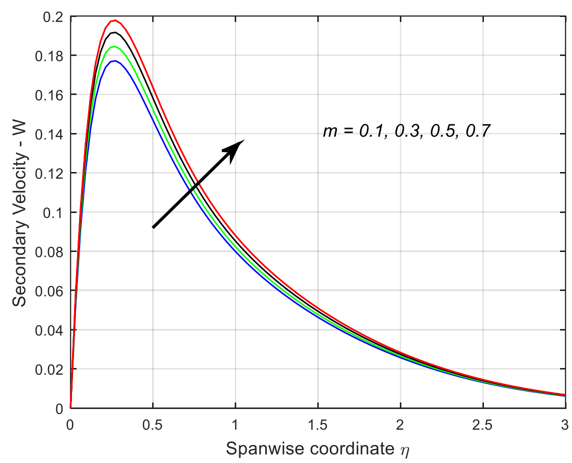

Fig 5: Influence of Hall current parameter on the secondary velocity.

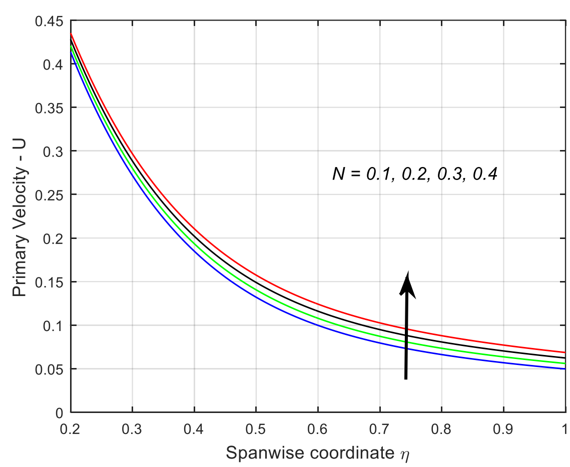

Fig 6: Influence of Buoyancy parameter on the primary velocity.

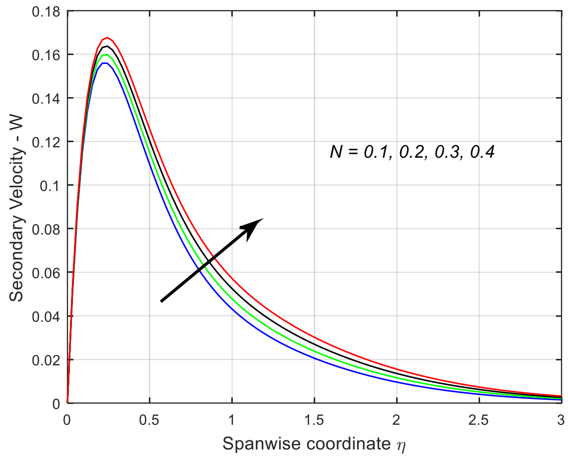

Fig 7: Influence of Buoyancy parameter on the secondary velocity.

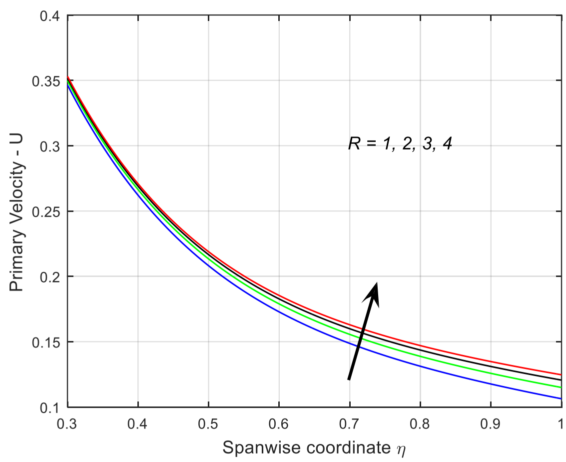

Fig 8: Influence of radiation parameter on the primary velocity.

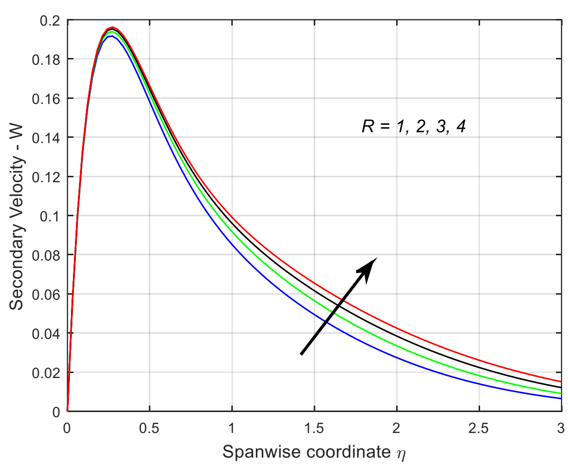

Fig 9: Influence of radiation parameter on the secondary velocity.

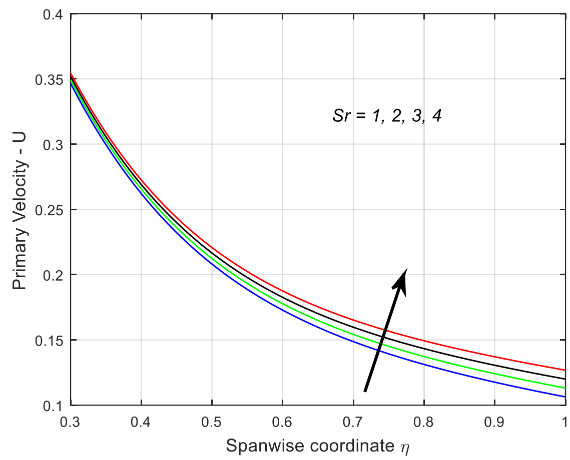

Fig 10: Influence of Soret number on the primary velocity.

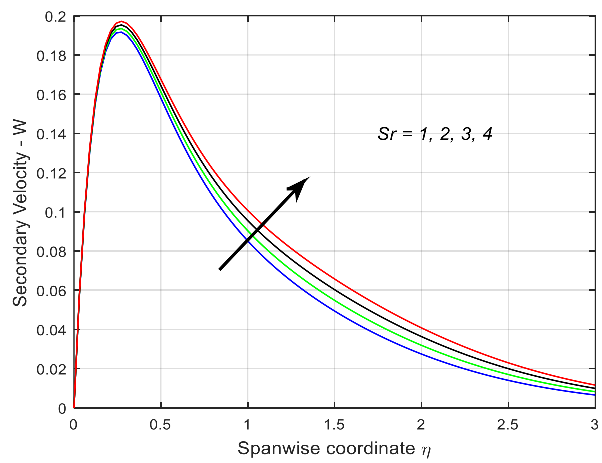

Fig 11: Influence of Soret number on the secondary velocity.

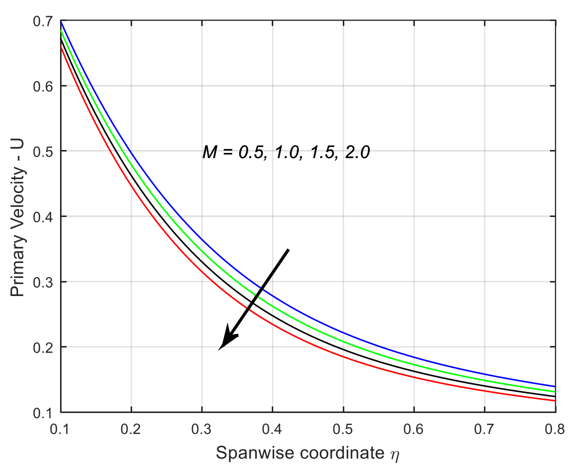

Fig 12: Influence of Magnetic parameter on the primary velocity.

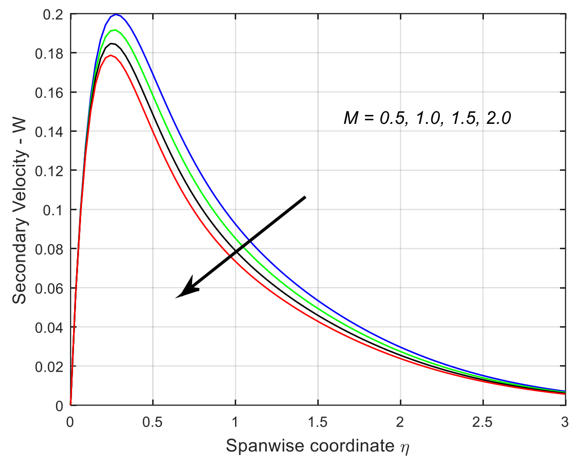

Fig 13: Influence of Magnetic parameter on the secondary velocity.

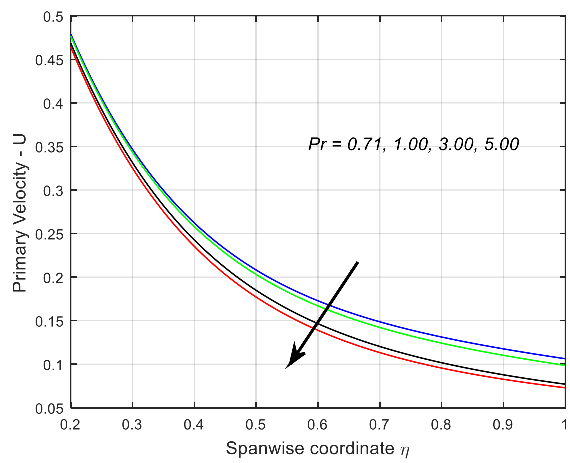

Fig 14: Influence of Prandtl number on the primary velocity.

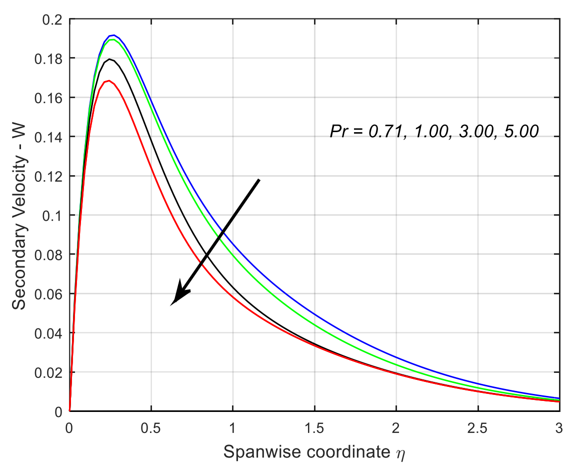

Fig 15: Influence of Prandtl number on the secondary velocity.

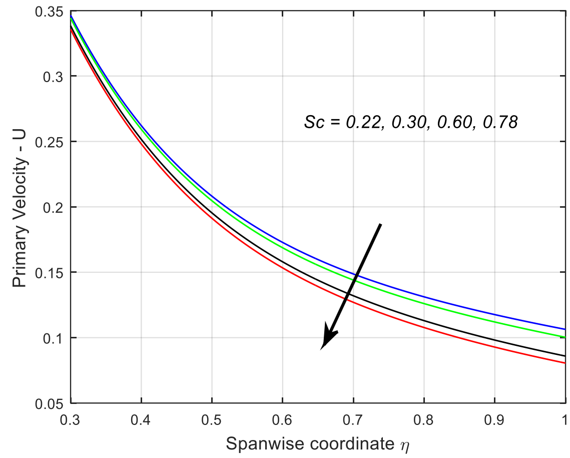

Fig 16: Influence of Schmidt number on the primary velocity.

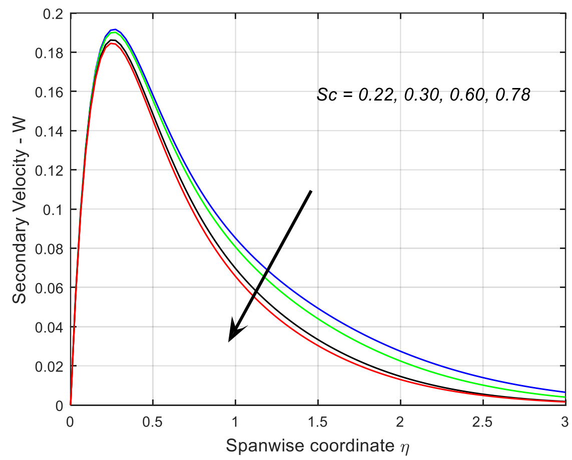

Fig 17: Influence of Schmidt number on the secondary velocity.

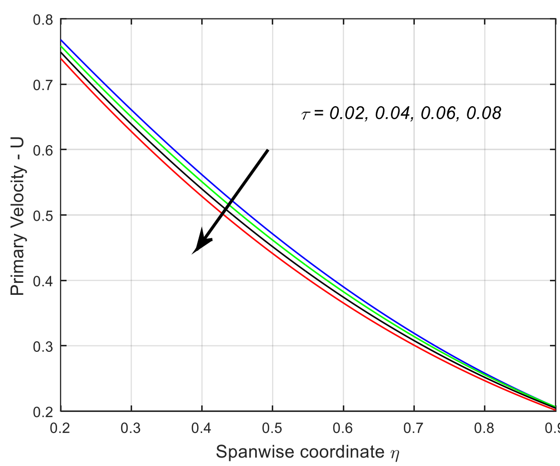

Fig 18: Influence of time on the primary velocity.

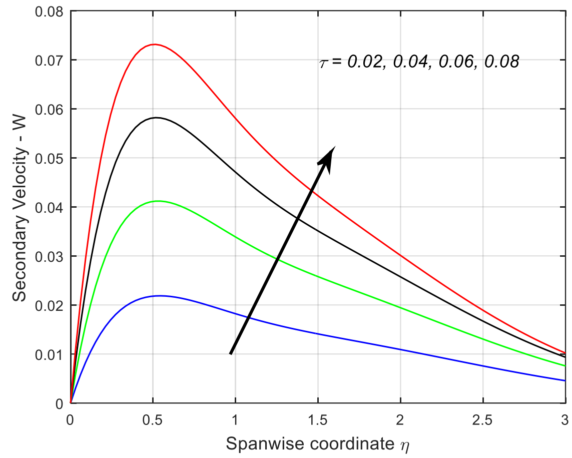

Fig 19: Influence of time on the secondary velocity.

5. Acknowledgements

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Nomenclature

u	fluid velocity in $x-$ direction
w	fluid velocity in $z-$ direction
g	acceleration due to gravity
t	dimensional time
K_1	permeability of porous medium
B_o	uniform magnetic field
T	fluid temperature
m	hall current parameter
k_T	thermal conductivity
C	species concentration
c_p	specific heat at constant pressure
D_M	chemical molecular diffusive
D_T	thermal diffusivity
q_r	radiating flux vector
T_w	temperature of the wall
C_w	concentration of the wall
T_∞	fluid temperature in the free stream
C_∞	species concentration in the free stream
a^*	mean absorption coefficient
N	buoyancy parameter
Pr	Prandtl number
T_M	mean temperature
R	radiation parameter
K^2	rotation parameter
M	magnetic parameter
Sc	Schmidt number
K	permeability parameter
Sr	Soret number
u_o	characteristic velocity
U	primary velocity
W	secondary velocity

Greek Symbols

ρ	fluid density
β_T	coefficient of thermal expansion
β_C	coefficient of concentration expansion
Ω	uniform angular velocity
σ	electrical conductivity
ν	kinematic coefficient of viscosity
ω_e	cyclotron frequency
τ_e	electron collision time
σ^*	Stefan- Boltzmann constant
θ	non dimensional temperature
ϕ	non dimensional concentration

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