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Some value distribution results of meromorphic and entire functions

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Abstract
The core objective of this article is to investigate zero distribution for q-shift linear differential-difference polynomials, monomials of entire (meromorphic) functions. The finding of entire (meromorphic) functions is either of zero or finite order which may be viewed as the analogues of Hayman Conjecture. These results generalise and improve the results due to Luo L and Xu J and Dhar R.S.

Keywords: Entire function, meromorphic function, monomial, difference polynomial, q-shift, zero order

Introduction

The present paper extracts the definition of entire(meromorphic) function $f(z)$ as given in complex plane. We think all the readers are familiar with the fundamentals of Nevanlinna theory (see [7, 14, 13]) inclusive of $T(r,f), m(r,f), N(r,f)$ etc. As per the need of results, we use various terminologies such as $S(r,f)$ which is equivalent to $O\{T(r,f)\}$ as well as a small function $\alpha(z) \in f$ if *T(r,α(z))=O(T(r,f))* as $r \to \infty$ possibly outside *E*.

Definition 1: [4] A differential-difference monomial is defined for a meromorphic function $f(z)$:

$$M[f] = \prod_{i=0}^{k} \prod_{j=0}^{m} [f^{(j)}(z + c_{ij})]^{n_{ij}}$$

where $c_{ij} \in C$ and $n_{ij} \in N$, with i varying from 0 to k and j from 0 to m. Hence, $M[f]$'s total degree is nothing but total of product of all right-hand side powers. Therefore, total degree of $M[f]$ is defined:

$$d = Max_{j \in \Delta} d_{M_j}$$

For our convenience, we define,
$F_1^* = \{f(z) : f$ is a transcendental entire function of finite or zero order$\}$.

$F_2^* = \{f(z) : f$ is a transcendental meromorphic function of finite or zero order$\}$.

Many mathematicians have contributed to distribution of zeros for difference-differential polynomials as well as its derivatives. The famous conjecture of Hayman [6] is discussed below.

Conjecture: A meromorphic function $f(z)$ can take number of zeros infinitely for a positive integer n, if $f^n(z) f'(z) \neq 1$.

Hayman [7] himself solved the conjecture when $n \geq 3$, then by Mues [9], Bergweiler and Eremenko [1] and Chen-Fang [3] respectively for $n = 2, 1$. These mathematicians paved a way to the other works or publications in this domain.

In 2012, Chen and Chen [2] investigated about the zeros of $f^n\ (f^l - 1)\ \prod_{j=1}^{t}\ [f\ (z\ +\ c_j)]^{\mu_j} = \alpha$, with $\alpha(z) \in f(z)$ and hence the below result.

Theorem 1.1. [2] Consider $f \in F_2^*,\ c_j(j = 1, 2, \ldots, t) \in C - \{0\}$ and $n, l, t, \mu_j(j = 1, 2, \ldots, t) \in N$. Subsequently $f^n\ (f^l - 1) \prod_{j=1}^{t}\ [f\ (z\ +\ c_j)]^{\mu_j} - \alpha$ has number of zeros infinitely for $n\ \geq\ 2$.

Recently, in the year 2020 J. Xu and L.Luo [8] investigated zero distribution of entire functions with q-shift for the generalized above result.

Theorem 1.2: [8] Consider $f \in F_1^*,\ q_j(j = 1, 2, \ldots, t)$ be finite non-zero constant and $\sigma = \sum_{j=1}^{t} \mu_j$, where $\mu_j \in N$. If $P(w) = a_l w^l + a_{l-1} w^{l-1} + \cdots + a_0 (\neq 0)$ be a polynomial, with $a_l, \ldots, a_0$ as constants, then $f^n\ P(f) \prod_{j=1}^{t}\ (f(q_j z))^{\mu_j} - \alpha(z)$ takes number of zeros infinitely when $n\ \geq\ 2$.

In the following two theorems, RS Dhar (2020) [4] replaced shift and difference operator by linear difference polynomials

Theorem 1.3: [4] Consider $f \in F_1^*,\ P[f]\ =\ c_0 f(z) + c_1 f(z + c) + \cdots + c_n f(z + nc)$ be a polynomial of linear difference such that $T(r, P[f])$ and $S(r, f)$ are not equal, $c_j, j = 0, 1, \ldots, n$ and $c\ \neq 0 \in C$, then $f^l P[f] - \alpha(z), (\alpha(z) \neq\ 0, \infty)\ for\ l > 2n + 1$, has several zeros infinitely.

Theorem 1.4: [4] Consider $f \in F_1^*$ and $P[f]\ =\ c_0 f(z) + c_1 f(z + c) + \cdots + c_n f(z + nc)$ be a polynomial of linear difference such that $T(r, P[f])$ and $S(r, f)$ are not equal, $c_j,\ j = 0, 1, \ldots, n$ and $c \neq 0 \in C$. Subsequently $f^l P[f] - \alpha(z)$ for $l \geq 4n + 3$ has infinitely several zeros.

J. Xu and L. Luo [11], in 2022, examined the distribution of zeros for linear q-shift difference polynomials to obtain the following result.

Theorem 1.5: [11] Consider $f \in F_2^*,\ b(z)(\neq \infty) \in f(z)$ be a small function. Then $f^n(z) + P(f) = b(z)$ takes many zeros infinitely, when $n \geq l + 3$.

Based on the above literature, we arrived at the following questions and also, we have tried to answer for the same in the third section

- **Question 1.1:** Can we verify theorems 1.1-1.2 for differential-difference monomials to get extended results?
- **Question 1.2:** Can we further generalise theorems 1.3-1.4 for q-shift linear difference polynomials?
- **Question 1.3:** Can we extend theorem 1.5 for higher order differential polynomials along with the monomial functions?

Theorem 1.6: Let $f(z) \in F_1^*$ and q_t with $t = 1, 2, \ldots, l$ be a finite non-zero constants. Let $\lambda = \sum_{t=1}^{l} \mu_t$, where $\mu_t \in N$ and as defined in definition 1, $M[f]$ be a differential-difference monomial. Then, $f^n(z)\ M[f] \prod_{t=1}^{l}\ (f(q_t z))^{\mu_t} - \alpha(z)$ has several zeros infinitely for $n \geq 2$.

Theorem 1.7: Let $f(z) \in F_1^*$ and a linear q-shift difference polynomial $P(f) = \sum_{j=1}^{t} a_j(z) f(q_j z + c_j)$ where $c_j\ (j = 1, 2, \ldots, t),\ q_j \neq\ 0 \in C$ and $P(f) \neq 0$. If $\alpha(z)$ and $a_j(z)(j = 1, 2, \ldots, t) \in f(z)$. Then $f^n\ (z) + P(f) - \alpha(z)$ takes number of zeros infinitely for $n \geq 1$.

Theorem 1.8: Let $f(z) \in F_2^*$ and a linear q-shift difference polynomial $P(f) = \sum_{j=1}^{t} a_j(z) f(q_j z + c_j)$ where $q_j \neq 0$ and $c_j\ (j = 1, 2, \ldots, t) \in C$. $P(f) \neq 0$ and $\alpha(z),\ a_j(z)\ (j = 1, 2, \ldots, t) \in f(z)$. Then $f^n(z) P(f) - \alpha(z)$ takes number of zeros infinitely when $n \geq 1$.

Theorem 1.9: Let $f(z) \in F_2^*,\ \alpha(z)(\neq \infty) \in f(z)$. Let $P(z) = c(z - \alpha_1)^{n_1}\ (z - \alpha_2)^{n_2}\ \cdots (z - \alpha_h)^{n_h},\ c \in C$ be a polynomial of degree p and $M[f] = \prod_{j=1}^{t} [f(q_j z)]^{\mu_j}$ be a monomial. Then $P(f^{(k)}) + M[f]\ -\ \alpha(z)$ has infinitely many zeros for $p(k+1)-\sigma-2\geq 0$.

Example 1.1 Let $f(z)=e^{iz}+1,\ c=\pi,\ q=1$ and $P(f)=f(z+c)$ then $f(z)+P(f)-2$ hold for $n=1$.

Example 1.2 Let $f(z)=\frac{e^{z}+1}{e^{z}-1},\ c=i\pi, q=1$ and $P(f)=f(z+c)$ then $f(z)P(f)-1$ holds for $n=1$.

2 Preliminaries

Lemma 2.1: [5] Consider $f(z)\in F_2^*$ and $q,c\in C-\{0\}$. Then, $m\left(r,\ \frac{f(qz+c)}{f(z)}\right)=S(r,f)$.

Whenever $r\ \rightarrow\ \infty$ is possibly outside E

Lemma 2.2: [12] Consider $f(z)\in F_1^*,\ c\in C-\{0\}$. Then

$T(r,\ f(z+c))=T(r,f)+S(r,f)$.

$N(r,f(z+c))=N(r,f)+S(r,f)$.

$N(r,0,f(z+c))=N(r,0,f)+S(r,f)$.

Lemma 2.3: [8] Suppose $f(z)\in F_1^*$ and $P(f)$ be a polynomial of degree p. Then $T(r,P(f))=pT(r,f)+S(r,f)$.

Lemma 2.4: [10] If $f(z)\in F_2^*$ and $n(\geq 1),\ s,\ \mu_j\ (j=1,2,\ldots,s)\in Q^+$ and $c_j\ (j=1,2,\ldots,s)$ be finite distinct complex numbers and $f_1=P(f(z))\prod_{j=1}^{s}f(z+c_j)^{\mu_j}$. So $T(r,f_1)=(n+\sigma)T(r,f)+S(r,f)$.

3 Proof of Theorems

Proof of Theorem 1.6

Let $H_1(z)=f^n\ (z)M[f]\prod_{t=1}^{l}[f(q_tz)]^{\mu_t}$, using Lemma 2.4 $T(r,f\ n+\lambda M[f])\ =\ (n\ +\ \lambda\ +\ d)T(r,f)\ +\ S(r,f)$.

We have $N(r,f)=0$, therefore

$T(r,f\ n+\lambda M[f])\ =\ m(r,f\ n+\lambda M[f])\ +\ S(r,f)$.

From Lemma 2.1

$m\left(r,\frac{f(z)}{f(q_tz)}\right)=S(r,f)$.

Hence

$m\left(r,f^{n+\lambda}M[f]\right)\ \leq m(r,H_1)+m\ \left(r,\ \frac{f^{n+\lambda}M[f]}{H_1}\right)+S(r,f)$

$\leq m(r,H_1)+\sum_{t=1}^{l}m\left(r,\ \frac{f(z)^{\mu_t}}{f(q_tz)^{\mu_t}}\right)+S(r,f)$

$\leq m(r,H_1)+\sum_{t=1}^{l}\mu_t m\left(r,\frac{f(z)}{f(q_tz)}\right)+S(r,f)$

$\leq\ T(r,H_1)+S(r,f)$.

One can get the following as a result of the above inequalities

$$(n+\lambda+d)T(r,f)\leq T(r,H_1)+S(r,f) \qquad (3.1)$$

Consider

$$T(r,H_1) = T\left(r,\ f^n(z)M[f]\prod_{t=1}^{l}(f(q_t z))^{\mu_t}\right)$$

$$\leq\ nT(r,f) + dT(r,f) + \lambda T(r,f) + O(1)T(r,f).$$

Therefore

$$T(r,H_1) \leq \left(n + \lambda + d + O(1)\right) T(r,f). \tag{3.2}$$

One can get from (3.1) and (3.2)

$$T(r,H_1) = \left(n + \lambda + d + O(1)\right) T(r,f).$$

We can deduce by the Nevanlinna's second fundamental theorem that

$$T(r,H_1) \leq \bar{N}\left(r,\ \frac{1}{H_1}\right) + \bar{N}\left(r,\ \frac{1}{H_1-\alpha}\right) + S(r,f)$$

$$\leq\ N\left(r,\ \frac{1}{f}\right) + N\left(r,\ \frac{1}{M[f\}}\right) + \sum_{t=1}^{l} N\left(r,\ \frac{1}{f(q_t z)}\right) + \bar{N}\left(r,\ \frac{1}{H_1-\alpha}\right) +\ S(r,f) \tag{3.3}$$

By using Lammas 2.2, 2.3 and Nevanlinna's first fundamental theorem

$$N\left(r,\ \frac{1}{f}\right) \leq T\left(r,\ \frac{1}{f}\right) = T(r,f) + O(1). \tag{3.4}$$

$$N\left(r,\ \frac{1}{M[f\}}\right) \leq\ T\left(r,\ \frac{1}{M[f\}}\right) = T(r,M[f]) + O(1) \leq dT(r,f) + O(1). \tag{3.5}$$

$$\sum_{t=1}^{l} N\left(r,\ \frac{1}{f(q_t z)}\right) \leq \sum_{t=1}^{l} T\left(r,\ \frac{1}{f(q_t z)}\right) = \sum_{t=1}^{l} T(r,\ f(q_t z)) \leq lT(r,f) + O(1) \tag{3.6}$$

From (3.3)-(3.6)

$$T(r,H_1) \leq (1 + d + l)T(r,f) +\ \bar{N}\left(r,\ \frac{1}{H_1-\alpha}\right) +\ S(r,f). \tag{3.7}$$

One can get by using (3.1) and (3.7) that

$$\bar{N}\left(r,\frac{1}{H_1-\alpha}\right) \geq (n + \lambda - l - 1)T(r,f) + S(r,f).$$

As $n \geq 2,\ \lambda \geq t$, we obtain $n + \lambda - l - 1 \geq 1$. Hence $H_1 - \alpha$ has several zeros infinitely.

Proof of Theorem 1.7

Let $H_2(z) = f^n + P(f)$.

$$T\left(r,H_2(z)\right) = T(r,f^n + P(f))$$

$$= T\ (r,f^n + a_1(z)f(q_1 z + c_1) + a_2(z)f(q_2 z + c_2) + \cdots + a_t(z)f(q_t z + c_t))$$

$$\leq T(r,f^n + f) + T\left(r,\ \left[a_1(z) + \frac{a_2(z)f(q_2 z+c_2)}{f(q_1 z+c_1)} + \cdots + \frac{a_t(z)f(q_t z+c_t)}{f(q_1 z+c_1)}\right]\right) +\ S(r,f).$$

But

$$T\left(r,\left[a_1(z) + \frac{a_2(z)f(q_2 z + c_2)}{f(q_1 z + c_1)} + \cdots + \frac{a_t(z)f(q_t z + c_t)}{f(q_1 z + c_1)}\right]\right)$$

$$\leq T\left(r,\frac{f(q_2z+c_2)}{f(q_1z+c_1)}\right)+T\left(r,\frac{f(q_3z+c_3)}{f(q_1z+c_1)}\right)+\cdots+T\left(r,\frac{f(q_tz+c_t)}{f(q_1z+c_1)}\right)+S(r,f)$$

$$=N\left(r,\frac{1}{f(q_1z+c_1)}\right)+N\left(r,\frac{1}{f(q_1z+c_1)}\right)+\cdots+N\left(r,\frac{1}{f(q_1z+c_1)}\right)+S(r,f).$$

Hence

$$T\left(r,\ a_1(z)+\frac{a_2(z)f(q_2z+c_2)}{f(q_1z+c_1)}+\cdots+\frac{a_t(z)f(q_tz+c_t)}{f(q_1z+c_1)}\right)\leq tT(r,f)+S(r,f).$$

Consequently

$$T(r,H_2(z))\leq(n+1)T(r,f)+tT(r,f)+S(r,f).$$

$$\therefore\ T(r,H_2(z))\leq(n+1+t)T(r,f)+S(r,f) \tag{3.8}$$

Consider

$$(n+1+t)T(r,f)=T(r,f^{n+1+t})$$

$$\leq T(r,f^n)+(1+t)T(r,f)+S(r,f)$$

$$\leq T(r,f^n+P(f))+S(r,f)$$

$$\leq T(r,H_2(z))+S(r,f). \tag{3.9}$$

By (3.8) – (3.9)

$$T(r,H_2(z))=(n+1+t)T(r,f).$$

As f is an entire function, using Nevanlinna's second fundamental theorem

$$(n+1+t)T(r,f)=T(r,H_2(z))$$

$$\leq\bar{N}\left(r,\frac{1}{H_2(z)}\right)+\bar{N}\left(r,\frac{1}{H_2(z)-\alpha(z)}\right)+S(r,f)$$

$$\leq N\left(r,\frac{1}{H_2(z)}\right)+N\left(r,\frac{1}{H_2(z)-\alpha(z)}\right)+S(r,f).$$

Therefore, one can get

$$(n+1+t)T(r,f)\leq N\left(r,\frac{1}{H_2-\alpha}\right)+S(r,f).$$

Hence $H_2(z)$ has number of zeros infinitely.

Proof of Theorem 1.8

Let $H_3(z)=f^nP(f)$.

$$T(r,H_3(z))=T(r,f^nP(f))$$

$$=T(r,f^n[a_1(z)f(q_1z+c_1)+a_2(z)f(q_2z+c_2)+\cdots+a_t(z)f(q_tz+c_t)])$$

$$\leq T(r,f^{n+1})+T\left(r,\ a_1(z)+\frac{a_2(z)f(q_2z+c_2)}{f(q_1z+c_1)}+\cdots+\frac{a_t(z)f(q_tz+c_t)}{f(q_1z+c_1)}\right)+S(r,f).$$

But

$$T\left(r,\ a_1(z)+\frac{a_2(z)f(q_2z+c_2)}{f(q_1z+c_1)}+\cdots+\frac{a_t(z)f(q_tz+c_t)}{f(q_1z+c_1)}\right)$$

$$\leq T\left(r,\frac{f(q_2z+c_2)}{f(q_1z+c_1)}\right)+T\left(r,\frac{f(q_3z+c_3)}{f(q_1z+c_1)}\right)+\cdots+T\left(r,\frac{f(q_tz+c_t)}{f(q_1z+c_1)}\right)+S(r,f)$$

$$=N\left(r,\frac{f(q_2z+c_2)}{f(q_1z+c_1)}\right)+N\left(r,\frac{f(q_3z+c_3)}{f(q_1z+c_1)}\right)+\cdots+N\left(r,\frac{f(q_tz+c_t)}{f(q_1z+c_1)}\right)+S(r,f).$$

Hence

$$T\left(r,\ a_1(z)+\frac{a_2(z)f(q_2z+c_2)}{f(q_1z+c_1)}+\cdots+\frac{a_t(z)f(q_tz+c_t)}{f(q_1z+c_1)}\right)\leq 2tT(r,f)+S(r,f).$$

As a result, we get

$$T(r,H_3(z))\leq(n+1)T(r,f)+2tT(r,f)+S(r,f).$$

$$\therefore\ T(r,H_3(z))\leq(n+l+2t)T(r,f)+S(r,f). \tag{3.10}$$

Consider

$$(n+1+2t)T(r,f)=T(r,f^{n+1+2t})$$

$$\leq T(r,f^n)+(1+2t)T(r,f)+S(r,f)$$

$$\leq T(r,f^nP(f))+S(r,f)$$

$$\leq T(r,H_3(z))+S(r,f). \tag{3.11}$$

From $(3.10)-(3.11)$

$$T(r,H_3(z))=(n+1+2t)T(r,f).$$

Since, f **being meromorphic, we get by using Nevanlinna's Second Fundamental theorem,**

$$(n+1+2t)T(r,f)=T(r,H_3(z))$$

$$\leq N(r,H_3)+N\left(r,\frac{1}{H_3}\right)+N\left(r,\frac{1}{H_3-\alpha}\right)+\ S(r,f).$$

$$\therefore\ (n+1+2t)T(r,f)\ \leq\ N\left(r,\frac{1}{H_3-\alpha}\right)\ +\ S(r,f).$$

Hence $H_3(z)-\alpha(z)$ **takes several zeros infinitely.**

Proof of Theorem 1.9

Let's consider $\sigma=Max_{j\epsilon\delta}d_{M_j}$ as total degree of $M[f]$ in f.

Suppose $M[f]=\alpha$ then $P\left(f^{(k)}\right)+M[f]-\alpha=P(f^{(k)})$.

Since

$$\frac{P(f^{(k)})}{f}=\frac{\alpha}{f}.$$

By Lemma 2.1

$$m\left(r,\frac{1}{f}\right) \leq m\left(r,\frac{P(f^{(k)})}{f}\right) + O(1) = S(r,f).$$

Using Nevanlinna's first fundamental theorem

$$T(r,f) = N\left(r,\frac{1}{f}\right) + S(r,f).$$

With respect to this and zero order transcendental function f, we get f which has number of zeros infinitely.

$\therefore\ P(f^{(k)}) + M[f] - \alpha(z)$ has infinitely several zeros.

Suppose $M[f] \neq \alpha$

Let

$$\psi = \frac{\alpha - M[f]}{P(f^{(k)})}. \tag{3.12}$$

Therefore, we have $\psi \neq 0$. as $M[f] \neq \alpha$.

Now it only remains to prove $\psi - 1$ has many zeros infinitely.

$$p(k+1)m(r,f) = m\left(r,P(f^{(k)})\right)$$

$$\leq m\left(r,\frac{1}{\psi}\right) + m(r,\alpha - M[f]). \tag{3.13}$$

$$p(k+1)N(r,f) = N\left(r,P(f^{(k)})\right)$$

$$\leq N\left(r,\frac{1}{\psi}\right) + N(r,\alpha - M[f]) - N_0(r) - N_1(r) \tag{3.14}$$

Where $N_1(r)$ and $N_0(r)$ represent common poles and zeros of ψ and $\alpha - M[f]$ respectively.

From (3.13) and (3.14)

$$p(k+1)T(r,f) \leq T\left(r,\frac{1}{\psi}\right) + T(r,\alpha - M[f]) - N_0(r) - N_1(r)$$

$$\leq T(r,\psi) + \sigma T(r,f) - N_0(r) - N_1(r) + S(r,f). \tag{3.15}$$

The poles of ψ are common poles of ψ and $\alpha - M[f]$ and zeros of f (From (3.12).

Therefore

$$\overline{N}(r,\psi) \leq \overline{N}\left(r,\frac{1}{f}\right) + N_1(r). \tag{3.16}$$

Again from (3.12), zeros of ψ are common zeros of ψ and $\alpha - M[f]$ and the poles of f.

Therefore

$$\overline{N}\left(r,\frac{1}{\psi}\right) \leq \overline{N}(r,f) + N_0(r). \tag{3.17}$$

Using second fundamental theorem and (3.16) – (3.17)

$$T(r,\psi) \leq \bar{N}\left(r,\frac{1}{\psi}\right) + \bar{N}(r,\psi) + \bar{N}\left(r,\frac{1}{\psi-1}\right) + S(r,f).$$

$$\therefore\ T(r,\psi) \leq \bar{N}(r,f) + \bar{N}\left(r,\frac{1}{f}\right) + N_0(r) + N_1(r) + \bar{N}\left(r,\frac{1}{\psi-1}\right) + S(r,f). \qquad (3.18)$$

From (3.15) and (3.18) we get,

$$(p(k+1) - \sigma - 2)T(r,f) \leq \bar{N}\left(r,\frac{1}{\psi-1}\right) + S(r,f).$$

Hence $\psi - 1$ takes infinitely several zeros.

Conflict of interests

The authors declare that they have no conflict of interest.

Availability of data and material

No data were used to study this.

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