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Some common soft fixed point theorems for soft compatible mappings

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Abstract
In this paper, a new class of generalized soft contractive condition is defined and some common fixed soft point results for four soft mappings in soft S-metric space are proved. Some consequences and illustrative example are provided to substantiate the main result.

Keywords: Soft S-metric Space, common fixed soft point, soft compatible mapping

1.1 Introduction

1.2 Preliminaries

In the theory of fixed point, Banach Contraction Principal, proved by Stefan Banach [1] in 1922, plays an important tool for proving the existence and unique solutions of the self-maps on metric space. Recently there has been numerous generalizations of Banach Contraction Principal by weakening its hypothesis while retaining the convergence property of successive iterates to the unique fixed point of the mapping. Some of these generalizations were made by Boyd and Wong [2], M. Edelstein [7], B. E. Rhoades [12] and many more. Theses work was also carried out by various researchers in different metric spaces. S. Ghaler [8] introduced 2-metric space. G-metric spaces was given by Mustafa and Sims [11] while S-metric space was introduced by Sedghi *et al.* [13].

In the meantime, Molodtsov [10] originated the idea of soft set in 1999. Soft set theory is a new mathematical tool for dealing with uncertainties and is a set associated with parameters and has been applies in several directions. Many research works on soft topological spaces, soft metric spaces, soft real sets, soft normed spaces, soft fixed point theory etc. can be found in ([5, 6, 9] and so on.). Recently, in 2018, Aras *et al.* [3, 4] introduced soft S-metric spaces and also discussed its important properties. They also established some results on soft mapping with a contractive condition and proved that in some conditions, these mappings possess unique fixed point. In the present paper we are going to prove common soft fixed point for four soft mapping in soft S-metric space. We begin with some basic definitions and results that will be needed in our main result.

Definition 1.1 [10]**:** "A pair (F, E) is called a soft set over a given universal set X, if and only if F is a mapping from a set of parameters E (each parameter could be a word or a sentence) into the power set of X denoted by $P(X)$. That is, $F: E \rightarrow P(X)$. Clearly, a soft set over X is a parameterized family of subsets of the given universe X".

Definition 1.2 [9]**:** "A soft set (F, E) over X is said to be a null soft set denoted by $\tilde{\Phi}$, if for all $e \in E, F(e) = \phi$ (null set)".

Definition 1.3 [9]**:** "A soft set (F, E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in E, F(e) = X$."

Definition 1.4 [5]**:** "Let $\mathbb{R}$ be the set of real numbers and $\mathcal{B}(\mathbb{R})$ the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters. Then a mapping $F: E \to \mathcal{B}(\mathbb{R})$ is called a soft real set. If a real soft set is a singleton soft set, it will be called a soft real number and denoted by $\bar{r}, \bar{s}, \bar{t}$ etc $\bar{0}$ *and* $\bar{1}$ are the soft real numbers where $\bar{0}(e) = 0$, $\bar{1}(e) = 1$, for all $e \in E$ respectively."

Definition 1.5 [5]**:** "**(Properties of Soft Real Numbers):** Let $\bar{r}$ and $\bar{s}$ be two soft real numbers. Then the following statements hold:

- $\bar{r} \tilde{\leq} \bar{s}$ if $\bar{r}(e) \tilde{\leq} \bar{s}(e)$ for all $e \in E$,
- $\bar{r} \tilde{\geq} \bar{s}$ if $\bar{r}(e) \tilde{\geq} \bar{s}(e)$ for all $e \in E$,
- $\bar{r} \tilde{<} \bar{s}$ if $\bar{r}(e) \tilde{<} \bar{s}(e)$ for all $e \in E$,
- $\bar{r} \tilde{>} \bar{s}$ if $\bar{r}(e) \tilde{>} \bar{s}(e)$ for all $e \in E$."

Definition 1.6 [6]**:** "A soft set (F, E) over X is said to be a soft point if there is exactly one $e \in E$ such that F(e) ={u}, for some $u \in \tilde{X}$ and F(e′) = ϕ, ∀ e′ ∈ E−{e}. It will be denoted by F_e^u or $\hat{u}_e$."

"The soft point $\hat{u}_e$ is said to be belonging to the soft set (F, E), denoted by $\hat{u}_e \in (F, E)$, if $\hat{u}_e(e) \in F(e)$, i.e., $\{u\} \subseteq F(e)$."

Definition 1.7 [3]**:** "A soft S-metric on $\tilde{X}$ is a mapping $S: SP(\tilde{X}) \times SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ which satisfies the following conditions:

($\bar{S}_1$) $S(\hat{u}_a, \hat{v}_b, \hat{w}_c) \tilde{\geq} \bar{0}$;

($\bar{S}_2$) $S(\hat{u}_a, \hat{v}_b, \hat{w}_c) = \bar{0}$, if and only if $\hat{u}_a = \hat{v}_b = \hat{w}_c$;

($\bar{S}_3$) $S(\hat{u}_a, \hat{v}_b, \hat{w}_c) \tilde{\leq} S(\hat{u}_a, \hat{u}_a, \hat{t}_d) + S(\hat{v}_b, \hat{v}_b, \hat{t}_d) + S(\hat{w}_c, \hat{w}_c, \hat{t}_d)$,

for all $\hat{u}_a, \hat{v}_b, \hat{w}_c, \hat{t}_d \in SP(\tilde{X})$, then the soft set $\tilde{X}$ with a soft S-metric S is called soft S-metric space and denoted by $(\tilde{X}, S, E)$."

Lemma 1.8 [3]**:** "Let $(\tilde{X}, S, E)$ is a soft S-metric space. Then we have

$S(\hat{u}_a, \hat{u}_a, \hat{v}_b) = S(\hat{v}_b, \hat{v}_b, \hat{u}_a)$."

Definition 1.9 [4]**:** "A soft sequence $\{\hat{u}_{a_n}^n\}$ in $(\tilde{X}, S, E)$ converges to $\hat{v}_b$ if and only if $S(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{v}_b) \to \bar{0}$ *as* $n \to \infty$ and we denote this by $\lim_{n\to\infty} \hat{u}_{a_n}^n = \hat{v}_b$."

Definition 1.10 [4]**:** "A soft sequence $\{\hat{u}_{a_n}^n\}$ in $(\tilde{X}, S, E)$ is called a Cauchy sequence if for $\tilde{\varepsilon} > \bar{0}$, there exists $n_0 \in \mathbb{N}$ such that $S(\hat{u}_{a_n}^n, \hat{u}_{a_n}^n, \hat{u}_{a_m}^m) < \tilde{\varepsilon}$ for each $m, n \geq n_0$".

Definition 1.11 [4]**:** "A soft S-metric space $(\tilde{X}, S, E)$ is said to be complete if every Cauchy sequence is converging to some soft point of $(\tilde{X}, S, E)$."

Definition 1.12 [4]**:** "Let $f_\varphi: (\tilde{X}, S, E) \to (\tilde{Y}, S', E')$ be a soft mapping from soft S-metric space $(\tilde{X}, S, E)$ to a soft S-metric space $(\tilde{Y}, S', E')$. Then f_φ is soft continuous at a soft point $\hat{u}_a \in SP(\tilde{X})$ if and only if $(f, \varphi)(\{\hat{u}_{a_n}^n\}) \to (f, \varphi)(\hat{u}_a)$."

Definition 1.13 [4]: "Let $(\tilde{X}, S, E)$ be a soft S-metric space. A map $(T,\varphi): (\tilde{X}, S, E) \to (\tilde{X}, S, E)$ is said to be a soft contraction mapping if there exists a soft real number $\bar{k} \in \mathbb{R}(E), \bar{0} \leq \bar{k} < \bar{1}$ (where $\mathbb{R}(E)$ denotes the soft real number set) such that

$S((T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{u}_a),(T,\varphi)(\hat{v}_b)) \leq \bar{k}\ S(\hat{u}_a, \hat{u}_a, \hat{v}_b)$, for all $\hat{u}_a, \hat{v}_b \in SP(\tilde{X})$".

2. Main Results

Definition 2.1: Let $(\tilde{X}, S, E)$ is a soft S-metric space and f_φ, g_φ be two soft self mapping on $(\tilde{X}, S, E)$. Then the pair $\{f_\varphi, g_\varphi\}$ is said to be soft compatible if and only if

$$\lim_{n\to\infty} S((f_\varphi g_\varphi)(\hat{u}^n_{a_n}), (f_\varphi g_\varphi)(\hat{u}^n_{a_n}), (g_\varphi f_\varphi)(\hat{u}^n_{a_n})) = \bar{0},$$

Whenever $\{\hat{u}^n_{a_n}\}$ is a soft sequence in $\tilde{X}$ such that $\lim_{n\to\infty} f_\varphi(\hat{u}^n_{a_n}) = \lim_{n\to\infty} g_\varphi(\hat{u}^n_{a_n}) = \hat{v}_b$, for some $\hat{v}_b \in SP(\tilde{X})$.

Lemma 2.2: Let $(\tilde{X}, S, E)$ be a soft S-metric space. If there exists two soft sequences $\{\hat{u}^n_{a_n}\}$ and $\{\hat{v}^n_{b_n}\}$ such that $\lim_{n\to\infty} S(\hat{u}^n_{a_n}, \hat{u}^n_{a_n}, \hat{v}^n_{b_n}) = \bar{0}$, whenever $\{\hat{u}^n_{a_n}\}$ is a soft sequence in $\tilde{X}$ such that $\lim_{n\to\infty} \hat{u}^n_{a_n} = \hat{t}_d$ for some $\hat{t}_d \in SP(\tilde{X})$, then $\lim_{n\to\infty} \hat{v}^n_{b_n} = \hat{t}_d$.

Proof: From ($\bar{S_3}$) of soft S-metric space, we have

$$S(\hat{v}^n_{b_n}, \hat{v}^n_{b_n}, \hat{t}_d) \lesssim 2\, S(\hat{v}^n_{b_n}, \hat{v}^n_{b_n}, \hat{u}^n_{a_n}) + S(\hat{t}_d, \hat{t}_d, \hat{u}^n_{a_n}).$$

Now, by taking the upper limit when $n \to \infty$ in (2.1) we obtain

$$\lim_{n\to\infty} sup S(\hat{v}^n_{b_n}, \hat{v}^n_{b_n}, \hat{t}_d) \lesssim 2 \lim_{n\to\infty} S(\hat{v}^n_{b_n}, \hat{v}^n_{b_n}, \hat{u}^n_{a_n}) + \lim_{n\to\infty} S(\hat{t}_d, \hat{t}_d, \hat{u}^n_{a_n}) = \bar{0}.$$

Hence $\lim_{n\to\infty} \hat{v}^n_{b_n} = \hat{t}_d$.

Theorem 2.3: Suppose that $f_\varphi, g_\varphi, R_\varphi, and\ T_\varphi$ are four soft self mapping on a complete soft S-metric space $(\tilde{X}, S, E)$, with $f_\varphi(\tilde{X}, S) \subseteq T_\varphi(\tilde{X}, S), g_\varphi(\tilde{X}, S) \subseteq R_\varphi(\tilde{X}, S)$ and the pair $\{f_\varphi, R_\varphi\}$ and $\{g_\varphi, T_\varphi\}$ are soft compatible. If

$$\begin{aligned} S(f_\varphi(\hat{u}_a), f_\varphi(\hat{v}_b), g_\varphi(\hat{w}_c)) \lesssim \tilde{k} \max\{ & S(R_\varphi(\hat{u}_a), R_\varphi(\hat{v}_b), T_\varphi(\hat{w}_c)), \\ & S(f_\varphi(\hat{u}_a), f_\varphi(\hat{u}_a), R_\varphi(\hat{u}_a)), \\ & S(g_\varphi(\hat{w}_c), g_\varphi(\hat{w}_c), T_\varphi(\hat{w}_c)), \\ & S(f_\varphi(\hat{v}_b), f_\varphi(\hat{v}_b), g_\varphi(\hat{w}_c))\}, \end{aligned} \tag{2.1}$$

For each $\hat{u}_a, \hat{v}_b, \hat{w}_c \in SP(\tilde{X})$ with $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$.

Then $f_\varphi, g_\varphi, R_\varphi, and\ T_\varphi$ has a unique common fixed soft point in $\tilde{X}$ provided that $R_\varphi\ and\ T_\varphi$ are soft continuous.

Proof: Let $\hat{u}^0_{a_0} \in SP(\tilde{X})$. Since $f_\varphi(\tilde{X}) \subseteq T_\varphi(\tilde{X})$ there exist $\hat{u}^1_{a_1} \in SP(\tilde{X})$ such that $f_\varphi(\hat{u}^0_{a_0}) = T_\varphi(\hat{u}^1_{a_1})$ and also as $g_\varphi(\hat{u}^1_{a_1}) \in R_\varphi(\tilde{X})$, we choose $\hat{u}^2_{a_2} \in SP(\tilde{X})$ such that $g_\varphi(\hat{u}^1_{a_1}) = R_\varphi(\hat{u}^2_{a_2})$. In general, $\hat{u}^{2n+1}_{a_{2n+1}} \in SP(\tilde{X})$ is chosen such that $f_\varphi(\hat{u}^{2n}_{a_{2n}}) = T_\varphi(\hat{u}^{2n+1}_{a_{2n+1}})$ and $\hat{u}^{2n+2}_{a_{2n+2}} \in SP(\tilde{X})$ such that $g_\varphi(\hat{u}^{2n+1}_{a_{2n+1}}) = R_\varphi(\hat{u}^{2n+2}_{a_{2n+2}})$, we obtain a soft sequence

$\{\hat{v}_{b_n}^{n}\}$ in $\tilde{X}$ such that

$\hat{v}_{b_{2n}}^{2n} = f_\varphi(\hat{u}_{a_{2n}}^{2n}) = T_\varphi(\hat{u}_{a_{2n+1}}^{2n+1}),$

$\hat{v}_{b_{2n+1}}^{2n+1} = g_\varphi(\hat{u}_{a_{2n+1}}^{2n+1}) = R_\varphi(\hat{u}_{a_{2n+2}}^{2n+2}),\ n \geq 0.$

Now, we show that $\{\hat{v}_{b_n}^{n}\}$ is a Cauchy Sequence. For this we have

$S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1}) = S\left(f_\varphi(\hat{u}_{a_{2n}}^{2n}), f_\varphi(\hat{u}_{a_{2n}}^{2n}), g_\varphi(\hat{u}_{a_{2n+1}}^{2n+1})\right)$

$\precsim \bar{k} \max\{S(R_\varphi(\hat{u}_{a_{2n}}^{2n}), R_\varphi(\hat{u}_{a_{2n}}^{2n}), T_\varphi(\hat{u}_{a_{2n+1}}^{2n+1})),$

$S\left(f_\varphi(\hat{u}_{a_{2n}}^{2n}), f_\varphi(\hat{u}_{a_{2n}}^{2n}), R_\varphi(\hat{u}_{a_{2n}}^{2n})\right),$

$S(g_\varphi(\hat{u}_{a_{2n+1}}^{2n+1}), g_\varphi(\hat{u}_{a_{2n+1}}^{2n+1}), T_\varphi(\hat{u}_{a_{2n+1}}^{2n+1})),$

$S(f_\varphi(\hat{u}_{a_{2n}}^{2n}), f_\varphi(\hat{u}_{a_{2n}}^{2n}), g_\varphi(\hat{u}_{a_{2n+1}}^{2n+1}))\}$

$= \bar{k} \max\{S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n}}^{2n}), S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n-1}}^{2n-1}),$

$S(\hat{v}_{b_{2n+1}}^{2n+1}, \hat{v}_{b_{2n+1}}^{2n+1}, \hat{v}_{b_{2n}}^{2n}), S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1})\}$

$= \bar{k} \max\{S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n}}^{2n}), S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1})\}.$

Now if $S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1}) \succsim S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n}}^{2n})$, then by above inequality we have

$S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1}) \precsim \bar{k}\, S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1}),$ which is contradiction.

Hence, $S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1}) \precsim S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n}}^{2n}),$ therefore by above inequality we get

$S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n+1}}^{2n+1}) \precsim \bar{k}\, S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n}}^{2n})$ (2.2)

By similar argument we have

$S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n}}^{2n}) = S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n-1}}^{2n-1})$

$= S\left(f_\varphi(\hat{u}_{a_{2n}}^{2n}), f_\varphi(\hat{u}_{a_{2n}}^{2n}), g_\varphi(\hat{u}_{a_{2n-1}}^{2n-1})\right)$

$\precsim \bar{k} \max\{S(R_\varphi(\hat{u}_{a_{2n}}^{2n}), R_\varphi(\hat{u}_{a_{2n}}^{2n}), T_\varphi(\hat{u}_{a_{2n-1}}^{2n-1})),$

$S\left(f_\varphi(\hat{u}_{a_{2n}}^{2n}), f_\varphi(\hat{u}_{a_{2n}}^{2n}), R_\varphi(\hat{u}_{a_{2n}}^{2n})\right),$

$S(g_\varphi(\hat{u}_{a_{2n-1}}^{2n-1}), g_\varphi(\hat{u}_{a_{2n-1}}^{2n-1}), T_\varphi(\hat{u}_{a_{2n-1}}^{2n-1})),$

$S(f_\varphi(\hat{u}_{a_{2n}}^{2n}), f_\varphi(\hat{u}_{a_{2n}}^{2n}), g_\varphi(\hat{u}_{a_{2n-1}}^{2n-1}))\}$

$= \bar{k} \max\{S(\hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-1}}^{2n-1}, \hat{v}_{b_{2n-2}}^{2n-2}), S(\hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n}}^{2n}, \hat{v}_{b_{2n-1}}^{2n-1}),$

$S(\hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n-2}_{b_{2n-2}}), S(\hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n-1}_{b_{2n-1}})\}$

$= \bar{k} \max\{S(\hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n-2}_{b_{2n-2}}), S(\hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n-1}_{b_{2n-1}})\}$.

Now if $S(\hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n-1}_{b_{2n-1}}) \gtrsim S(\hat{v}^{2n-2}_{b_{2n-2}}, \hat{v}^{2n-2}_{b_{2n-2}}, \hat{v}^{2n-1}_{b_{2n-1}})$, then by above inequality we have

$S(\hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n-1}_{b_{2n-1}}) \precsim \bar{k}\, S(\hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n}_{b_{2n}}, \hat{v}^{2n-1}_{b_{2n-1}})$, which is contradiction.

Hence, $S(\hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n}_{b_{2n}}) \lesssim S(\hat{v}^{2n-2}_{b_{2n-2}}, \hat{v}^{2n-2}_{b_{2n-2}}, \hat{v}^{2n-1}_{b_{2n-1}})$, therefore by above inequality we get

$$S(\hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n-1}_{b_{2n-1}}, \hat{v}^{2n}_{b_{2n}}) \lesssim \bar{k}\, S(\hat{v}^{2n-2}_{b_{2n-2}}, \hat{v}^{2n-2}_{b_{2n-2}}, \hat{v}^{2n-1}_{b_{2n-1}}) \quad (2.3)$$

From 2.2 and 2.3 we have

$S(\hat{v}^{n}_{b_{n}}, \hat{v}^{n}_{b_{n}}, \hat{v}^{n-1}_{b_{n-1}}) \lesssim \bar{k}\, S(\hat{v}^{n-1}_{b_{n-1}}, \hat{v}^{n-1}_{b_{n-1}}, \hat{v}^{n-2}_{b_{n-2}}), n \geq 2,$ where $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$.

Hence, it follows that

$$S(\hat{v}^{n}_{b_{n}}, \hat{v}^{n}_{b_{n}}, \hat{v}^{n-1}_{b_{n-1}}) \lesssim \dots \lesssim \bar{k}^{n-1} S(\hat{v}^{1}_{b_{1}}, \hat{v}^{1}_{b_{1}}, \hat{v}^{0}_{b_{0}}). \quad (2.4)$$

From ($\bar{S}_3$) of soft S-metric space, for $n > m$ we have

$S(\hat{v}^{n}_{b_{n}}, \hat{v}^{n}_{b_{n}}, \hat{v}^{m}_{b_{m}}) \lesssim 2\, S(\hat{v}^{m}_{b_{m}}, \hat{v}^{m}_{b_{m}}, \hat{v}^{m+1}_{b_{m+1}})$

$+2\, S(\hat{v}^{m+1}_{b_{m+1}}, \hat{v}^{m+1}_{b_{m+1}}, \hat{v}^{m+2}_{b_{m+2}})$

$+ \cdots + S(\hat{v}^{n-1}_{b_{n-1}}, \hat{v}^{n-1}_{b_{n-1}}, \hat{v}^{n}_{b_{n}})$

$\lesssim 2\, S(\hat{v}^{m}_{b_{m}}, \hat{v}^{m}_{b_{m}}, \hat{v}^{m+1}_{b_{m+1}}) + 2\, S(\hat{v}^{m+1}_{b_{m+1}}, \hat{v}^{m+1}_{b_{m+1}}, \hat{v}^{m+2}_{b_{m+2}})$

$+ \cdots + 2\, S(\hat{v}^{n-1}_{b_{n-1}}, \hat{v}^{n-1}_{b_{n-1}}, \hat{v}^{n}_{b_{n}})$.

Hence from (2.4) and as $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$ we have

$S(\hat{v}^{n}_{b_{n}}, \hat{v}^{n}_{b_{n}}, \hat{v}^{m}_{b_{m}}) \lesssim 2\,(\bar{k}^{m} + \bar{k}^{m+1} + \cdots + \bar{k}^{n-1})\, S(\hat{v}^{1}_{b_{1}}, \hat{v}^{1}_{b_{1}}, \hat{v}^{0}_{b_{0}})$

$\lesssim 2\,(1 + \bar{k} + \bar{k}^2 + \cdots)\, S(\hat{v}^{1}_{b_{1}}, \hat{v}^{1}_{b_{1}}, \hat{v}^{0}_{b_{0}})$

$\lesssim 2\, \frac{\bar{k}^{m}}{1-\bar{k}} S(\hat{v}^{1}_{b_{1}}, \hat{v}^{1}_{b_{1}}, \hat{v}^{0}_{b_{0}}) \to \bar{0}$,

As $m \to \infty$.

It follows that sequence $\{\hat{v}^{n}_{b_{n}}\}$ is a Cauchy Sequence. Since $(\tilde{X}, S, E)$ is a complete soft S-metric space, so there is some $\hat{w}_c \in SP(\tilde{X})$ such that

$$\lim_{n\to\infty} f_{\varphi}(\hat{u}^{2n}_{a_{2n}}) = \lim_{n\to\infty} T_{\varphi}(\hat{u}^{2n+1}_{a_{2n+1}}) = \lim_{n\to\infty} g_{\varphi}(\hat{u}^{2n+1}_{a_{2n+1}}) = \lim_{n\to\infty} R_{\varphi}(\hat{u}^{2n+2}_{a_{2n+2}}) = \hat{w}_c.$$

Now we show that $\hat{w}_c$ is a common fixed point of $f_{\varphi},\ g_{\varphi},\ R_{\varphi}$ and T_{φ}.

Since R_φ is soft continuous it follows that

$$\lim_{n\to\infty} R_\varphi{}^2\left(\hat{u}^{2n+2}_{a_{2n+2}}\right) = R_\varphi(\widehat{w}_c),$$

$$\lim_{n\to\infty} R_\varphi f_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right) = R_\varphi(\widehat{w}_c).$$

And since R_φ *and* f_φ are compatible, $\lim_{n\to\infty} S\left(f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), R_\varphi f_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right)\right) = \bar{0}.$

So, by Lemma 2.1 $\lim_{n\to\infty} f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right) = R_\varphi(\widehat{w}_c).$

Putting $\hat{u}_a = \hat{v}_b = R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right)$ *and* $\widehat{w}_c = \hat{u}^{2n+1}_{a_{2n+1}}$ in condition (2.1), we get

$$S\left(f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right)$$

$$\lesssim \bar{k}\max\{S\left(R_\varphi{}^2\left(\hat{u}^{2n}_{a_{2n}}\right), R_\varphi{}^2\left(\hat{u}^{2n}_{a_{2n}}\right), T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right),$$

$$S\left(f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), R_\varphi{}^2\left(\hat{u}^{2n}_{a_{2n}}\right)\right), S\left(g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right),$$

$$S\left(f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right)\}. \qquad (2.5)$$

Now, by taking the upper limit when $n \to \infty$ in (2.5) we get

$$S\left(R_\varphi(\widehat{w}_c), R_\varphi(\widehat{w}_c), \widehat{w}_c\right) = \lim_{n\to\infty} S\left(f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right)$$

$$\lesssim \bar{k}\max\{S\left(R_\varphi(\widehat{w}_c), R_\varphi(\widehat{w}_c), \widehat{w}_c\right), \bar{0}, \bar{0}, S\left(R_\varphi(\widehat{w}_c), R_\varphi(\widehat{w}_c), \widehat{w}_c\right)\}$$

$$= \bar{k}\, S\left(R_\varphi(\widehat{w}_c), R_\varphi(\widehat{w}_c), \widehat{w}_c\right).$$

Consequently, $S\left(R_\varphi(\widehat{w}_c), R_\varphi(\widehat{w}_c), \widehat{w}_c\right) \lesssim \bar{k}\, S\left(R_\varphi(\widehat{w}_c), R_\varphi(\widehat{w}_c), \widehat{w}_c\right)$, as $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$ it follows that $R_\varphi(\widehat{w}_c) = \widehat{w}_c$.

In similar way, since T_φ is soft continuous, we obtain that

$$\lim_{n\to\infty} T_\varphi{}^2\left(\hat{u}^{2n+1}_{a_{2n+1}}\right) = T_\varphi(\widehat{w}_c),$$

$$\lim_{n\to\infty} T_\varphi g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right) = T_\varphi(\widehat{w}_c).$$

And since g_φ *and* T_φ are compatible, $\lim_{n\to\infty} S\left(g_\varphi T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), g_\varphi T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), T_\varphi g_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right) = \bar{0}.$

So, by Lemma 2.1 $\lim_{n\to\infty} g_\varphi T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right) = T_\varphi(\widehat{w}_c).$

Putting $\hat{u}_a = \hat{v}_b = \hat{u}^{2n}_{a_{2n}}$ *and* $\widehat{w}_c = T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)$ in condition (2.1), we get

$$S\left(f_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), g_\varphi T_\varphi\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right) \lesssim$$

$$\bar{k}\max\{S\left(R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), T_\varphi{}^2\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right), S\left(f_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), f_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right), R_\varphi\left(\hat{u}^{2n}_{a_{2n}}\right)\right),$$

$S\left(g_{\varphi}T_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), g_{\varphi}T_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), T_{\varphi}{}^{2}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right), S\left(f_{\varphi}\left(\hat{u}^{2n}_{a_{2n}}\right), f_{\varphi}\left(\hat{u}^{2n}_{a_{2n}}\right), g_{\varphi}T_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right)\}.$ (2.6)

Now, by taking the upper limit when $n \to \infty$ in (2.6) we get

$$S\left(\hat{w}_c, \hat{w}_c, T_{\varphi}(\hat{w}_c)\right) = \lim_{n\to\infty} S\left(f_{\varphi}\left(\hat{u}^{2n}_{a_{2n}}\right), f_{\varphi}\left(\hat{u}^{2n}_{a_{2n}}\right), g_{\varphi}T_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right)$$

$$\widetilde{\leq} \bar{k} \max\{S\left(\hat{w}_c, \hat{w}_c, T_{\varphi}(\hat{w}_c)\right), \bar{0}, \bar{0}, S\left(\hat{w}_c, \hat{w}_c, T_{\varphi}(\hat{w}_c)\right)\} = \bar{k}\, S\left(\hat{w}_c, \hat{w}_c, T_{\varphi}(\hat{w}_c)\right).$$

Thus, again it follows that $T_{\varphi}(\hat{w}_c) = \hat{w}_c$ as $\bar{0} \widetilde{\leq} \bar{k} \widetilde{\leq} \bar{1}$. Also we can apply condition (2.1) to obtain

$S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), g_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right) \widetilde{\leq} \bar{k} \max\{S\left(R_{\varphi}(\hat{w}_c), R_{\varphi}(\hat{w}_c), T_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right),$

$S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), R_{\varphi}(\hat{w}_c)\right),$

$S\left(g_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), g_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right), T_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right),$

$S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), g_{\varphi}\left(\hat{u}^{2n+1}_{a_{2n+1}}\right)\right)\}.$ (2.7)

And by taking the upper limit when $n \to \infty$ in (2.7), as $R_{\varphi}(\hat{w}_c) = T_{\varphi}(\hat{w}_c) = \hat{w}_c$, we have

$S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), \hat{w}_c\right) \widetilde{\leq} \bar{k} \max\{S\left(R_{\varphi}(\hat{w}_c), R_{\varphi}(\hat{w}_c), \hat{w}_c\right),$

$S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), \hat{w}_c\right), S(\hat{w}_c, \hat{w}_c, \hat{w}_c), S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), \hat{w}_c\right)\}$

$= \bar{k}\, S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), \hat{w}_c\right).$

As $\bar{0} \widetilde{\leq} \bar{k} \widetilde{\leq} \bar{1}$, it follows that $S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), \hat{w}_c\right) = \bar{0}$ which implies that $f_{\varphi}(\hat{w}_c) = \hat{w}_c$.

Finally, by using of condition (2.1) and as $R_{\varphi}(\hat{w}_c) = T_{\varphi}(\hat{w}_c) = f_{\varphi}(\hat{w}_c) = \hat{w}_c$, we obtain

$S\left(\hat{w}_c, \hat{w}_c, g_{\varphi}(\hat{w}_c)\right) = S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), g_{\varphi}(\hat{w}_c)\right)$

$\widetilde{\leq} \bar{k} \max\{S\left(R_{\varphi}(\hat{w}_c), R_{\varphi}(\hat{w}_c), T_{\varphi}(\hat{w}_c)\right), S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), R_{\varphi}(\hat{w}_c)\right),$

$S\left(g_{\varphi}(\hat{w}_c), g_{\varphi}(\hat{w}_c), T_{\varphi}(\hat{w}_c)\right), S\left(f_{\varphi}(\hat{w}_c), f_{\varphi}(\hat{w}_c), g_{\varphi}(\hat{w}_c)\right)\}$

$= \bar{k}\, S\left(\hat{w}_c, \hat{w}_c, g_{\varphi}(\hat{w}_c)\right),$

Which implies that $S\left(\hat{w}_c, \hat{w}_c, g_{\varphi}(\hat{w}_c)\right) = \tilde{0}$ as $\bar{0} \widetilde{\leq} \bar{k} \widetilde{\leq} \bar{1}$ and hence $g_{\varphi}(\hat{w}_c) = \hat{w}_c$.

Thus, we prove that

$R_{\varphi}(\hat{w}_c) = T_{\varphi}(\hat{w}_c) = f_{\varphi}(\hat{w}_c) = g_{\varphi}(\hat{w}_c) = \hat{w}_c.$

If there exist another common fixed soft point $\hat{v}_b$ in $SP(\tilde{X})$ of all $f_{\varphi}, g_{\varphi}, R_{\varphi}$ and T_{φ}, then

$S(\hat{v}_b, \hat{v}_b, \hat{w}_c) = S\left(f_{\varphi}(\hat{v}_b), f_{\varphi}(\hat{v}_b), g_{\varphi}(\hat{w}_c)\right)$

$\lesssim \bar{k}\max\{S(R_\varphi(\hat{v}_b),R_\varphi(\hat{v}_b),T_\varphi(\widehat{w}_c)),S(f_\varphi(\hat{v}_b),f_\varphi(\hat{v}_b),R_\varphi(\hat{v}_b)),$

$S(g_\varphi(\widehat{w}_c),g_\varphi(\widehat{w}_c),T_\varphi(\widehat{w}_c)),S(f_\varphi(\hat{v}_b),f_\varphi(\hat{v}_b),g_\varphi(\widehat{w}_c))\}$

$\lesssim \bar{k}\max\{S(\hat{v}_b,\hat{v}_b,\widehat{w}_c),S(\hat{v}_b,\hat{v}_b,\hat{v}_b),S(\widehat{w}_c,\widehat{w}_c,\widehat{w}_c),S(\hat{v}_b,\hat{v}_b,\widehat{w}_c)\}$

$= \bar{k}\ S(\hat{v}_b,\hat{v}_b,\widehat{w}_c),$

Which implies that $S(\hat{v}_b,\hat{v}_b,\widehat{w}_c) = \bar{0}$ as $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$ and hence $\hat{v}_b = \widehat{w}_c$. Thus, $\widehat{w}_c$ is unique common fixed soft point of $f_\varphi, g_\varphi,\ R_\varphi$ and T_φ.

Here completes the proof

Now we give an example to support our result.

Example 2.4: Let $E = \mathbb{N}$ be a parameter set and $\tilde{X} = [\bar{0},\bar{1}]$ be endowed with Soft S-metric space

$S(u_a,v_b,w_c) = |a-c| + |b-c| + |u-w| + |v-w|.$

Let $f_\varphi, g_\varphi, R_\varphi$ and T_φ be four soft self mapping on $(\tilde{X},S,E)$, define by

$f_\varphi(\hat{u}_a) = \left(\frac{u}{2}\right)^8_3,\ g_\varphi(\hat{u}_a) = \left(\frac{u}{2}\right)^4_3,$

$R_\varphi(\hat{u}_a) = \left(\frac{u}{2}\right)^2_3, T_\varphi(\hat{u}_a) = \left(\frac{u}{2}\right)_3,$

Where $f(u) = \left(\frac{u}{2}\right)^8, g(u) = \left(\frac{u}{2}\right)^4,$

$R(u) = \left(\frac{u}{2}\right)^2, T(u) = \left(\frac{u}{2}\right)$ and let $\varphi(a) = 3$ is a constant mapping.

Obviously $f_\varphi(\tilde{X}) \subseteq T_\varphi(\tilde{X})$ and $g_\varphi(\tilde{X}) \subseteq R_\varphi(\tilde{X})$. Moreover, the pair $\{f_\varphi, R_\varphi\}$ and $\{g_\varphi, T_\varphi\}$ are soft compatible mappings. Thus, for each $\hat{u}_a, \hat{v}_b, \widehat{w}_c \in SP(\tilde{X})$ we have

$S\left(f_\varphi(\hat{u}_a),f_\varphi(\hat{v}_b),g_\varphi(\widehat{w}_c)\right) = S\left(\left(\frac{u}{2}\right)^8_3,\left(\frac{v}{2}\right)^8_3,\left(\frac{w}{2}\right)^4_3\right)$

$= |3-3| + |3-3| + \left|\left(\frac{u}{2}\right)^8 - \left(\frac{w}{2}\right)^4\right| + \left|\left(\frac{v}{2}\right)^8 - \left(\frac{w}{2}\right)^4\right|$

$= \left|\left(\frac{u}{2}\right)^4 - \left(\frac{w}{2}\right)^2\right|\left|\left(\frac{u}{2}\right)^4 + \left(\frac{w}{2}\right)^2\right| +$

$\left|\left(\frac{v}{2}\right)^4 - \left(\frac{w}{2}\right)^2\right|\left|\left(\frac{v}{2}\right)^4 + \left(\frac{w}{2}\right)^2\right|$

$\leq \frac{5}{16}\left|\left(\frac{u}{2}\right)^4 - \left(\frac{w}{2}\right)^2\right| + \left|\left(\frac{v}{2}\right)^4 - \left(\frac{w}{2}\right)^2\right|$

$= \frac{5}{16}\left|\left(\frac{u}{2}\right)^2 - \left(\frac{w}{2}\right)\right|\left|\left(\frac{u}{2}\right)^2 + \left(\frac{w}{2}\right)\right| +$

$\left|\left(\frac{v}{2}\right)^2 - \left(\frac{w}{2}\right)\right|\left|\left(\frac{v}{2}\right)^2 + \left(\frac{w}{2}\right)\right|$

$\leq \frac{15}{64}\left|\left(\frac{u}{2}\right)^2 - \left(\frac{w}{2}\right)\right| + \left|\left(\frac{v}{2}\right)^2 - \left(\frac{w}{2}\right)\right|$

$\leq \frac{15}{64} S\left(\left(\frac{u}{2}\right)_3^2, \left(\frac{v}{2}\right)_3^2, \left(\frac{w}{2}\right)_3\right)$

$= \frac{15}{64} S\left(R_\varphi(\hat{u}_a), R_\varphi(\hat{v}_b), T_\varphi(\hat{w}_c)\right)$

$\leq \frac{15}{64} \max\{S\left(R_\varphi(\hat{u}_a), R_\varphi(\hat{v}_b), T_\varphi(\hat{w}_c)\right), S\left(g_\varphi(\hat{w}_c), g_\varphi(\hat{w}_c), T_\varphi(\hat{w}_c)\right),$

$S\left(f_\varphi(\hat{u}_a), f_\varphi(\hat{u}_a), R_\varphi(\hat{u}_a)\right), S\left(f_\varphi(v_b), f_\varphi(\hat{v}_b), g_\varphi(\hat{w}_c)\right)\},$

Where $\tilde{k} = \frac{\widetilde{15}}{64} \precsim \bar{1}$. Thus $f_\varphi, g_\varphi, R_\varphi$ *and* T_φ satisfy the conditions given in Theorem 2.2 and $\bar{0}$ is the unique common fixed soft point of $f_\varphi, g_\varphi, R_\varphi$ *and* T_φ.

Now we present the special cases of Theorem 2.2.

Corollary 2.5: Let $(\tilde{X}, S, E)$ be a complete soft S-metric space and let $f_\varphi, g_\varphi: (\tilde{X}, S, E) \to (\tilde{X}, S, E)$ are two soft mappings such that

$S(f_\varphi(\hat{u}_a), f_\varphi(\hat{v}_b), g_\varphi(\hat{w}_c)) \lesssim \bar{k} \max\{S(\hat{u}_a, \hat{v}_b, \hat{w}_c), S(f_\varphi(\hat{u}_a), f_\varphi(\hat{u}_a), \hat{u}_a),$

$S(g_\varphi(\hat{w}_c), g_\varphi(\hat{w}_c), \hat{w}_c)), S(f_\varphi(\hat{v}_b), f_\varphi(\hat{v}_b), g_\varphi(\hat{w}_c))\},$

For all $\hat{u}_a, \hat{v}_b, \hat{w}_c \in SP(\tilde{X})$ with $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$.

Then there exists a unique fixed soft point in $SP(\tilde{X})$ such that $f_\varphi(\hat{w}_c) = g_\varphi(\hat{w}_c) = \hat{w}_c$.

Proof: If we take R_φ and T_φ as identity map on $(\tilde{X}, S, E)$, then from Theorem 2.2 follows that f_φ and g_φ have a unique common fixed soft point.

Corollary 2.6: Let $(\tilde{X}, S, E)$ be a complete soft S-metric space and let $R_\varphi, T_\varphi: (\tilde{X}, S, E) \to (\tilde{X}, S, E)$ are two soft continuous mappings onto $(\tilde{X}, S, E)$ such that

$S(\hat{u}_a, \hat{v}_b, \hat{w}_c) \lesssim \bar{k} \max\{S(R_\varphi(\hat{u}_a), R_\varphi(\hat{v}_b), T_\varphi(\hat{w}_c)), S(\hat{u}_a, \hat{u}_a, R_\varphi(\hat{u}_a)), S(\hat{w}_c, \hat{w}_c, T_\varphi(\hat{w}_c)),$

$S(\hat{v}_b, \hat{v}_b, \hat{w}_c))\},$

For all $\hat{u}_a, \hat{v}_b, \hat{w}_c \in SP(\tilde{X})$ with $\bar{0} \lesssim \bar{k} \lesssim \bar{1}$.

Then R_φ *and* T_φ have a unique common fixed soft point.

Proof: If we take f_φ and g_φ as identity map on $(\tilde{X}, S, E)$, then from Theorem 2.2 follows that R_φ and T_φ have a unique common soft fixed point.

3. Conclusion

In this work, we introduced a new class of generalized soft contractive conditions and established common fixed soft point results for four self-mappings in soft S-metric spaces. The theoretical findings were further supported with illustrative examples and corollaries, highlighting the robustness of the proposed framework. These results extend classical fixed point theorems to a more generalized setting, thereby contributing to the growing field of soft set theory and its applications in handling uncertainties. The practical relevance lies in potential applications across decision-making, optimization, and computational mathematics where uncertainty is inherent. Future research may focus on exploring soft fixed point results under weaker conditions, hybrid structures, and interdisciplinary applications in computer science and engineering.

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