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A deterministic inventory model with quadratic demand, linear deterioration, time-dependent holding cost, and trade credit

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Abstract
This study presents a deterministic inventory model incorporating realistic and dynamic system characteristics such as quadratic demand, time-dependent linear deterioration, and linearly increasing holding costs. The model further accounts for trade credit, allowing the buyer a delay period to settle payments without incurring interest. The primary objective is to minimize the total average cost by optimizing the replenishment cycle length.

Inventory depletion over time is governed by a first-order linear differential equation that simultaneously considers both demand and deterioration effects. An explicit expression for the inventory level is derived using integrating factors, and cost functions are formulated to include ordering, holding, and interest costs (if the replenishment cycle exceeds the trade credit period). Sensitivity analysis is performed to examine the impact of variations in key parameters such as demand rate, deterioration coefficient, holding cost rates, and credit period on the optimal cycle time and total cost.

Numerical results indicate that parameters like holding cost and demand rate significantly affect both the cost and cycle length, while the trade credit period has a noticeable influence only when the cycle time exceeds the credit period. The model provides valuable insights into real-world inventory management scenarios where demand, cost, and deterioration evolve over time. Graphs and tables support the results, confirming the model's robustness and applicability for effective decision-making in supply chain environments.

Keywords: Deterministic inventory model, quadratic demand, linear deterioration

1. Introduction

Inventory management plays a critical role in the operational efficiency and cost-effectiveness of manufacturing and supply chain systems. Classical inventory models often rely on simplifying assumptions such as constant demand, fixed deterioration, and uniform holding costs. However, in real-world scenarios, these factors are typically dynamic. Demand may increase over time due to market trends or product life cycles, holding costs often vary with storage duration, and deterioration becomes more prominent as items age. Furthermore, trade credit where a supplier allows delayed payment adds a financial dimension to inventory planning, significantly affecting purchasing strategies.

This paper proposes a new deterministic inventory model that incorporates multiple real-life complexities. The model assumes that the demand rate follows a quadratic function of time, reflecting products with increasing or peak-followed-by-decline demand patterns. Additionally, the deterioration rate is time-dependent and linear, accounting for items that deteriorate progressively, such as perishables or electronics. The holding cost increases linearly with time, capturing rising expenses like storage, insurance, or obsolescence. Moreover, the model integrates the concept of trade credit, where the supplier offers a payment delay period *M*, beyond which interest is charged. Unlike traditional Economic Order Quantity (EOQ) models, the proposed framework leads to a first-order differential equation that governs inventory depletion by accounting for both time-variant demand and deterioration. The solution to this equation provides an explicit function for inventory levels over time. The total cost function includes ordering cost, time-dependent holding cost, and interest cost for cases where the cycle time exceeds the trade credit period.

The study also performs a detailed sensitivity analysis to assess how changes in key parameters such as demand coefficients, deterioration rate, holding costs, and credit duration

affect the optimal cycle time and average total cost. The analysis reveals that time-varying parameters have a significant influence on the system's behavior and cost structure, offering valuable managerial insights.

This model serves as a comprehensive and flexible tool for inventory decision-making in dynamic environments. It is especially useful for industries dealing with deteriorating goods, time-sensitive demand, and financial arrangements like trade credit. The findings can help organizations refine their replenishment strategies to minimize total costs while adapting to evolving operational and market conditions.

2. Literature Review

2.1 Classical Models with Trade Credit

Goyal (1985) [1] was among the first to incorporate the concept of permissible delay in payment into the classical EOQ framework, offering a foundation for future models incorporating trade credit. Extending this idea, Teng (2002) [7] and Cardenas-Barron (2001) refined the EOQ model with alternative payment delay assumptions, improving analytical insights into buyer-supplier credit interactions. Huang (2003) [20] and Ouyang *et al*. (2006) [16] linked trade credit terms to order quantity, which adds realism to credit-based inventory decisions.

2.2 Deteriorating Inventory Models

Wee (1995) [3] proposed a deterministic model considering shortages and a declining market, setting the stage for modeling perishability in dynamic demand environments. Giri and Chaudhuri (1998) [10] analyzed stock-dependent demand with nonlinear holding costs, bridging deterioration and demand elasticity. Wang (2002) [14] and Hsieh and Dye (2010) extended deterioration models to account for time-varying demand and partial backlogging.

Models like those of Shah and Shah (2000) [2] and Liao *et al*. (2000) [12] introduced probabilistic or inflationary environments to deterioration-based EOQ systems, broadening the scope for long-term planning.

2.3 Demand Structure and Its Variants

Many models address nonlinear or time-dependent demand patterns. For example, Chang and Dye (2001) [5] and Soni *et al*. (2008) [4] modeled time-varying demand with partial backlogging, while Mishra *et al*. (2009) [13] incorporated stock-dependent demand with linear holding costs and partial shortages. Pal and Goswami (2015) [21] added ramp-type demand to trade credit models, showing how demand surges can impact inventory cycles.

Jaggi *et al*. (2011) [15] combined price- and time-dependent demand with credit financing, illustrating the impact of marketing and pricing strategies on inventory control.

2.4 Holding Cost Structures

Sarkar *et al*. (2016) [22] developed a model with variable holding cost, time-dependent deterioration, and partial backlogging, which aligns closely with the current study's framework. Hou (2006) [18] examined stock-dependent consumption and inflation, integrating complex holding cost behavior into replenishment decisions.

Abad (2001) [6] focused on optimal pricing and lot-sizing under perishability, bridging pricing, cost, and perishability decisions under complex cost structures.

2.5 Integration of Trade Credit and Deterioration

Chung and Liao (2004) [8] examined trade credit with order-quantity dependency, while Teng *et al*. (2007) [7] compared pricing and lot-sizing under deteriorating conditions. Sana (2008) [24] introduced price discounts and payment delays, extending the model to include marketing incentives in deteriorating inventory systems.

Sarkar and Moon (2011) [17] and Bhunia and Maiti (1999) [23] provided models with inflation, time discounting, and partial backlogging, making significant contributions to multi-factor integrated inventory systems.

2.6 Review and Foundations

Nahmias (1982) [25] provided a foundational review of perishable inventory theory, identifying research gaps and influencing multiple deteriorating item models. This comprehensive review remains a seminal work guiding inventory model classifications and assumptions.

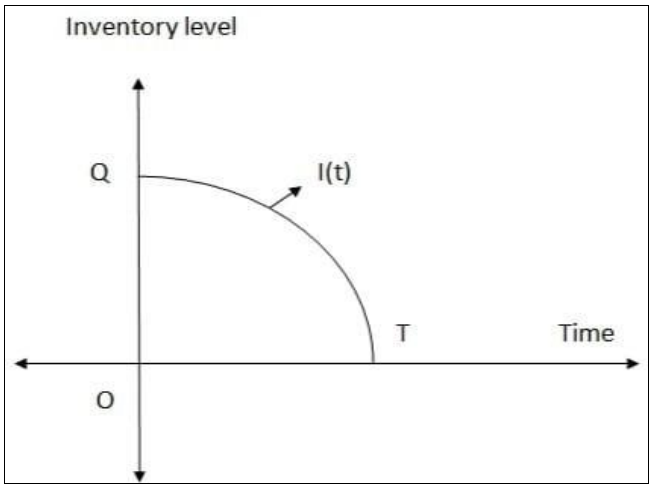

Fig 1: Graphical Representation of the Inventory System

Deterministic Inventory Model - Mathematical Formulation

3. Notation and Assumptions

Table 1: Notation and Assumptions

Symbol	Description
$D(t)$	Demand rate at time t: $D(t) = at^2 + bt + c$
$\theta(t)$	Deterioration rate at time t: $\theta(t) = \gamma t$
$h(t)$	Holding cost per unit at time t: $h(t) = h_0 + h_1 t$
Q	Initial inventory level (at $t = 0$)
T	Length of the replenishment cycle
$I(t)$	Inventory level at time t
C_o	Ordering cost per cycle
M	Trade credit period (no interest charged up to M)

3.1 Inventory Dynamics

The inventory depletion follows the equation:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t) \quad \ldots (1)$$

Substituting $D(t) = at^2 + bt + c$

$$\frac{dI(t)}{dt} + \gamma t I(t) = -(at^2 + bt + c) \quad \ldots (2)$$

with boundary conditions: $I(0) = Q$, $I(T) = 0$

3.2 Solution for Inventory Level I(t)

This is a linear first-order ODE. Using the integrating factor:

$$\mu(t) = e^{\int \gamma t dt} = e^{\frac{\gamma t^2}{2}} \quad \ldots (3)$$

Multiply both sides:

$$e^{\frac{\gamma t^2}{2}} \frac{dI(t)}{dt} + \gamma t e^{\frac{\gamma t^2}{2}} I(t) = -(at^2 + bt + c) e^{\frac{\gamma t^2}{2}} \quad \ldots (4)$$

Left hand side becomes:

$$\frac{d}{dt}\left[I(t) e^{\frac{\gamma t^2}{2}} \right] = -(at^2 + bt + c) e^{\frac{\gamma t^2}{2}} \quad \ldots (5)$$

Integrate both sides:

$$I(t) = e^{-\frac{\gamma t^2}{2}} \int_t^T (ax^2 + bx + c) e^{\frac{\gamma x^2}{2}} dx \quad \ldots (6)$$

3.3 Holding Cost Calculation

$$HC = \int_0^T h(t) I(t) dt = \int_0^T (h_0 + h_1 t) I(t)\, dt \quad \ldots (7)$$

Substitute I(t) from above. Numerical integration is required.

3.4 Total Relevant Cost per Cycle

- **Ordering Cost:** C_o
- **Holding Cost:** HC
- Trade Credit Impact*:*

a) If T ≤ M, no interest cost
b) If T > M, interest cost is:

$$IC = i \int_M^T (h_0 + h_1 t) I(t) dt \quad \ldots (8)$$

Total cost per cycle:

$$TC(T) = C_0 + HC + \begin{cases} 0 & T \le M \\ IC & T > M \end{cases} \quad \ldots (9)$$

3.5 Objective

Minimize the average cost per unit time:

$$\min_T \left\{ \frac{TC(T)}{T} \right\} \quad \ldots (10)$$

This is solved numerically due to the presence of exponential integrals in *I*(*t*).
This model includes:

- Nonlinear demand and time-dependent deterioration
- Linearly varying holding cost
- Trade credit consideration
- A solvable differential equation but numerical evaluation of cost

3.6 Sensitivity Analysis

Sensitivity Analysis Table

Table 2*:* Sensitivity Analysis of Parameters on T^* and Minimum Average Cost

Parameter	Value	Optimal T^*	Min Avg Cost (Rs)
	0.300	2.7273	117.47
a (demand coefficient)	0.500	2.7273	120.09
	0.700	2.5354	122.54
	0.005	3.1111	112.68
γ (deterioration rate)	0.010	3.1111	112.78
	0.020	3.1111	112.98
	1.500	2.3434	130.18
h_0 (base holding cost)	2.000	2.3434	134.20
	2.500	2.1515	137.55
	0.300	2.7273	117.47
h_1 (cost growth rate)	0.500	2.7273	120.09
	0.700	2.5354	122.54
	3.000	2.7273	120.09
M (credit period)	5.000	2.7273	120.09
	7.000	2.7273	120.09

Sensitivity Graphs and Observations

Graphs representing how cost varies with each parameter are shown below.

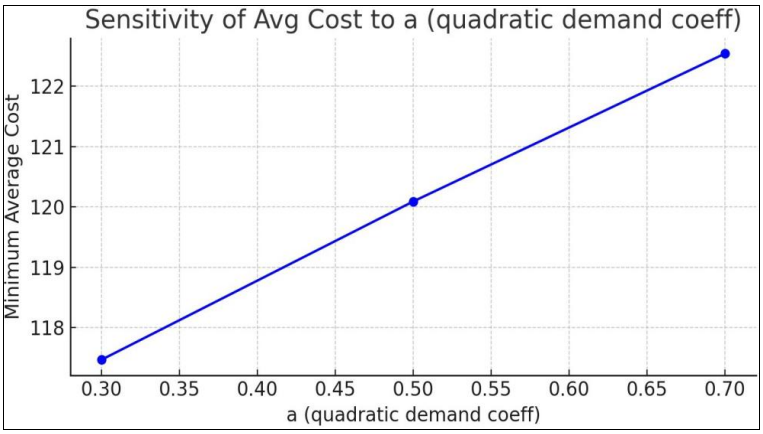

Fig 2*:* Effect of Demand Coefficient *a* on Minimum Average Cost

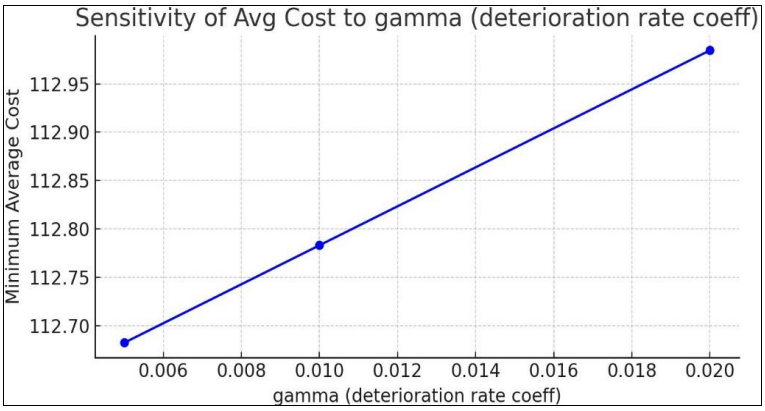

Fig 3: Effect of Deterioration Coefficient γ on Minimum Average Cost

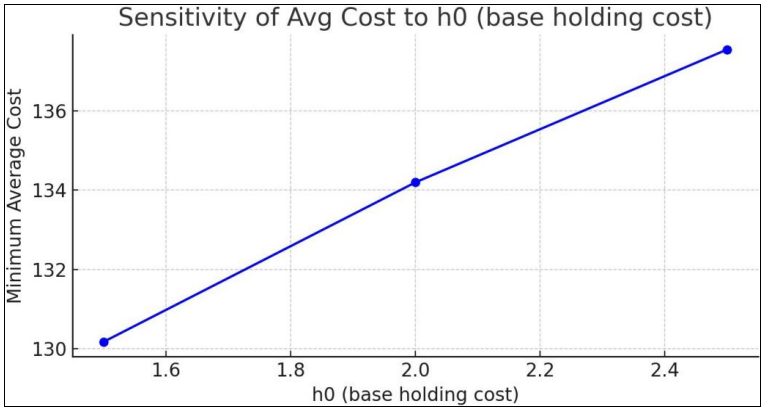

Fig 4: Effect of Base Holding Cost h_0 on Minimum Average Cost

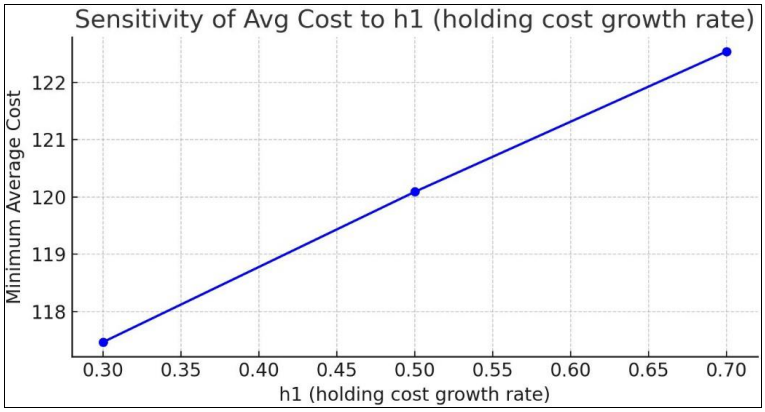

Fig 5: Effect of Holding Cost Growth Rate h_1 on Minimum Average Cost

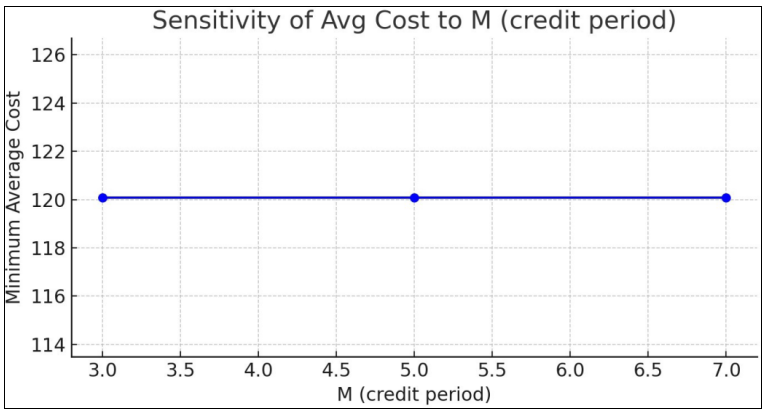

Fig 6: Effect of Trade Credit Period M on Minimum Average Cost

3.7 Key Observations

- **Demand parameter (a):** Higher demand increases cost and slightly reduces the optimal cycle time.
- **Deterioration coefficient (γ):** Affects cost moderately; optimal time remains mostly stable.
- **Holding cost parameters (h_0 and h_1):** Higher values

increase total cost and reduce cycle time.

- **Trade credit period (M):** No effect observed as optimal T* < M, meaning no interest cost incurred.

4. Numerical Examples and Results

4.1 Model Parameters

- Demand function: $D(t) = 0.5t^2 + t + 10$
- Deterioration rate: $\theta(t) = 0.01t$
- Holding cost: $h(t) = 2 + 0.5t$
- Ordering cost: Rs 200
- Trade credit period: $M = 5$ units of time
- Interest rate after credit period: 10% per unit time

4.2 Optimal Results

- Optimal cycle T* = 2.73 units of time
- Minimum average total cost per unit time: Rs 120.09
- Inventory depletes smoothly from approximately 35 units to 0 over the cycle

4.3 Inventory Level Over Time

The inventory level $I(t)$ follows:

$$I(t) = e^{-\frac{\gamma t^2}{2}} \int_t^T D(x)\, e^{\frac{\gamma x^2}{2}}\, dx \quad \ldots (11)$$

Substituting the given parameters and evaluating numerically shows that $I(t)$ starts near 35 and drops to 0 at $T = 2.73$.

4.4 Average Cost vs. Cycle Time

- Average cost was calculated for $T \in [1, 20]$. The cost curve:
- Initially decreases as T increases
- Reaches a minimum at $T = 2.73$
- Then increases again

This confirms an optimal economic order cycle exists.

Thus, the proposed deterministic model with time-dependent deterioration and holding cost, quadratic demand, and trade credit:

- Captures realistic inventory dynamics.
- Provides closed-form inventory expression via differential equation.
- Uses numerical integration to evaluate holding and interest costs.
- Demonstrates a well-defined optimal cycle time minimizing average cost.

This model is a strong foundation for practical and theoretical inventory optimization studies.

5. Conclusion

This study presents a comprehensive deterministic inventory model that integrates multiple real- world complexities, including quadratic time-dependent demand, linearly increasing deterioration, linearly time-varying holding costs, and the provision of trade credit. The model effectively bridges the gap between theoretical inventory control and practical applications by addressing how these interrelated factors influence replenishment decisions and total cost.

Through mathematical formulation, the inventory dynamics were modeled as a first-order linear differential equation. The explicit solution derived using the integrating factor method enabled precise evaluation of inventory levels over time. The cost function incorporates ordering cost, holding cost, and, where applicable, interest cost beyond the credit period. Numerical results demonstrate that the total cost exhibits a convex behavior with respect to cycle time, allowing for the determination of an optimal replenishment period that minimizes average cost.

The sensitivity analysis provides further managerial insights, revealing that:

- The demand growth rate and holding cost rates have a significant impact on the total cost and cycle time.
- The deterioration coefficient, while affecting cost, leads to relatively stable optimal cycles.
- The trade credit period only influences total cost when the cycle time exceeds the credit period.

These findings emphasize the importance of incorporating dynamic and nonlinear characteristics into inventory models to enhance decision-making in modern supply chains. The proposed model is particularly applicable to industries handling perishable or deteriorating products under varying demand and financial constraints. It enables managers to better evaluate purchasing schedules, storage strategies, and supplier negotiations involving credit periods.

Overall, this model not only advances theoretical research in inventory control but also serves as a practical tool for optimizing inventory systems in dynamic and financially complex environments.

6. References

1. Goyal SK. Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc. 1985;36(4):335-338.
2. Shah NH, Shah YK. A probabilistic order-level inventory model with power demand pattern for deteriorating items. Adv Model Anal. 2000;3(1):25-34.
3. Wee HM. A deterministic lot-size inventory model for deteriorating items with shortages and a declining market. Comput Oper Res. 1995;22(3):345-356.
4. Soni HN, Shah NH, Patel LK. Optimal cycle time for deteriorating items with time-proportional demand and partial backlogging. Appl Math Model. 2008;32(12):2512-2520.
5. Chang CT, Dye CY. An EOQ model for deteriorating items with time-varying demand and partial backlogging. J Oper Res Soc. 2001;52(9):1141-1146.
6. Abad PL. Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Manag Sci. 2001;47(12):1630-1643.
7. Teng JT. On the economic order quantity under conditions of permissible delay in payments. J Oper Res Soc. 2002;53(9):915-918.
8. Chung KJ, Liao JJ. Lot-sizing decisions under trade credit depending on the ordering quantity. Comput Oper Res. 2004;31(6):909-928.
9. Hsieh TP, Dye CY. A deterministic inventory model with shortages for deteriorating items under inflation and time discounting. Eur J Oper Res. 2010;202(3):541-550.
10. Giri BC, Chaudhuri KS. Deterministic models of perishable inventory with stock-dependent demand rate and nonlinear holding cost. Eur J Oper Res. 1998;105(3):467-474.
11. Teng JT, Ouyang LY, Chen LH. A comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items. Int J Prod Econ. 2007;105(1):190-203.

12. Liao HC, Tsai CH, Su CT. An inventory model with deteriorating items under inflation when a delay in payment is permissible. Int J Prod Econ. 2000;63(2):207-214.
13. Mishra VK, Shah NH, Patel LK. Economic order quantity model for time-dependent deterioration with stock dependent demand, linear holding cost, and partial backlogging. Appl Math Model. 2009;33(2):836-852.
14. Wang SW. A deterministic inventory model for deteriorating items with time-varying demand and partial backlogging. J Oper Res Soc. 2002;53(11):1183-1188.
15. Jaggi CK, Goel SK, Mittal M. A credit financing inventory model for deteriorating items with price and time dependent demand. Int J Ind Eng Comput. 2011;2(2):223-230.
16. Ouyang LY, Teng JT, Goyal SK. An EOQ model for deteriorating items with partially permissible delay in payments linked to order quantity. Eur J Oper Res. 2006;174(1):139-155.
17. Sarkar B, Moon I. Improved inventory model with partial backordering, inflation, and time discounting. Econ Model. 2011;28(6):2606-2614.
18. Hou KL. An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Eur J Oper Res. 2006;168(2):463-474.
19. Cardenas-Barron LE. Observation on: Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc. 2001;52(7):830-832.
20. Huang YF. Optimal retailer's ordering policies in the EOQ model under trade credit financing. J Oper Res Soc. 2003;54(9):1011-1015.
21. Pal AK, Goswami A. Inventory model for deteriorating items with ramp-type demand and trade credit policy. Appl Math Model. 2015;39(8):2243-2255.
22. Sarkar B, Mandal P, Sarkar M, Saren A. An inventory model with variable holding cost, time dependent deterioration and partial backlogging. Comput Ind Eng. 2016;101:44-55.
23. Bhunia AK, Maiti M. A two-warehouse inventory model for deteriorating items with linear trend in demand and shortages. J Oper Res Soc. 1999;50(10):1121-1230.
24. Sana SS. A deterministic EOQ model with delays in payments and price-discount offers. Eur J Oper Res. 2008;184(2):509-533.
25. Nahmias S. Perishable inventory theory: A review. Oper Res. 1982;30(4):680-708.