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Non-static plane-symmetric barrow holographic dark energy model in $f(Q)$ gravity

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Abstract

We construct Barrow holographic dark energy in the non-static, plane-symmetric universe. BHDE is based on the Barrow relation for horizon entropy (considering the Hubble horizon as the IR cut-off). We deal with the issue of locating solutions to the $f(Q)$ theory of gravity for non-static plane-symmetric cosmological models. Where the nonmetricity Q is responsible for the gravitational interaction. We obtain the equation of state parameter (EoS), the deceleration parameter, for the BHDE model to determine the cosmological evolution of the model.

Keywords: Barrow HDE, non-static plane-symmetric, $f(Q)$ gravity

Introduction

The curvature imported from Riemannian geometry, which is characterized by the Ricci scalar R , is the fundamental quantity in General Relativity (GR). A modification of general relativity (GR) is the modified $f(R)$ gravity, which substitutes a general function of R for the Ricci scalar R . In addition, there are various alternatives to General Relativity (GR), like the Teleparallel Equivalent of GR (TEGR), where the idea of torsion T describes the gravitational interactions. Whereas in teleparallelism the Weitzenböck link relates to torsion but zero curvature, the Levi-Civita connection in GR is related to curvature but zero torsion. The symmetric teleparallel equivalent of GR (STEGR) is a recently suggested theory of gravity in which the concept of non-metricity Q with zero torsion and curvature characterizes the gravitational interactions. The covariant derivative of the metric tensor in Weyl geometry is formally dictated by the nonmetricity tensor rather than taking equal to zero. Because modified theories of gravity explain present phenomena in the universe, scholars have been drawn to them in recent decades. Thus, many geometries have been used to derive the gravitational interactions.

According to cosmological data from the Cosmic Microwave Background (CMB), Large Scale Structure (LSS), and type Ia Supernova, the universe is currently expanding faster than before. A new form of energy with a negative pressure, commonly referred to as "dark energy" (DE) in the literature, must be introduced to explain such an acceleration within the parameters of orthodox cosmology. The acceleration of the expansion of the universe has been explained by a wide range of DE scenarios. The Equation of State (EoS) parameter, which represents the ratio of spatially homogenous pressure to the energy density of DE, is widely used to classify the ultimate nature of DE. In cosmology, holographic consideration is widely used, particularly for the explanation of the late time dark energy era also referred to as holographic dark energy, or HDE. Recently, from a holographic point of view, Nojiri *et al.* offered a unified cosmological model of the universe from inflation to dark energy (with intermediary radiation and a matter-like era). One notable extension is Barrow holographic dark energy, which is based on the traditional holographic principle but utilizes the newly proposed Barrow entropy instead of the Bekenstein–Hawking entropy. Saridakis demonstrated that the BHDE encompasses basic HDE as a special case when Barrow entropy reduces to the standard Bekenstein–Hawking entropy. However, in general, it presents a new scenario that reveals a richer and more intriguing phenomenology. Mamon has explored the implications of gravity-thermodynamics within the BHDE model by considering the dynamical apparent horizon as

the cosmological boundary. Anagnostopoulos *et al.* [25] have shown that the BHDE aligns well with observational data, making it a strong candidate for describing dark energy. Barrow [26] proposed that quantum-gravitational effects could create complex, fractal structures on the surface of a black hole, thereby altering its actual horizon area. This resulted in a new relationship for black hole entropy as

$$S_B = \left(\frac{A}{A_0}\right)^{1+\Delta/2} \quad (1)$$

where A is the usual horizon area and A_0 is the Planck area. The new exponent Δ quantifies the quantum-gravitational deformation. It is found that $\Delta = 1$ represents maximal deformation, while $\Delta = 0$ indicates the simplest horizon structure and in this case, we obtain the standard Bekenstein-Hawking entropy [27]. Thus, using this extended entropy relation as the foundation for holographic dark energy leads to the Barrow holographic dark energy model, which provides an enhanced phenomenological framework compared to standard holographic dark energy scenarios. It is important to highlight that the entropy described in equation (1) resembles Tsallis non-extensive entropy [28], though the underlying physical principles and foundations are entirely distinct.

If we need the expansion history to align with that of Λ CDM, we can analytically derive the appropriate functional form of $f(Q)$. Khyllip *et al.* [29] has investigated the evolution index of matter perturbations and the cosmological explanations for a polynomial functional form of $f(Q)$ gravity. Harko *et al.* explored the coupling of matter in $f(Q)$ gravity by assuming a power-law function. Shekh *et al.* [30] has examined both holographic and Renyi holographic dark energy model inflation using the Hubble and Granda-Oliveros cut-offs in $f(Q)$ gravity. And the analysis revealed that these models are excluded by Galaxy Clustering Statistics within the range $-1.33 \leq \omega \leq -0.79$.

The present paper is organized as follows: In section 2 we present the formalism of $f(Q)$ gravity. Non-static, plane-symmetric Metric and its field equations are discussed in section III. The solution of the field equations and physical and kinematical parameters are obtained in section IV. Lastly, the conclusion is given in section V.

2. Basic of $f(Q)$ gravity

The action of symmetric teleparallel gravity $f(Q)$ has been extensively covered in the current literature. For a more comprehensive discussion, readers are encouraged to consult the work cited in the references [31, 32]. We begin with the action which is given by

$$S = \int \left[\frac{1}{2} f(Q) + L_m \right] d^4x \sqrt{-g} \quad (2)$$

Here, $f(Q)$ represents an arbitrary function of the non-metricity scalar Q , g is the determinant of the metric tensor $g_{\mu\nu}$, and L_m is the matter of Lagrangian density. The non-metricity scalar Q is derived from the disformation tensor $L_{\mu\nu}^Y$ as follows:

$$Q \equiv -g^{\mu\nu} \left(L_{\alpha\mu}^\beta L_{\nu\beta}^\alpha - L_{\alpha\beta}^\beta L_{\mu\nu}^\alpha \right) \quad (3)$$

The disformation tensor $L_{\mu\nu}^Y$ is given by

$$L_{\alpha\gamma}^\beta = -\frac{1}{2} g^{\beta\eta} (\nabla_\gamma g_{\alpha\eta} + \nabla_\alpha g_{\eta\gamma} - \nabla_\eta g_{\alpha\gamma}) \quad (4)$$

The non-metricity tensor is written as

$$Q_{\beta\mu\nu} = \nabla_\beta g_{\mu\nu} \quad (5)$$

the trace of the non-metricity tensor has the form

$$Q_\beta = g^{\mu\nu} Q_{\beta\mu\nu} \widetilde{Q}_\beta = g^{\mu\nu} Q_{\mu\beta\nu} \quad (6)$$

The superpotential tensor is defined as

$$P_{\mu\nu}^\beta = -\frac{1}{2} L_{\mu\nu}^\beta + \frac{1}{4} (Q_\beta - \widetilde{Q}_\beta) g_{\mu\nu} - \frac{1}{4} \delta_{(\mu}^\beta \delta_{\nu)}^{\quad} \quad (7)$$

The non-metricity scalar using equations (6) and (7) can be written as

$$Q = -Q_{\beta\mu\nu} P^{\beta\mu\nu} \quad (8)$$

Now the energy-momentum tensor for the matter can be expressed as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}} \quad (9)$$

By varying the action (2) with respect to the metric tensor, the corresponding field equation can be obtained as

$$\frac{2}{\sqrt{-g}} \nabla_\beta (f_Q \sqrt{-g} P_{\mu\nu}^\beta) + \frac{1}{2} f g_{\mu\nu} + f_Q (P_{\mu\beta i} Q_\nu^{\beta i} - 2Q_{\beta i \mu} P_\nu^{\beta i}) = -T_{\mu\nu} \quad (10)$$

Where $f = f(Q)$, $f_Q = \frac{\partial f}{\partial Q}$ and ∇_β is a covariant derivative.

The energy-momentum tensor $T_{\mu\nu}$ and $\bar{T}_{\mu\nu}$ for matter and BHDE is given as follows

$$T_{\mu\nu} = \rho_m u_\mu u_\nu = [-1, 0, 0, 0] \rho_m \quad (11)$$

$$\bar{T}_{\mu\nu} = (\rho_B + p_B) u_\mu u_\nu + g_{\mu\nu} p_B = [-1, \omega_B, \omega_B, \omega_B] \rho_B \quad (12)$$

where p_B , ρ_B and ρ_m represent the pressure of BHDE, energy density for BHDE and matter. Also $\omega_B = p_B / \rho_B$ relates Equation of State parameter (EoS). The deviation from isotropy is obtained by setting deviation along y and z axis using skewness parameter δ from x -axis. Thus equation (12) can be parametrized as

$$\bar{T}_{\mu\nu} = [-1, \omega_B, (\omega_B + \delta), (\omega_B + \delta)] \rho_B \quad (13)$$

3. Metric and field equations

The line element for a non-static, plane-symmetric universe is given by

$$ds^2 = e^{2h} (dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2) \quad (14)$$

where r, θ and z are the cylindrical polar co-ordinates and $h(t)$ and $s(t)$ are the metric potentials which are the function of cosmic time t only.

The spatial volume (V) and the related scale factor (a) is obtained as,

$$V = [a(t)]^3 = s e^{4h} \quad (15)$$

The directional Hubble's parameter of the space-time (14) is $H_i (i = r, \theta, z)$

$$H_1 = 2\dot{h}, H_2 = 2\dot{h}, H_3 = \frac{\dot{s}}{s} \quad (16)$$

Let the deceleration parameter q is a function of the Hubble parameter as

$$q = -\frac{a \ddot{a}}{\dot{a}^2} = -1 + \frac{d}{dt} \left(\frac{1}{H} \right) \quad (17)$$

The rationale for assuming a variable deceleration parameter lies in the observation that the universe's expansion transitions from an initial decelerating phase to the current accelerating phase. The deceleration parameter is a dimensionless variable that characterizes the evolution of the cosmos. A positive value of q signifies cosmic deceleration, while a negative value indicates an accelerating expansion of the universe.

The non-metricity scalar Q for non-static, the plane-symmetric universe is given by

$$Q = -\left(6\dot{h}^2 + \frac{4\dot{h}\dot{s}}{s} \right) e^{-2h} \quad (18)$$

Applying Barrow entropy (1) within the holographic framework yields a dark energy density of the form as

$$\rho_B = CL^{\Delta-2} \quad (19)$$

with L as the holographic horizon length and C as a parameter with the dimension $[L]^{-2-\Delta}$. Equation (15) gives the standard holographic dark energy as $\rho_B = CL^{-2}$ with $C = 3c^2 M_p^2$ where M_p is the Planck mass and c^2 is the standard parameter of order one that is present in all holographic dark energy models.

We consider that the non-static, plane-symmetric universe is filled with BHDE and matter as well then, the field equation (10) of line element (14) using energy momentum tensors (11-13) obtained as

$$\frac{f}{2} + f_Q \left[2\dot{h} + 4\dot{h}^2 + 4\frac{\dot{h}\dot{s}}{s} + \frac{\dot{s}^2}{s} \right] e^{-2h} + e^{-2h} \left(2\dot{h} + \frac{\dot{s}}{s} \right) f_{QQ} \dot{Q} = -\omega_B \rho_B \tag{20}$$

$$\frac{f}{2} + f_Q \left[2\dot{h} + 4\dot{h}^2 + 2\frac{\dot{h}\dot{s}}{s} \right] e^{-2h} + e^{-2h} (2\dot{h}) f_{QQ} \dot{Q} = -(\omega_B + \delta) \rho_B \tag{21}$$

$$\frac{f}{2} + f_Q \left[6\dot{h}^2 + 4\frac{\dot{h}\dot{s}}{s} \right] e^{-2h} = \rho_m + \rho_B \tag{22}$$

here the overhead dot (.) denotes the derivative with respect to cosmic time t and f_Q, f_{QQ} denotes first and second-order derivative of f with respect to non-metricity Q .

4. Solution of the Field Equation

Here non-static, plane-symmetric metric generates three differential equations (20)-(22) with seven unknowns namely $h, s, f, \rho_m, \rho_B, \omega_B$ and δ .

Redshift studies indicate a correlation between the shear and the Hubble parameter in regions close to our galaxy, suggesting certain constraints in these areas. Collins *et al.* (1980) observed that in a homogeneous and anisotropic universe, the relationship between the Hubble parameter and shear remains constant. Vinutha *et al.* [33] have studied the non-static plane-symmetric dark energy string cosmological model in Barber's second self-creation theory of gravitation. Shekh *et al.* [34] used the non-static plane symmetric space-time for the dynamic study of the holographic dark energy model as a candidate for IR cut-offs (specifically Hubble's and Granda-Oliveros cut-offs). A study of non-static plane-symmetric cosmological models filled with a perfect fluid within the framework of $f(R, T)$ gravity is carried out by U. M. Rao & D. Neelima. In the literature, the various investigators use the following assumptions that can be made to solve this system

$$e^h \propto s^n, \quad n > 0 \tag{23}$$

Where m is a positive constant. The solution of the field equation is obtained by considering the quadratic form of the $f(Q)$ function as

$$f(Q) = \lambda Q, \lambda \text{ is a constant} \tag{24}$$

To solve the field equation, we used power law and exponential law cosmology which can be used in [35] to describe the epoch-based evolution of the Universe because of the constancy of the deceleration parameter. Consider the following ansatz referred to, hereafter, as hybrid expansion law (HEL)

$$a(t) = a_0 \left(\frac{t}{t_0} \right)^\gamma e^{\xi \left(\frac{t}{t_0} - 1 \right)} \tag{25}$$

where γ and ξ are non-negative constants. Further, a_0 and t_0 respectively denote the scale factor and age of the Universe today. The cosmological parameters: Hubble parameter, DP and jerk parameter are respectively given by:

$$H = \frac{\xi}{t_0} + \frac{\gamma}{t} \tag{26}$$

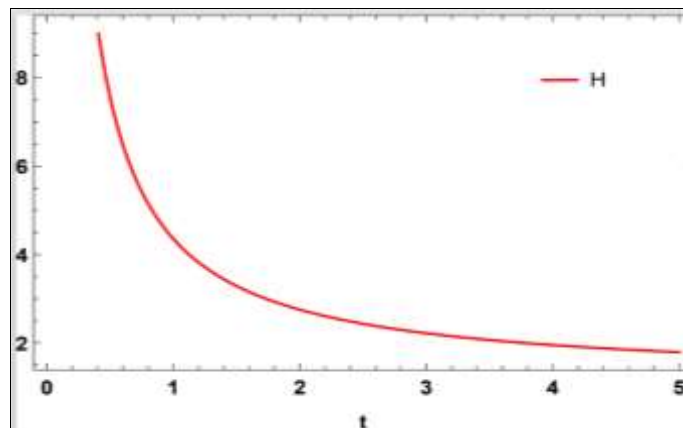


Fig 1: Plot of Hubble Parameter (H) Vs Cosmic time (t)

The plot of the Hubble parameter (H) with respect to time (t) shows that the Hubble parameter steadily decreases approaches towards a small positive value. It is clear that the trajectory of $H > 0$ throughout evolution indicates the expansion of the universe, but at a slow rate.

$$q = \frac{\gamma t_0^2}{(\xi t + \gamma t_0)^2} - 1 \tag{27}$$

$$j = \frac{(2t_0 - 3\xi t - 3\gamma t_0)\gamma t_0^2}{(\xi t + \gamma t_0)^3} + 1 \tag{28}$$

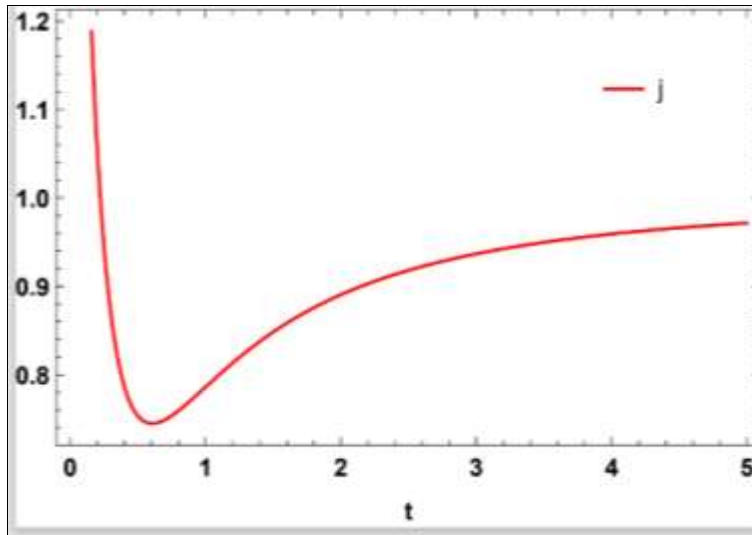


Fig 2: Plot of the deceleration parameter (q) vs. cosmic time (t)

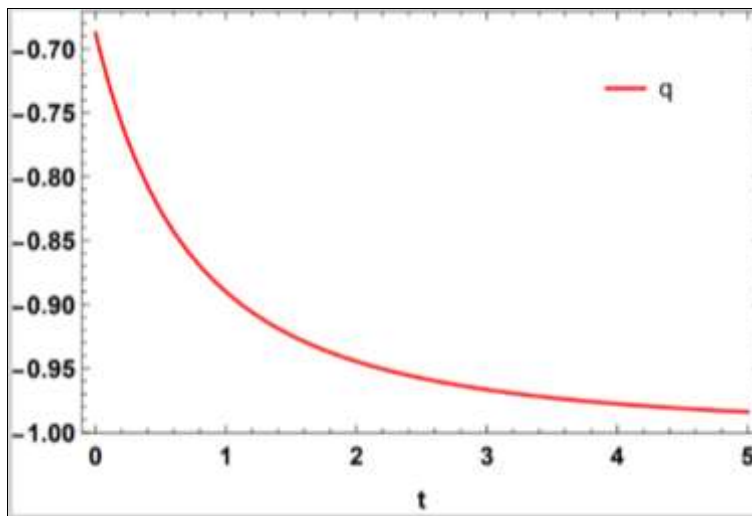


Fig 3: Plot of Jerk parameter (J) Vs. Cosmic time (t)

Figure 2 shows the behaviour of the deceleration parameter q with respect to cosmic time t . It is observed that the value of $q < 0$ throughout evolution which indicates, that the universe remains in accelerated expansion mode.

The plot for jerk parameter (j) versus cosmic time (t) is given by Figure 3 suggests that the current value of the jerk parameter is close to 1 which is consistent with the standard Λ CDM model. But it's sudden deviation indicates active exploration to test for modified gravity.

Now from the equation (15), (23) and (25) we get the value of the metric potential as

$$s(t) = \left[a_0 \left(\frac{t}{t_0} \right)^\gamma e^{\xi \left(\frac{t}{t_0} - 1 \right)} \right]^{\frac{3}{1+4n}} \tag{29}$$

$$h(t) = \frac{3n}{1+4n} \left[\log a_0 + \gamma \log \left(\frac{t}{t_0} \right) + \xi \left(\frac{t}{t_0} - 1 \right) \right] \tag{30}$$

Using the metric potential (29) and (30) in space-time (14), we can write

$$ds^2 = R^{\frac{6n}{1+4n}}(dt^2 - dr^2 - r^2 d\theta^2 - R^{\frac{6}{1+4n}} ndz^2) \tag{31}$$

The non-metricity scalar Q using equation (18), (29) and (30) is obtained as

$$Q = - \left[\frac{18n(n+2)}{(1+4n)^2} \left(\frac{\xi t + \gamma t_0}{t t_0} \right)^2 \right] e^{-2h} \tag{32}$$

We consider the Hubble Horizon ($L = H^{-1}$) to be the system's infrared cut-off, where H is the model's Hubble parameter. Consequently, the Borrow holographic dark energy density in symmetric teleparallel gravity using equation (19) converted to the following form:

$$\rho_B = CH^{2-\Delta} \tag{33}$$

The motives for using this new formula introduced by Borrow is that he thinks it is important in compensating the evolution of the universe for a large horizon length L , particularly an anisotropic universe. Using equations (26) and (33) we get the Barrow density as

$$\rho_B = C \left(\frac{\xi t + \gamma t_0}{t t_0} \right)^{2-\Delta} \tag{34}$$

The energy density of the matter using equation (22), (24), (32) and (34) is calculated as:

$$\rho_m = \frac{9\lambda n(n+2)}{(1+4n)^2} \left(\frac{\xi t + \gamma t_0}{t t_0} \right)^2 e^{-2h} - C \left(\frac{\xi t + \gamma t_0}{t t_0} \right)^{2-\Delta} \tag{35}$$

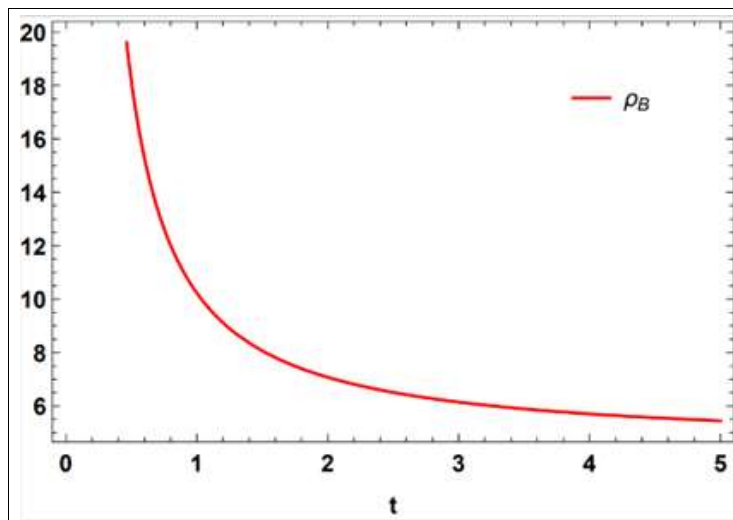


Fig 4(a): Plot of Barrow density (ρ_B) vs. Cosmic time (t)

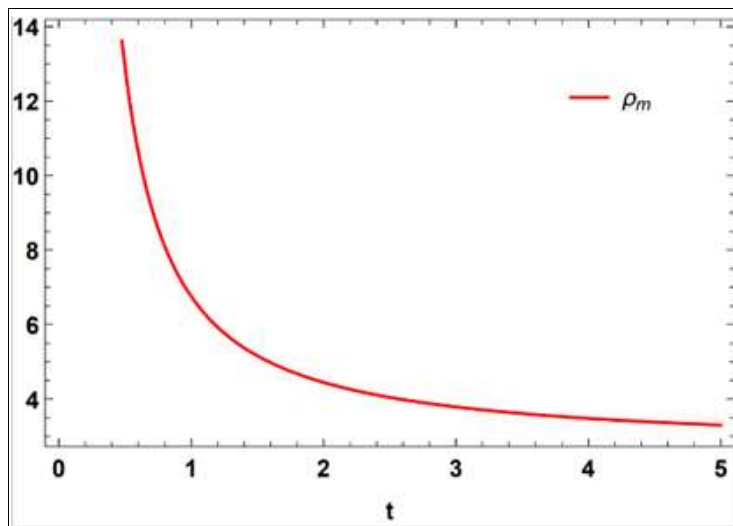


Fig 4(b): Plot of matter density (ρ_m) vs. Cosmic time (t)

The plot of energy density (ρ_B) of Barrow holographic dark energy and energy density (ρ_m) of matter versus time are given in the figures 4(a) and 4(b) respectively. It is clear that both energy densities decrease over time (t), which aligns well with standard model.

The equation of state parameter of Barrow holographic dark energy from equations (20), (24) and (34) is given by

$$\omega_B = \frac{9\lambda(n+2)}{c} \left(\frac{\xi t + \gamma t_0}{t t_0} \right)^{\Delta+2} \left[\frac{\{n(n+2)e^{-2h} - 4n^2 - 4n + 1\}(\xi t + \gamma t_0)^2 + 3\gamma t_0}{(1+4n)^2 t^2 t_0^2} \right] \tag{36}$$

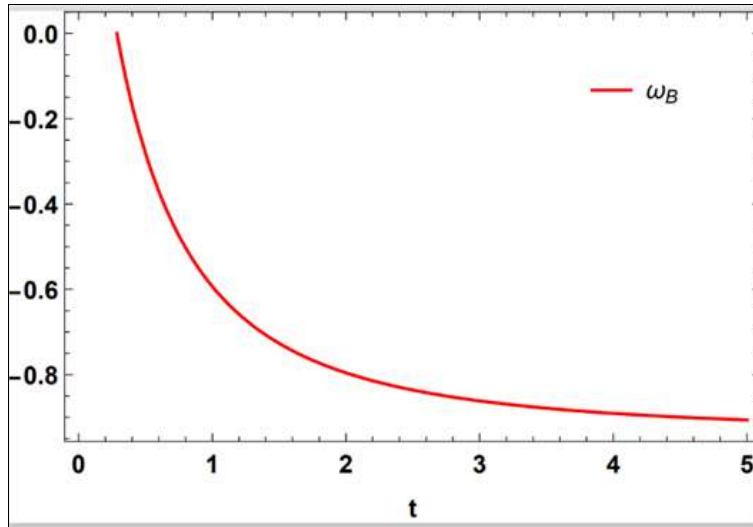


Fig 5: Plot of Barrow EoS parameter (ω_B) vs. Cosmic time (t)

Figure 5 represents the plot of Barrow EoS (ω_B) holographic dark energy with respect to time. The negative trajectory is due to the presence of DE. It does not cross -1 as time evolves, which represents that the given model does not cross the phantom divide line and lies in the quintessence era. At the late time, it may be predicted that the model gets aligned with the de-Sitter model.

The skewness parameter δ can be calculated as

$$\delta = \frac{9\lambda}{c} \left(\frac{\xi t + \gamma t_0}{t t_0} \right)^{\Delta+2} \left[\frac{9\{4n^2 + 4n + 1 - 2n(2n+1)e^{-2h}\}(\xi t + \gamma t_0)^2 - 27\gamma t_0 + 2n(1+4n)\gamma t_0^2 e^{-2h}}{(1+4n)^2 t^2 t_0^2} \right] \tag{37}$$

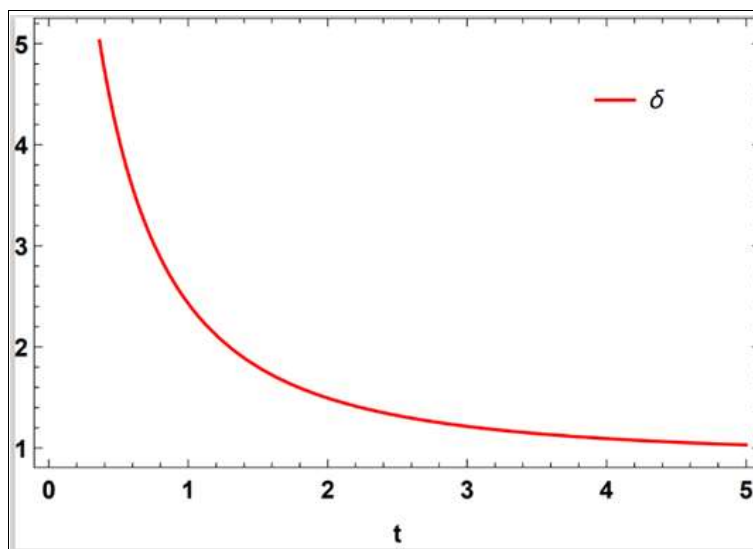


Fig 6: Plot of Skewness parameter (δ) vs. Cosmic time (t)

5. Conclusion

In this study, we explored the Barrow Holographic Dark Energy (BHDE) model within a non-static, plane-symmetric framework under $f(Q)$ gravity. By utilizing the Hubble horizon as the infrared cutoff and adopting a specific functional form $f(Q)$ we derived exact solutions to the field equations and investigated various cosmological parameters to understand the model's behavior and implications.

The analysis of the Hubble parameter (H) reveals a consistent decrease over cosmic time, confirming the ongoing expansion of the universe. This expansion, characterized by the positive values of the Hubble parameter throughout the evolution, indicates the universe remains in an accelerated phase, which aligns well with observational data.

The deceleration parameter (q) demonstrates a negative value across the cosmic timeline, supporting the accelerated expansion phase. This behavior highlights the model's effectiveness in describing the late-time dynamics of the universe, where dark energy dominates. Furthermore, the decreasing energy densities of BHDE (ρ_B) and matter (ρ_m) over time signify a smooth transition towards a stable phase of cosmic evolution, consistent with standard cosmological models.

The equation of state (EoS) parameter (ω_B) remains negative throughout the evolution, reflecting a quintessence-like behavior. Importantly, the parameter does not cross the phantom divide, suggesting the model does not enter unphysical regions and may converge towards a de Sitter state at late times. The dynamics of the skewness parameter (δ) capture the anisotropic nature of the universe, while the jerk parameter aligns with Λ CDM at present, with deviations hinting at potential signatures of modified gravity.

In conclusion, the BHDE model under $f(Q)$ gravity offers a compelling framework for examining anisotropic cosmologies. It effectively bridges modified gravity and holographic dark energy theories, enriching our understanding of cosmic evolution. Future work could focus on applying observational constraints and extending the model to include additional forms of entropy or higher-order corrections in $f(Q)$ gravity.

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