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A mathematical approach to smooth marketing effects in product adoption models

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Abstract
The strategic intricacies of technology-driven and service-oriented businesses are sometimes overlooked by traditional diffusion models that highlight pricing as a key adoption driver in the contemporary marketplace. In the current study we suggest a modified diffusion model that takes into account five important strategic factors such as distribution reach, product quality and performance, customer service and after-sales support, advertising intensity, and innovation capabilities. The goal is to create a more thorough, mathematically based model that accounts for the real-world factors that influence decision-making when innovations spread under fixed price conditions. In this study's diffusion model which incorporates five important non-price components. The model is particularly suitable for industries like electric vehicles, smartphones, and digital services etc, where pricing strategies are relatively stable over time.

Keywords: Diffusion model, Cobb-Douglas function, marketing impact optimization, innovation diffusion, smooth approximation, constraints.

1. Introduction

Traditional models of product adoption need to be significantly modified in the quickly changing technological context of today. Traditional diffusion model formulations make the assumption that pricing and advertising changes are the main factors influencing adoption rates. However, a lot of contemporary businesses, such digital services, smartphones, and electric vehicles, increasingly use fixed price methods and prioritize customer service, innovation cycles, and brand awareness. In marketing science and technology management, the spread of innovations and new goods has long been a major focus. Consumer adoption patterns have been effectively captured by pioneering models like the Bass Diffusion Model (Bass, 1969) and its generalized versions over time. Nevertheless, these models frequently make the assumption that the main forces behind adoption are cost and word-of-mouth. This assumption is no longer valid in many contemporary sectors, especially those involving electric vehicles, smartphones, over-the-top (OTT) platforms, and renewable energy technology. Price frequently stays the same throughout a product's lifecycle in today's competitive and innovation-driven market, but non-price elements including advertising, customer service, product quality, innovation rate, and distribution reach significantly influence consumer choices. For example, Apple and Tesla drive adoption by consistent innovation and improved service infrastructure while maintaining pricing consistency.

In the light of above, we suggests a Modified diffusion model that takes into account five important strategic factors -distribution reach, product quality and performance, customer service and after-sales support, advertising intensity, and innovation capabilities . We use smooth approximation approaches that enable gradient-based optimization because our modified diffusion model which is marketing impact function not differentiable at all places. The developed approach directs strategic planning under practical restrictions in addition to estimating the adoption trajectory .This makes it possible to depict the impact of several strategic inputs on the diffusion rate in a realistic and adaptable way. Crucially, this research integrates financial limitations into the model, allowing businesses to ascertain the best way to distribute limited marketing across different tasks.

The theoretical development of diffusion modelling as well as the useful toolbox for marketing managers and policymakers looking to optimize return on investment in markets driven by innovation is both advanced by this research.

2. Literature Review

The foundation of diffusion modeling lies in Bass's (1969) [1] seminal work, which introduced the concept of innovators and imitators to explain new product adoption. Extensions to the Bass model by Dodson and Muller (1978) [2] and Mahajan *et al*. (1990) [3] introduced repeat purchases and multivariate influences. Subsequent research by Horsky and Simon (1983) [4] and Bass *et al*. (1994) [5] highlighted the role of advertising in accelerating diffusion curves. Peres *et al*. (2010) [6] emphasized the social influence component, while Zhou *et al*. (2018) [7] demonstrated how policy and government incentives affect innovation adoption-especially in EV markets. More recent studies, such as Kim and Park (2020) [8], stress the significance of customer service. Bass, Jain, and Krishnan (2000) [9] to find optimization approaches for the GBM that do not require fixing the initial levels of decision variables, we revisit the question of optimal dynamic advertising when a new product diffuses as specified by the GBM. Using a variational approach, we identify both the optimal initial level and the optimal subsequent trajectory of advertising. Krishnan and Jain (2006) [10] handled this challenge by transforming the objective function, turning the constraint on the control variable into a constraint on the state variable. This approach, however, does not allow the identification of the optimal level of marketing support at launch. Mesak and Clark (1998) [11] obtain a somewhat more nuanced result where monotonicity depends on advertising effectiveness. This work lays the groundwork for the present study's modified GBM, which integrates multiple dimensions of marketing and strategic investment to reflect modern consumer behavior and market dynamics.

3. Proposed Model

Let us suppose the new product diffusion follows the GBM structure and that the initial level of advertising is predetermined and not under the control of the decision maker, Krishnan and Jain (2006) [10] reached the following conclusions. (1) If the initial expenditures are low, then the optimal policy is to increase advertising throughout the entire planning horizon. (2) If the initial expenditures are high, then the optimal policy is to first decrease but later increase advertising. In short, the recommendation for how advertising should evolve over time within the GBM hinges critically on the initial level of advertising. The Generalized Bass Model (GBM) and its general form have been extensively used to understand the diffusion of innovative products over time. These models assume that the marketing impact function $x(t)$ primarily depends on price and advertising, implying that fluctuations in price directly influence the consumer decision to adopt a product. But in today's technologically advanced and highly competitive markets, this assumption has limitation. In industrial organization [12] like, electric vehicles (EVs), smart watches, smartphones and other consumer electronics, companies often use a fixed pricing strategy throughout a product's lifecycle. Instead of changing prices, they focus on launching new and improved versions of the product regularly. This approach aims to maintain consumer interest, strengthen brand positioning through price stability, and gain a competitive advantage. For example, companies such as Apple and Tesla usually keep a steady price for each product generation and rely on innovation to boost sales. In this market environment, price is no longer the main factor influencing consumer choices, especially when customers are already comfortable with the price point. So, until a significantly better version is released, consumer adoption mainly depends on non-price factors, particularly advertising intensity and the quality of customer service and support after the sale. Advertising is crucial for raising brand awareness and generating interest among potential buyers. Meanwhile, a strong service system builds consumer trust and satisfaction, leading to positive word-of-mouth and imitation-based adoption. Recognizing this change in behavior, the current study suggests a modified model designed for today's market realities. This study is based on the premise that pricing will not change until the company releases a superior, next-generation product. The marketing impact function $x(t)$ depend on advertising expenditure and market exposure $A(t)$, customer service quality and after-sales support $S(t)$, product quality and performance $Q(t)$, Innovation capability $I(t)$ and distribution reach and availability $D(t)$ at time t. Therefore a general marketing impact function at each time t is given by:

$$x(t) = f(A(t), S(t), Q(t), I(t), D(t))$$

A modified Bass diffusion model [13] based on the proposed model is given by:

$$\frac{d\,n(t)}{dt} = (Z(t) - n(t))[p + q\,\frac{N(t)}{Z(t)}]\,x(t) \tag{1}$$

Where

$$(t) = 1 + \gamma_1 \max\{0, \frac{[A(t)-A(t-1)]}{A(t-1)}\} + \gamma_2 \max\{0, \frac{[S(t)-S(t-1)]}{S(t-1)} + \gamma_3 \max\{0, \frac{[Q(t)-Q(t-1)]}{Q(t-1)} + \gamma_4 \max\{0, \frac{[I(t)-I(t-1)]}{I(t-1)} + \gamma_5 \max\{0, \frac{[D(t)-D(t-1)]}{D(t-1)}] \tag{2}$$

$\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$ are coefficient capturing the percentage increases in diffusion speed resulting from a percentage in increasing in advertising, customer service, product quality, innovation capability and distribution reach availability respectively.

For an industry their goal is to maximize $x(t) > 1$ under a cost constraint for the optimum trajectories to achive the higest adoption influence without exceeding the total marketing budget.

Suppose $f(t) = \max\{0, z(t)\}$ then $x(t)$ is not differential at 0 so $x(t)$ defined in (2) is not differentiable to make this function differentiable we have to use approximation it with

$$x_{smooth}(t) = \sqrt{\{z(t)\}^2 + \epsilon} \tag{3}$$

This function is smooth function so it is differential there for optimization

$$\frac{d}{dt}x_{smooth}(t) = \frac{z(t)z(t)'}{\sqrt{\{z(t)\}^2+\epsilon}}$$

So in our context we define smoothness properties of the marketing impact function $x(t)$ from (3) we get

$$x(t)_\epsilon = 1+\gamma_1\sqrt{(z(t)_A)^2+\epsilon}+\gamma_2\sqrt{(z(t)_S)^2+\epsilon}+\gamma_3\sqrt{(z(t)_Q)^2+\epsilon}+\gamma_4\sqrt{(z(t)_I)^2+\epsilon}+\gamma_5\sqrt{(z(t)_D)^2+\epsilon} \quad (4)$$

Where

$$z(t)_A = \frac{A(t)-A(t-1)}{A(t-1)},$$

$$z(t)_S = \frac{S(t)-S(t-1)}{S(t-1)},$$

$$z(t)_Q = \frac{Q(t)-Q(t-1)}{Q(t-1)},$$

$$z(t)_I = \frac{I(t)-I(t-1)}{I(t-1)},$$

$$z(t)_D = \frac{D(t)-D(t-1)}{D(t-1)}$$

Now differentiating (4) with respect to $A(t)$

$$\frac{dx(t)}{dA(t)_\epsilon} = \gamma_1\frac{z_A(t)}{\sqrt{\{z_A(t)\}^2+\epsilon}}\cdot\frac{d}{dA(t)}z_A(t)$$

$$\frac{dx(t)}{dA(t)_\epsilon} = \gamma_1\frac{z_A(t)}{\sqrt{\{z_A(t)\}^2+\epsilon}}\cdot\frac{d}{dA(t)}\left[\frac{A(t)-A(t-1)}{A(t-1)}\right]$$

$$= \gamma_1\frac{z_A(t)}{\sqrt{\{z_A(t)\}^2+\epsilon}}\cdot\frac{1}{A(t-1)}$$

Similarly we can find

$$\frac{dx(t)}{dS(t)_\epsilon}, \frac{dx(t)}{dQ(t)_\epsilon}, \frac{dx(t)}{dI(t)_\epsilon}, \frac{dx(t)}{dD(t)_\epsilon}$$

then the updated equation

$$A^{k+1}(t) = A^k(t) + \Omega\,\gamma_1\frac{z_1^k}{\sqrt{\{z_1^k(t)\}^2+\epsilon}}\frac{1}{A^k(t-1)}$$

Where

$$z_1^k(t) = \frac{A^k(t)-A^k(t-1)}{A^k(t-1)}$$

$$A^{k+1}(t) = A^k(t) + \Omega\,\frac{dx(t)}{dA(t)_\epsilon}$$

$$S^{k+1}(t) = S^k(t) + \Omega\,\frac{dx(t)}{dS(t)_\epsilon}$$

$$Q^{k+1}(t) = Q^{k}(t) + \Omega \frac{dx(t)}{dQ(t)_{\epsilon}}$$

$$I^{k+1}(t) = I^{k}(t) + \Omega \frac{dx(t)}{dI(t)_{\epsilon}}$$

$$D^{k+1}(t) = D^{k}(t) + \Omega \frac{dx(t)}{dD(t)_{\epsilon}}$$

We can use smooth form of function x(t), where $\Omega, k, \in$ are learning rate (small value 0.01), iteration index ($k = 1,2,3,4,5,6,7,8,9 \ldots\ldots\ldots..$) and smoothing parameter (very small $.0000001$) respectively.
Therefore, for achieving optimum trajectory smooth marketing impact function is

$$\max x(t)_{\epsilon} = 1 + \gamma_1 \sqrt{(z(t)_A)^2 + \in} + \gamma_2 \sqrt{(z(t)_S)^2 + \in} + \gamma_3 \sqrt{(z(t)_Q)^2 + \in} + \gamma_4 \sqrt{(z(t)_I)^2 + \in} + \gamma_5 \sqrt{(z(t)_D)^2 + \in}$$

Subject to constraints,

$$c_1 A(t) + c_2 S(t) + c_3 Q(t) + c_4 I(t) + c_5 D(t) = b$$

$$A(t), S(t), Q(t), I(t), D(t) \geq 0$$

$c_1, c_2, c_3 c_4, c_5$ are cost per unit inputs for each components and 'b' is the total market budget.
Since the above problem is nonlinear programming problem (NLLP) so we can use Lagrangian method by writing the following Lagrangian function

$$L(A, S, Q, I, D, \lambda) = x(t)_{\epsilon} - \lambda (c_1 A(t) + c_2 S(t) + c_3 Q(t) + c_4 I(t) + c_5 D(t) - B)$$

$$\frac{\partial L}{\partial A} = \gamma_1 \frac{A(t) - A(t-1)}{\{A(t-1)\}^2 \sqrt{(z(t)_A)^2 + \in}} - \lambda C_1$$

Equating $\frac{\partial L}{\partial A} = 0$

$$\gamma_1 \frac{A(t) - A(t-1)}{\{A(t-1)\}^2 \sqrt{(z(t)_A)^2 + \in}} = \lambda C_1 \tag{5}$$

Similarly if $\frac{\partial L}{\partial S}, \frac{\partial L}{\partial Q}, \frac{\partial L}{\partial I}, \frac{\partial L}{\partial D}$ to zero we get

$$\gamma_2 \frac{S(t) - S(t-1)}{\{S(t-1)\}^2 \sqrt{(z(t)_S)^2 + \in}} = \lambda C_2 \tag{6}$$

$$\gamma_3 \frac{Q(t) - Q(t-1)}{\{Q(t-1)\}^2 \sqrt{(z(t)_Q)^2 + \in}} = \lambda C_3 \tag{7}$$

$$\gamma_4 \frac{I(t) - I(t-1)}{\{I(t-1)\}^2 \sqrt{(z(t)_I)^2 + \in}} = \lambda C_4 \tag{8}$$

$$\gamma_5 \frac{D(t) - D(t-1)}{\{D(t-1)\}^2 \sqrt{(z(t)_D)^2 + \in}} = \lambda C_5 \tag{9}$$

From equation (5) (6) (7) (8) & (9) we get

$$\lambda = \frac{\gamma_1}{C_1} \frac{A(t) - A(t-1)}{\{A(t-1)\}^2 \sqrt{(z(t)_A)^2 + \in}} = \frac{\gamma_2}{C_2} \frac{S(t) - S(t-1)}{\{S(t-1)\}^2 \sqrt{(z(t)_S)^2 + \in}} = \frac{\gamma_3}{C_3} \frac{Q(t) - Q(t-1)}{\{Q(t-1)\}^2 \sqrt{(z(t)_Q)^2 + \in}}$$

$$= \frac{\gamma_4}{C_4} \frac{I(t) - I(t-1)}{\{I(t-1)\}^2 \sqrt{(z(t)_I)^2 + \epsilon}} = \frac{\gamma_5}{C_5} \frac{D(t) - D(t-1)}{\{D(t-1)\}^2 \sqrt{(z(t)_D)^2 + \epsilon}}$$

From here we can easily get optimum value $A^*(t), S^*(t), Q^*(t), I^*(t), D^*(t)$ that maximize the marketing impact function without exceeding the total market budget.

4. Application of the Model

4.1. Electric Vehicle (EV) Industry

Because of established manufacturer tactics and legislative incentives, price in the EV market frequently stays constant. Innovation (such as battery range), government subsidies, and service infrastructure are more responsive to consumers. EV awareness campaigns, after-sales service, product quality (range, charging speed), and innovation (autonomous features) are all used in this study to optimize the marketing mix. Evaluate the effects of eliminating subsidies on x(t) and the subsequent slowdown in n(t). policy at the state level comparing media expenditures and infrastructure.

4.2 Smartphone and Consumer Electronics

Companies like Apple and Samsung keep prices constant while emphasizing innovation based on generational differences and retaining customers through quality and service. Using historical sales data and the elasticity of elements like innovation and service support, the current study predicts the acceptance of the new model. When competitors introduce enhancements at the same time, they assess $x(t)$ trends and strike a balance between digital advertising and distribution network coverage.

4.3 Renewable Energy Adoption

Adoption is influenced by more than just price; it also depends on installation service, awareness, efficiency innovation, and regional availability. Create nationwide campaigns to encourage the use of solar power by allocating elasticity-weighted funds for awareness and service training. Use dynamic $x(t)$ to examine adoption under different subsidy timelines.

4.4 Online Platforms and Digital Subscriptions:

Services like SAAS, EdTech, and OTT platforms (like Netflix) frequently prioritize reach, quality, support, and innovative content over cost. Adjust the platform-wide advertising vs. content improvement budget with the aid of the current study. Analyse client attrition against repeat adoption rates and forecast seasonal user growth using time-variant $x(t)$.

4.5 Government Policy Design and Public Goods:

Trust, service accessibility, and communication are more important than price in public programs like UPI (Unified Payments Interface), immunization campaigns, and health insurance enrolment. The model that is being presented assesses the effects of marketing initiatives such as mobile outreach and IEC campaigns. Public adoption is modelled using dynamic $x(t)$, which represents steady market expansion and awareness.

5. Conclusion

This research suggests a mathematically sound diffusion model that incorporates five major non-price factors of marketing-advertising intensity, customer service, product quality, capability for innovation, and reaches for distribution-into a smooth function framework. In contrast to the conventional diffusion models placing primary importance on pricing and word-of-mouth, the suggested model is more in tune with modern, fixed-price markets like electric vehicles, smartphones, digital platforms, and public services. Through the incorporation of smooth approximation methods and optimization, the model allows companies to obtain optimal marketing effect functions given budget constraints. Not only does this help allocate efficient strategic planning, but it also maximizes the marginal utility of every marketing element, which can guide companies in prioritizing investments for highest return on adoption influence. The model's ability to adapt across industries such as EVs, renewable energy, OTT platforms, and government programs proves its usability in real-world scenarios. In addition, through the integration of real-time constraints and elasticity analysis, the model facilitates data-driven marketing strategy, public policy-making, and resource optimization. Subsequent research may expand upon this model by incorporating dynamic variables, probabilistic uncertainty, and nonlinear constraints to manage sophisticated, rapidly changing market conditions. Generally, this paper provides a useful, scalable, and forward-looking tool for companies and policymakers who seek to maximize product adoption in an economy driven by innovation.

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