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An analytical study on Srinivasa Ramanujan's contributions in the field of mathematics

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Abstract
The Indian govt. celebrated $125th$ anniversary of the great Mathematician of Indian soil Srinivasa Ramanujan on 22 December in the year 2012. Without any formal education and extreme poverty conditions, he emerged as one of great mathematician of India. His mathematical ideas transformed and reshaped 20^{th} century mathematics and their ideas are inspiration for $21st$ century mathematicians.
Srinivasa Ramanujan made substantial contributions to the analytical theory of numbers and worked on ‘elliptic functions’, ‘continued fractions’, and ‘infinite series’. He was a great Mathematician, who became world famous at the tender age of twenty-six. He was born into a family that had a humble background and that had no distinguished professional achievement, yet his mathematical ideas transformed and reshaped century mathematics and continues to inspire modern day mathematicians.
Considered to be a mathematical genius, Srinivasa Ramanujan, was regarded at par with the likes of Leonhard Euler and Carl Jacobi. In spite of having almost no formal training in mathematics, Ramanujan’s knowledge of mathematics was astonishing. Even though he had no knowledge of the modern developments in the subject, he effortlessly worked out the Riemann series, the elliptic integrals, hypergeometric series, and the functional equations of the zeta function. The purpose of this paper is to introduce some of the contributions of Srinivasa Ramanujan in the field of mathematics.

Keywords: Excellence, knowledge, integrals, hypergeometric series, results

Introduction

Ramanujan, one of the elegant Mathematician of India was born in Erode on 22nd December 1887.Erode is a small village (In that time), 400 Km away from Tamilnadu’s present capital Chennai. His father was a clerk in Kumbkonam. At the age of five, Srinivasa Ramanujan made his first appearance in school as a student. It was only a matter of time before it came to be known that he had extraordinary talent. He showed flashes of brilliance which were not to be seen in any ordinary kid at that age. He completed his primary education in a couple of years and then went to Town High School for further studies. He showed extraordinary liking for mathematics. When he was yet in school, he mathematically calculated the approximate length of earth's equator. He very clearly knew the values of the square root of two and the pie value. At the age of 16, he got scholarship. But his love only for mathematics cost him the scholarship as he neglected and failed in other subjects.

His loss of scholarship was a great blow to him. He could not afford to study on his own. He had to find work and leave studies for good. He found a job of an accounts clerk in the office of the Madras Port Trust. Despite being rejected two times, his work was recognized by both G. H. Hardy and J. E. Littlewood and he went to England in 1914.

In 1916, he was awarded with a degree of B.Sc. (later named Ph.D.) by Cambridge University for his work on highly composite number. In 1916, when he was at his best while working with his colleagues Hardy & Littlewood, he met with health problems. He was hospitalized in Cambridge and was diagnosed with T.B. and vitamin deficiency. After two years struggle, in 1919, he showed some recovery and he decided to return back to India.

However, the improvement was temporary and after his arrival at Bombay, his health deteriorated again and finally he passed away on 26^{th} April, 1920. His main contributions in mathematics lie in the field of Analysis, Infinite Series, Number Theory & Game Theory. His geniusness was that he discovered his own theorems.

Due to his great achievements in the field of Mathematics, Indian govt. decided to celebrated his birthday 22^{nd} December as Mathematics Day.

ISTE, New Delhi and NBHM, Mumbai have taken initiative to hold Mathematical competition for students as well as teachers of colleges on the name of Srinivasa Ramanujan from 2012 to till date so that students and teachers of India know about the legacy of such great mathematician of India.

Goldbach's Conjecture

Goldbach's conjecture is one of the important illustrations of Ramanujan contribution towards the proof of the conjecture. The statement is every even integer >2 is the sum of two primes, that is, 6=3+3. Ramanujan and his associates had shown that every large integer could be written as the sum of at most four (Example: 43=2+5+17+19).

Theory of Equations

Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quadratic. He derived the formula to solve biquadratic equations. The following year, he tried to provide the formula for solving quintic but he couldn't as he was not aware of the fact that quintic could not be solved by radicals.

Ramanujan-Hardy Asymptotic formula

Ramanujan's one of the major work was in the partition of numbers. By using partition function, he derived a number of formulae in order to calculate the partition of numbers. In a joint paper with Hardy, Ramanujan gave an asymptotic formula for. Infact, a careful analysis of the generating function for leads to the Hardy-Ramanujan asymptotic formula given by

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}},\ n \to \infty$$

In their proof, they discovered a new method called the 'circle method' which made fundamental use of the modular property of the Dedekind η-function. We see from the Hardy-Ramanujan formula that has exponential growth. It had the remarkable property that it appeared to give the correct value of and this was later proved by Rademacher using special functions and then KenOno gave the algebraic formula to calculate partition function for any natural number.

Ramanujan's congruences

Ramanujan's congruences are some remarkable congruences for the partition function. He discovered the congruences

$$p(5n+4) \equiv 0 (mod\ 5)$$

$$p(7n+5) \equiv 0 (mod\ 7)$$

$$p(11n+6) \equiv 0\ (mod\ 11),\ \forall n \in N.$$

In his 1919 paper, he gave proof for the first two congruences using the following identities using Pochhammer symbol notation. After the death of Ramanujan, in 1920, the proof of all above congruences extracted from his unpublished work.

Highly Composite Numbers

A natural number n is said to be highly composite number if it has more divisors than any smaller natural number. If we denote the number of divisors of n by d(n), then we say is called a highly composite if $d(m) < d(n)\ \forall m < n$ where ϵ N. For example, n=36 is highly composite because it has d(36)=9 and smaller natural numbers have less number of divisors. If $n = 2^{k_2}3^{k_3}.....p^{k_p}$ (by Fundamental theorem of Arithmetic) is the prime factorization of a highly composite number then the primes 2,3,……,p form a chain of consecutive primes where the sequence of exponents is decreasing; i.e. $k_2 \geq k_3 \geq \ldots.. \geq k_p$ and the final exponent is k_p is 1, except for n=4 and n=36.

A variant of Fermat's last theorem

Ramanujan's number has another implication also. Pierre de Fermat (1601-1665) stated the famous conjecture that when n > 2, the equation $x^n + y^n = z^n$ has no integral solution except x = y = z = 0. This is popularly known as Fermat's last theorem. A special case of the above equation is $x^3 + y^3 = z^3$. Euler proved that there is no non-trivial integral solution for this equation. A variant of this special case is the equation $x^3+y^3 = z^3 +1$. Ramanujan's number provides the solution $9^3+10^3 = 12^3+1$ to the above Diophantine equation.

The Diophantine equation $X^3 + Y^3 + Z^3 = W^3$

Ramanujan found out the following parametric solution to the Diophantine equation $X^3 + Y^3 + Z^3 = W^3$:

$X = 3a^2 + 5ab - 5b^2$

$Y = 4a^2 - 4ab + 6b^2$

$Z = 5a^2 - 5ab - 3b^2$

$W = 6a^2 - 4ab + 4b^2$

Ramanujan Magic Squares

In his school days, he used to enjoy solving magic squares. Magic squares are the cells in 3 rows and 3 columns, filled with numbers starting from 1 to 9. The numbers in the cells are arranged in such a way that the sum of numbers in each row is equal to the sum of numbers in each column is equal to the sum of numbers in each diagonal. Ramanujan gave a general formula for solving the magic square of dimension 3×3,

C+Q	A+P	B+R
A+R	B+Q	C+P
B+P	C+R	A+Q

Where A, B, C and P, Q, R are in arithmetic progression. The following formula was also given by him.

Applications of Ramanujan's discoveries in science and technology

The mathematical contributions of Ramanujan have also been widely used in solving various problems in higher scientific fields of specialisation. The diverse specialised higher scientific fields include the particle physics, statistical mechanics, computer science, space science, cryptology, polymer chemistry, medical science. Ramanujan's mathematical methods are being used in designing better blast furnaces for smelting metals and splicing telephone cables for communication, as well.

The most celebrated applications of the "Ramanujan conjecture" is explicit construction of Ramanujan graphs by Lubotzky, Philips and Sarnak. The connection of this conjecture with other conjectures of A. Weil in algebraic geometry opened up new areas of research.

Ramanujan developed exceptionally efficient ways of calculating pi that were latter incorporated into computer algorithms, which is also used in various formulae of physics and engineering to describe such periodic phenomenon as the motion of pendulums, the vibration of strings and alternating electric currents.

Ramanujan's Master Theorem is a technique that provides an analytic expression for the Mellin transform of an analytic function. Ramanujan widely used it to calculate definite integrals and infinite series. Higher dimensional versions of this theorem also appear in quantum-physics.

Ramanujan's different identities are used in simple physical models and 2-D models.

Ramanujan Modular functions provide symmetry which gives evidence about possibility of multiple dimensions of universe i.e. 10 or more.

Ramanujan ideas of number theory have the best application in Cryptography.

A new formula, inspired by the mysterious work of Srinivasa Ramanujan could improve our understanding of black holes. Devised by Ken Ono of Emory university in Atlanta, Georgia the formula concerns a type of function called Mock Modular form. These functions are now used to complete the entropy of black holes. This property is linked to the startling prediction by Stephan Hawking that black holes emit radiation.

Conclusion

Ramanujan independently compiled nearly 3,900 results (mostly identities and equations), during his short life. Approximately all his claims have now been proven correct. His original and highly unconventional results for example prime and the Ramanujan theta function, have inspired a vast amount of further research. His excellence can be realized from the fact that he discovered some results which are supposed to be true but have not been proved till date.

His foresightedness was so scintillating that he gave those ideas in Mathematics that no one can imagine to invent them. The facts, achievements and contributions presented by Srinivasa Ramanujan have not just been acknowledged within India, but also globally by leading mathematicians.

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