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Seema Verma
Professor, NITTTR, Bhopal, Madhya Pradesh, India

Savita Kumari Sheoran
Professor, Department of CSE, Indira Gandhi University Meerpur, Rewari, Haryana, India

Rakesh Kumar Sheoran
Research Scholar (Physics) Banasthali Vidyapith, Rajasthan, India

Corresponding Author:
Rakesh Kumar Sheoran
Research Scholar (Physics) Banasthali Vidyapith, Rajasthan, India

Topological quantum error correction with semions

Seema Verma, Savita Kumari Sheoran and Rakesh Kumar Sheoran

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Abstract
Topological quantum error correction is important class of error correction codes which are essential for development of a robust and scalable quantum system. Semion are anyonic particles with bizarre properties have ability to encode the qubit into fusion and braiding properties to protect it from local perturbances. This paper studied the application of semions in topological quantum error correction task. The performance results reveal that semion based codes can reduce the quantum errors below a threshold to facilitate a practicable quantum computing to happen.

Keywords: Quantum computing, quantum error correction, semion code, topological codes

Introduction

The dawn of twenty first century witnessed a powerful technology leveraging the benefits of century long development in quantum mechanics and computer science [1]. Ideally, this promising technology possesses huge potential to bring a phenomenal change in computational landscape but its physical realization is limited by the inherent errors [2]. The errors appearing in quantum systems are more complex and non-trivial in nature than the primitive computational regime [3]. To overcome the issue, various quantum error correction schema has been designed but none of them is an ultimate solution for all type of system. Initially, some simpler codes which can be modelled by Pauli matrices (X,Y,Z,I); have been developed to mitigate the quantum errors but they suffer from the issues such as encoding overhead, decoding complexity, limit of fault-tolerant gates and sensitivity towards depolarizing noise. The initial quantum error codes are uncapable to mitigate the non-Pauli noise $e^{i\theta X}$ which require non-Pauli modelling [1, 4].

Topological quantum error correction (QEC) employs geometrical properties for formulate quantum error correction codes (QECC). Semion are elementary anyon which follow the fractional statistics and bizarre topological properties. Its fusion and braiding properties can be used to encode the qubit in the global topological properties to protect it from quantum errors which are local in nature [5, 6]. This paper studied the QEC ability of semion codes which is essential for development of effective non-Pauli code based on abelian and non-abelian group properties. The rest of this paper is organized as follow. Section 2 present brief background of the study regions. The essentials of topological QEC and properties of semions are presented with their error handling capabilities. Section 3 present the topological QEC using semionic properties. The performance of model is evaluated in term of its ability to reduce the logical errors per unit physical errors. Finally, section 4 concluded the paper along with a projected direction for future research in this regime.

Background

The topological QEC using semion properties is a complex task. The essential elements to explore this innovative concept are presented in sub-sections of this section.

Topological Quantum Error Correction

It is fault-tolerant approach to encode the qubits in to topologically protected states governed by global properties to preserve from local errors and noise. Stabilizer codes such as Toric codes and surface codes are the example of topological codes. Such codes can't be described by Pauli matrices, hence, fall under non-Pauli codes which can be constructed using the non-Clifford gates. Topological QEC relies on stabilizer formalism to describe the QECC.

The stabilizer group (S) can be expressed as Pauli subgroup P_n acting on 'n' qubits as $S = \langle S_1, S_2, \ldots. S_m \rangle$ where S_i is commuting Pauli operator satisfying $S_i^2 = 1 \forall i = 1,2, \ldots. m$. the stabilizer code space will be +1 eigen state of all stabilizer such that $\mathcal{H} = \{|\psi\rangle : s|\psi\rangle \rightarrow |\psi\rangle \; \forall \; s\epsilon S\}$. An error (E) can be detected only if, its anti-commute with any one of the stabilizers such that $sE|\psi\rangle = -Es|\psi\rangle$. The topological codes use the geometrical properties of lattice to place physical qubits at vertices and plaquettes. A vertices stabilizer $(A_v = \prod_{e\epsilon v} X_e)$ measure parity around vertex and plaquette stabilizer $(B_p = \prod_{e\epsilon p} Z_e)$ measures parity around plaquette. Stabilizers are local elements which acts on small cluster of qubits in lattice geometry while the logical operators are non-local wrapping around the geometry of lattice.

Semion and its Topological Properties

Semions are elementary anyons with statistical properties somewhat intermediate of fermion and bosons. These particles possess non-trivial topology and obey fractional statistics capable of braiding the quantum information in global properties to protection from local perturbations (Quantum error). Their exotic statistical properties introduce phase change of $e^{\pm i\pi/2}$ during wave function exchange in 2-dimensional systems [7, 8, 9]. Logical qubits are encoded using the fusion and braiding properties of semions to protect from errors. Few of the important properties of semions codes which useful in QEC are-

Fractional Statistics

The braiding of two semions $(s_1 and\ s_2)$ in 2-dimensional space shift the wave function $(|\psi\rangle)$ by a phase factor of $\pi/2$ *i.e.* $|\psi(s_1, s_2)\rangle = e^{\pm i\pi/2}$. In general term this property can be expressed as equation (i)

$$|\psi(s_1, s_2)\rangle = e^{\pm i\varphi} \qquad \text{(i)}$$

where $\varphi = 0$ for fermions, $\varphi = \pi$ for bosons, $\varphi = arbitrary$ for anyons and $\varphi = \pi/2$ for semions

The braiding of semions in 2-dimensional space depends solely on topological path. From equation (i), the braiding matrix (B) for two semions s_1, s_2 can be described as equation (ii). This braiding matrix represent a unitary transformation behaving as topological gate $U(B) = e^{i\pi}$.

$$B = \begin{bmatrix} 0 & e^{i\pi/2} \\ e^{-i\pi/2} & 0 \end{bmatrix} \qquad \text{(ii)}$$

Similarly, semion group braided with N particles can be represented as $\sigma_i|\psi\rangle = e^{i\pi/2}|\psi\rangle$ and $\sigma_i^{-1}|\psi\rangle = e^{-i\pi/2}|\psi\rangle$ which must satisfy braiding relation $(\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1})$ and commutativity relation $(\sigma_i\sigma_j = \sigma_j\sigma_i \; \forall |i-j| > 1)$.

Fusion Rule

The combination of multiple semions follow a specific topological order. Two semions combine to form a fermion whose further combination can create a vacuum state (1). For instance, let a fermion (f) and semion (s) whose fusion will be held as per equation (iii).

$$s \times s = f, s \times f = s, f \times f = 1, s \times 1 = s \qquad \text{(iii)}$$

This fusion rule describes algebraic structure of semion which can be used to encode the qubit to protect from local errors.

Self-Exchange

The complete rotation of semion $(\varphi = 2\pi)$ introduce a phase factor of equal to fermion with entirely different rotational properties *i.e.* $|\psi\rangle \rightarrow e^{2i\varphi}|\psi\rangle \rightarrow e^{i\pi} \rightarrow -|\psi\rangle$.

Chirality

The exchange of semion by phase of $\pm\pi/2$ assign a preferred direction during braiding. The phase of $+\pi/2$ offer clockwise (right) alignment while phase of $-\pi/2$ introduce left alignment. It means semions are not identical in mirror image and possess inherent chirality while the bosons and fermions are achiral.

QEC with Semion

The process of quantum error correction involve encoding, syndrome measurement and decoding as essential steps. The topological nature of semionic braiding protect information from local interactions [7, 9]. Semion systems can be explained by modular tensor categories with fusion and braiding properties as depicted in equation (iv).

$$S = \begin{bmatrix} 1 & \sqrt{-1} \\ \sqrt{-1} & 1 \end{bmatrix} \text{ and } T = \begin{bmatrix} 1 & e^{i\pi} \\ 0 & 1 \end{bmatrix} \qquad \text{(iv)}$$

The higher dimensional Hilbert space for logical qubit can be constructed by manifolding the ground state Hamiltonian of 2-dimensional topological ordered system. Let a lattice $(\mathcal{L})$ encodes qubits at 'N' sites, then Hilbert space will be $\mathcal{H}_{semion} = span\{|s_1, s_2, \ldots\ldots. s_N|\}$

where $s_i \in \{0,1\} \; \forall \; 0 = presence\ and\ 1 = absence\ od\ semion\ on\ sites$. The logical qubits are encoded into topologically generated ground states such that $|0_L\rangle, |1_L\rangle \in \mathcal{H}_{logical} \subset \mathcal{H}_{semion}$ [8]. The final Hamiltonian for topological QEC can be obtained by 2.2. Levin-Wen String-Net model as presented in equation (v).

$$\mathcal{H} = -\sum_v A_v - \sum_v B_p \qquad \text{(v)}$$

The basis states for semion codes can be constructed as per the semion locations on the sits such that $|\psi\rangle = \sum_n c_n |S_n\rangle$ where $|S_n\rangle$ is semion configuration and c_n is its co-efficient. To improve the robustness topological entanglement entropy is introduced such that $S_{Topo} = \ln \mathcal{D}$ where '$\mathcal{D}$' signify the total quantum dimensions $\mathcal{D} = \sqrt{\sum_a d_a^2}$. The degeneracy of code increases with genus (g) of surface. For semionic codes it scales upto 2^g similar to Toric codes as shown in Figure 1. The ground state degeneracy for a d-dimensional quantum system will be $\mathcal{D} = d^{2g}$.

The semion error correction model can be characterized braiding and fusion properties. Two semions fuse into 1-dimensional vacuum space through fusion operator $F(|s_1\rangle, |s_2\rangle) = |1\rangle$. The fusion co-efficient can be expressed as $N_{ss}^1 = 1$ and $N_{s1}^s = 1$ Similarly braiding operator acting on semion pair can be represented as

$B(|s_1\rangle, |s_2\rangle) = e^{i\pi}|s_1, s_2\rangle$ which satisfy Young-Baxter equation $B_1B_2B_1 = B_2B_1B_2$ where B_1 and B_2 are operator acting on different semions. The braiding operation with 'R'matrix can be represented as $R^{ss} = e^{i\pi}$ and $R^{s1} = 1$. The permutation operator exchanging particle '*i*' and '*j*' for semion apply as $B_{i,j} = R_{i,j}P_{i,j}$ which is converted into $B = e^{i\pi}.P$. The logical error rate in semion codes depend on the number of physical qubits, code distance and errors in the physical qubits. For a code distance 'd' the logical error rate $P_L \propto p^{d/2}$. The decoder interprets the syndrome to construct the error pattern. To prevent logical error to occur error must have the total number of errors less than $(d-1)/2$. The threshold p_{th} on physical error rates can be represented as $p_{th} \approx \text{interpolation of } P_L(d,p)$. Figure 2 depicts the performance of semion based quantum error correction model studies in this research.

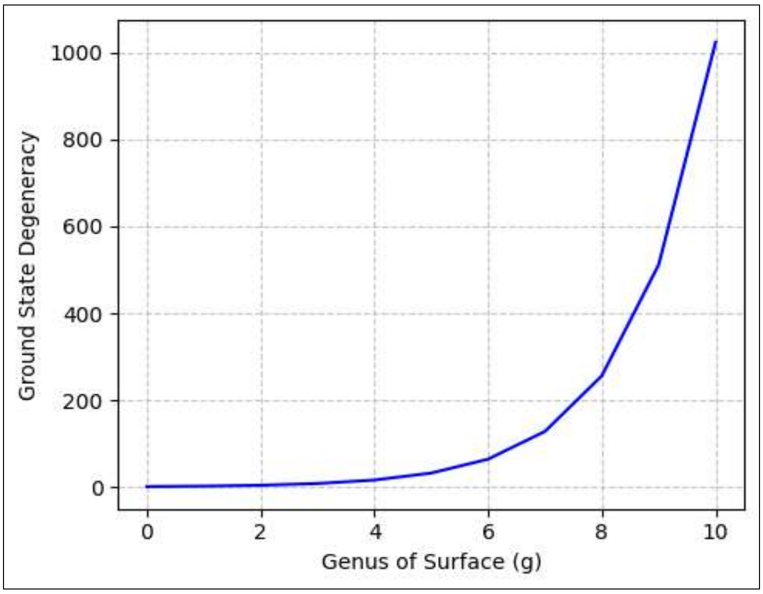

Fig 1: Ground State Degeneracy of Semion Codes versus Genus of Surface

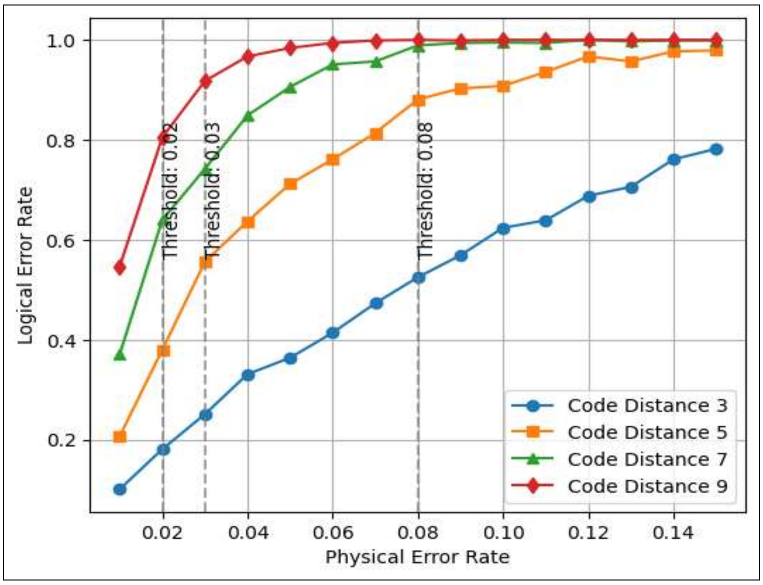

Fig 2: Performance of Semion-based QEC Codes

Conclusion

This research paper explored the QEC ability of semionic codes based on the bizarre fractional statistical properties. The study reveals that the topology of semion codes support the encoding of quantum information in global topological properties. It is observed that semion support easy formulation of Hamiltonian, Hilbert space and degeneracy to effectuate the computational model. The ground state degeneracy in semion model depends on genus surface and semion based codes such as Toric codes possess vast space to enhance the degeneracy of quantum states. Further, semion model effectively mitigate the logical errors below a threshold for quantum computing to happen in a robust and scalable manner. However, this research presented deep theoretical analysis about applicability of semionic properties for quantum error correction but decoding the QECC with fractional statistics is a herculean task. However, there is a ray of hope in neural network which is data-centric learning approach to solve the complex problems through statistical predictions. Therefore, exploring the neural network for semion decoding may yield a fruitful technological outcome. We are planning to implement semion model with neural network approach in our future researches.

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