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Non-static plane symmetric bulk viscous dark energy cosmological model

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Abstract
In this paper, we have investigated Non-static plane symmetric universe in the presence of pressure less dark matter and bulk viscous fluid of dark energy within the framework of general relativity. To find the solution of Einstein's field equations, we assumed that the hybrid expansion law. The physical and geometric properties of the cosmological model are also well discussed.

Keywords: Non-static plane symmetric metric, bulk viscous fluid, dark energy, hybrid expansion law

1. Introduction

The appearance of new cosmological models is connected with the discovery of the accelerated expansion of the universe. Cosmic acceleration can be introduced via dark energy or via modification of gravity (Nojiri and Odintsov [1]). A general review of dark energy cosmology was given in Bamba *et al.* [2]. Dark energy (DE) should have strong negative pressure and can be characterized by an equation of state parameter (EoS). Cosmological models that treat dark energy and dark matter as imperfect fluids with unusual equation of state are considered in Nojiri and Odintsov [3, 4], where viscous fluids are just one particular case. Bulk viscous cosmology is also an alternative to gravity modifying theories (Nojiri and Odintsov [1]) in that it alters the right hand side of Einstein' s field equations instead of the left hand side. Bulk viscosity characterizes deviations from local equilibrium which modifies the energy-momentum tensor. It is necessary to take into account viscosity effects when considering turbulence (Brevik *et al.* [5]) or other realistic situations.

Dissipative dark energy models in which the negative pressure, which is responsible for the current acceleration, is an effective bulk viscous pressure have been proposed in order to avoid the occurrence of the big rip (Barrow [6]; McInnes [7]). Influence of bulk viscosity in the cosmic fluid plays an important role in the big rip phenomenon (Brevik *et al.* [8]). In this scenario which is based on the Eckart theorem (Eckart [9]), we consider the DE fluid with viscous. Evolution of the universe involves a sequence of dissipative process. Recently, Velten *et al.* [10] have investigated phantom DE as an effect of bulk viscosity. It is worth noting that Brevik and Gorbunova [11] show that fluid which lies in the quintessence region can reduce its thermodynamical pressure and cross the barrier ωde = -1, and behave like a phantom fluid with the inclusion of a sufficiently large bulk viscosity. In recent years, many researchers have been inspired by the study of cosmological models with bulk viscous dark energy within the framework of general relativity. The theory of Einstein' s general relativity (GR) was only successful in describing the universe gravitational phenomena.

The contribution of bulk viscosity to the cosmic pressure plays the role of accelerating the universe. In an expanding system, relaxation processes associated with bulk viscosity effectively reduce the pressure as compared to the value prescribed by the equation of state. For a large bulk viscosity, the effective pressure becomes negative and could mimic a dark energy behavior. Recently Planck collaboration revealed that this property of isotropic and homogeneity of the universe is well defined by the CDM model in the FRW geometry. However, at low multi-poles the Λ CDM cosmology shows a poor fit to the CMB temperature power spectrum. This indicates that the isotropy and homogeneity were not the essential features of the early universe.

Moreover, the recent Planck data results motivate us to construct and analyze the cosmological models with anisotropic geometry to get a deeper understanding on the evolution of the universe. In this regard, Non-static plane symmetric space-time is of fundamental importance since it provides the stipulation framework. The bulk viscous driven inflation leads to a negative pressure term, which in process results in repulsive gravity and ultimately became a cause for the rapid expansion of the universe (Tripathy *et al.* [12]; Maartens [13]; Lima *et al.* [14]). The contribution of bulk viscosity to the cosmic pressure plays the role of accelerating the universe. In an expanding system, relaxation processes associated with bulk viscosity effectively reduce the pressure as compared to the value prescribed by the equation of state. For a large bulk viscosity, the effective pressure becomes negative and could mimic a dark energy behavior. Brevik *et al.* [15] have investigated viscous cosmology in the early universe for both homogeneous and inhomogeneous EoS and examined the bulk viscosity effects on the various inflationary observables. Since viscosity appears to be an important dissipative phenomena in Friedman-Robertson-Walker cosmology, therefore it is expected that cosmological model with bulk viscosity fluid would produce some results in the two fluid situations. Moreover, viscosity cosmological model indicates a substantial contribution of bulk viscosity at the inflationary phase (Barrow [16]; Zimdahl [17]; Bafaluy and Pavon [18]).

Inspired by the above mentioned works in the present paper we have obtained a non-static plane symmetric bulk viscous dark energy cosmological model with in the frame work of general reality. This paper is organized as follows. In section 2, we present the metric and the field equations. In section 3, we describe the solutions of the field equations. We discuss some physical and geometrical properties of the model in section 4. Finally, concluding remarks are summarized in section 5.

2. Metric and field equations

Now we consider the non-static plane symmetric metric of the form

$$ds^2 = e^{2A}[dt^2 - dr^2 - r^2 d\theta^2 - B^2 dz^2], \qquad (1)$$

Where A, B and are metric functions of cosmic time t alone.

We consider the universe filled with pressure less dark matter and viscous dark energy fluid. In this case the Einstein' s field equations in gravitational units (8πG=c=1) are given by

$$R_i^j - \frac{1}{2} R\delta_i^j = -\left(T_{(m)i}^j + T_{(de)i}^j\right) \qquad (2)$$

Where $T_{(m)i}^j$ and $T_{(de)i}^j$ are the energy momentum tensors of dark matter and viscous DE fluid, respectively. These are given by

$$T_{(m)i}^j = (\rho_m, 0,0,0), \qquad (3)$$

and

$$T_{(de)i}^j = (\rho_{de}, -p_{de}, -p_{de}, -p_{de})$$
$$= diag(1, -\omega_{de}, -\omega_{de}, -\omega_{de})\rho_{de}, \qquad (4)$$

Where ρ_m is the energy density of dark matter, ρ_{de} and p_{de} are, respectively, the energy density and pressure of viscous DE component while $\omega_{de} = \frac{p_{de}}{\rho_{de}}$ is the corresponding EoS parameter. The only change in the formalism because of bulk viscosity is that the thermodynamical pressure with the effective pressure Peff and effective of EoS parameter ω_{eff}, defined as

$$p_{eff} = p_{de} + \Pi; \quad \omega_{eff} = \frac{p_{eff}}{\rho_{de}} \qquad (5)$$

Where $\Pi = -3\zeta H$ is the bulk viscosity pressure, ζ is the coefficient of bulk viscosity. The form of the above equation was originally proposed by Eckart [9] in the context of relativistic dissipative process occurring in thermodynamic systems went out of local thermal equilibrium. Hu and Meng [19], Kremer and Devecchi [20], Cataldo and Cruz [21], Fabris *et al.* [22] have used Eckart approach to explain the current acceleration of the universe with bulk viscous fluid. This motivates us to use Eckart formalism on viscous term, especially when one tries to look at recent acceleration of the universe. Here ω_{eff} is referred to as the effective equation of state parameter of viscous dark energy. Based on Landau and Lifshitz [23] in an irreversible process the positive sign of the entropy changes, ζ has to be positive. In a co-moving coordinate system $(u^i = \delta_0^i)$, Einstein' s field equations (2) with (3) and (4) for non-static plane symmetric bulk viscous dark energy (1) subsequently lead to the following system of differential equations.

$$e^{-2A}\left[2\ddot{A} + \frac{\ddot{B}}{B} + \dot{A}^2 + \frac{2\dot{A}\dot{B}}{B}\right] = -p_{de}$$
$$e^{-2A}\left[2\ddot{A} + \dot{A}^2\right] = -p_{de},$$
$$e^{-2A}\left[3\dot{A}^2 + \frac{2\dot{A}\dot{B}}{B}\right] = \rho_m + \rho_{de},$$

Where the overhead dot denotes ordinary differentiation with respect to cosmic time t. The law of energy-conservation equation $(T_{;j}^{ij} = 0)$ from Eq.(2) yields

$$\dot{\rho}_m + 3\rho_m H = 0, \qquad (9)$$

$$\dot{\rho}_{de} + 3(1 + \omega_{de})\rho_{de} H = 0. \qquad (10)$$

3. Solution of the field equations

The field equations (6)-(8) have three differential equations with five unknowns namely A, B, Pde, Pm and Pde. In order to find a deterministic solution we take the following four physically valid conditions.

The shear scalar (σ) is proportional to expansion scalar (θ), which leads to a relationship between the metric potentials (Collins *et al.* [24]). That is

$$e^A = B^n \qquad (11)$$

Where n ≠ 1 is a positive constant and preserves the anisotropic character of the space time.

We consider hybrid expansion law (HEL) of the scale factor

$a(t)$, given by (Akarsu *et al.* [25]).

$$a(t) = t^{\alpha} e^{t\beta} \tag{12}$$

Where ν and μ are positive constants. The above form of the scale factor is more generalized as it is the product of both exponential and power functions of cosmic time t. and some of the authors who have investigated various DE models in different theories by taking this HEL given in the literature [26, 27].

Now from equations (11) and (12), we obtain the metric potentials as

$$A = \frac{3n}{4n+1}\log\left(t^{\alpha} e^{t\beta}\right) \tag{13}$$

$$B = \left(t^{\alpha} e^{t\beta}\right)^{\frac{3}{4n+1}} \tag{14}$$

Therefore, the metric (1) can be written as

$$ds^2 = \left(t^{\alpha} e^{t\beta}\right)^{\frac{6n}{4n+1}}\left[dt^2 - dr^2 - r^2 d\theta^2 - \left(t^{\alpha} e^{t\beta}\right)^{\frac{6}{4n+1}} dz^2\right]$$

(15)

The pressure is given by

$$\left.\begin{aligned} p_d \quad &= \frac{1}{(4n+1)^2 t^2}\left(18\left(t^{\alpha} e^{t\beta}\right)^{\frac{-6n}{4n+1}}\left(2\left(t^2\beta^2\left(n+\frac{1}{2}\right) + 2\left(n+\frac{1}{2}\right)t\alpha\beta + \alpha\left(\left(\alpha - \frac{4}{3}\right)n + \frac{\alpha}{2} - \frac{1}{3}\right)\right)\right)\right) \\ &\frac{1}{(4n+1)^2 t^2}\left(18\left(t^{\alpha} e^{t\beta}\right)^{\frac{-6n}{4n+1}}\left(n\left(t^{\alpha} e^{t\beta}\right)^{\frac{3n}{4n+1}} + n^2(t\beta+\alpha)^2\left(t^{\alpha} e^{t\beta}\right)^{\frac{6n}{4n+1}} + \frac{\beta^2 t^2}{2} + \beta\alpha t - \frac{2\alpha}{3}\left(n - \frac{3\alpha}{4} + \frac{1}{4}\right)\right)\right)\end{aligned}\right\}$$

The energy density of matter is given by

$$\rho_m = \frac{\kappa_1}{t^{3\alpha} e^{3t\beta}} \tag{17}$$

The energy density of bulk viscous of dark energy is given by

$$\rho_{de} = \frac{1}{(4n+1)^2 t^2}\left(18\left(\frac{3}{2}\left(t^{\alpha} e^{t\beta}\right)^{\frac{6n}{4n+1}} n + \left(t^{\alpha} e^{t\beta}\right)^{\frac{3n}{4n+1}}\right) n(t\beta+\alpha)^2\left(t^{\alpha} e^{t\beta}\right)^{\frac{-6n}{4n+1}} - 16\kappa_1 t^2 t^{-3\alpha} e^{-3\beta t}\left(n+\frac{1}{4}\right)^2\right) \tag{18}$$

4. Physical and geometrical properties of the model

We define the following physical and geometric parameters to be used in formulating the law and further in solving Einstein' s field equations for the metric (1). The spatial volume (V), Hubble parameter (H), expansion scalar (θ), average anisotropy parameter (Am), the shear scalar (σ^2) and deceleration parameter (q) for non-static plane symmetric space-time are defined as

$$V = a^3 = \left(t^{3\alpha} e^{3t\beta}\right), \tag{19}$$

$$H = \beta + \frac{\alpha}{t} \tag{20}$$

$$\theta = 3H = 3\left(\beta + \frac{\alpha}{t}\right), \tag{21}$$

$$A_m = \frac{2(-t\beta n - \alpha n - t\beta - \alpha)^2 + (2t\beta - 4t\beta n - 4\alpha n + 2\alpha)^2}{(4n+1)^2(\beta t+\alpha)^2}, \tag{22}$$

$$\sigma^2 = \frac{(t\beta+\alpha)^2}{t^2}\left[\frac{18n^2+9}{(4n+1)^2} - \frac{3}{2}\right] \tag{23}$$

$$q = \frac{\alpha}{(\beta t+\alpha)^2} - 1, \tag{24}$$

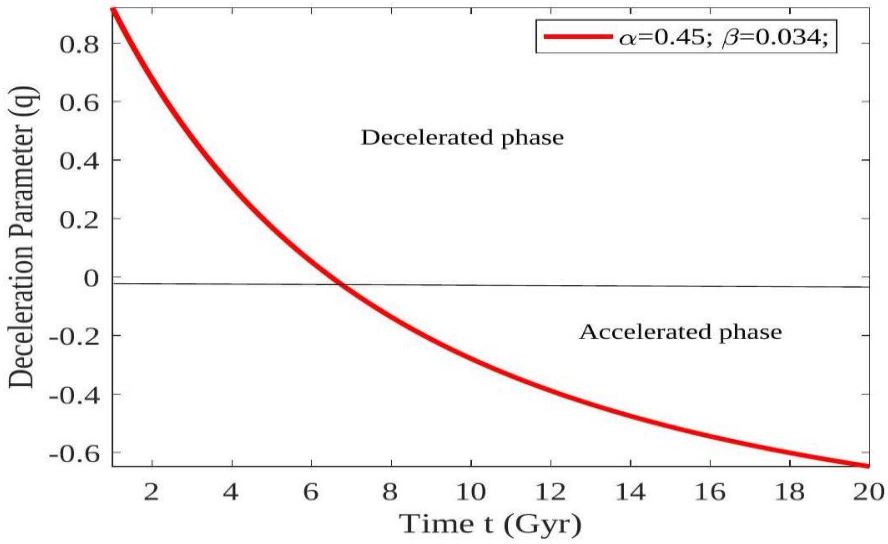

Fig 1: Plot of deceleration parameter versus time t(Gyr)

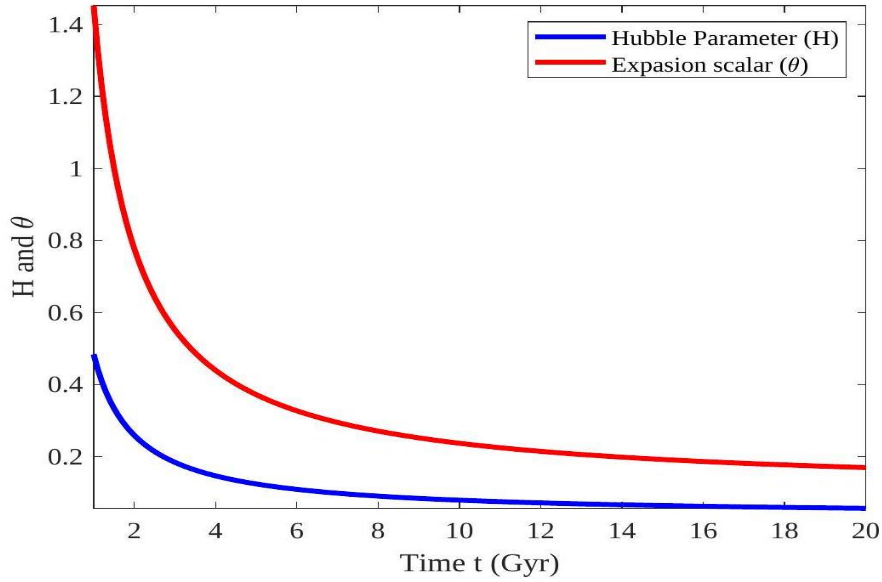

Fig 2: Plot of Hubble parameter and expansion scalar versus time t(Gyr)

From figure (1) we can observe that the deceleration parameter gives a nice transition form early deceleration parameter to present accelerated phase of the universe, which consistency recent observational data. The behavior of average Hubble parameter and expansion scalar versus cosmic
time (t) are plotted in figure (2). It can be seen that both are decreasing functions of time and become constant at late times.

5. Conclusion

In this paper, we have studied non-static plane symmetric universe filled by a pressure less dark matter and viscous dark energy in general relativity. To get the exact solutions of the Einstein' s field equations, we assumed that the hybrid expansion law. We have conclude the following points:

We observed that the spatial volume of the models tend to zero at $t = 0$. At this epoch, all the physical and kinematical parameters diverge. As $t \to \infty$ spatial volume become infinite. As $t \to \infty$, Hubble' s parameter H is constant hence the universe expands forever with constant rate. The mean anisotropy parameter Ah ≠ 0, the models are anisotropic throughout the evolution of the universe. Recent observations of SNe I_a, expose that the present universe is accelerating and the value of the deceleration parameter lies in the range of -1 ≤ q < 0. It is observed that in our model the deceleration parameter is time depended and exhibits a transition from early decelerated phase to current accelerated phase of the universe. It is realistic because of the fact that the energy densities of matter and viscous dark energy are always non-negative and decrease with increasing cosmic time. Finally, the models (15) obtained by using hybrid expansion law provide a very nice description of the transition of the universe from the early deceleration to present cosmic acceleration, which is an essential feature for evolution of the universe.

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