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Quantum interference in three-level atomic configuration and double slit

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Abstract
The main idea of lasing without inversion is the cancellation of absorption which leads to amplification (and laser) even if population inversion is not present. Such a situation is realized in three level atomic configurations where two coherent atomic transitions interfere destructively. In the present work, we describe the salient features where the situation is analogous to the double slit experiment.

Keywords: Quantum interference, double slit, eigen state, antibunched photon

Introduction

In the personal work we make a comparison between the basic concepts and physics behind the double slit experiment and three level atomic configurations producing quantum interference. It is worthy of remark that young double slit experiment is a well-known topic for producing interference. Similarly, three level atomic configuration (Λ-type and V-type) are used to produce quantum interference which leads to lasing without inversion (LWI). When two or more light waves from different sources meet together, then the distribution of energy due to one wave is disturbed by the other. This modification in the distribution of energy due to superposition of two light waves is called interference. Thomas Young devised his well-known double-slit experiment in 1801 to prove that light consists of waves which was proposed by Christian Huygens in the early part of seventeenth century. Nowadays the experiment which was performed by Thomas Young for light waves is also used for electrons, neutrons and even for molecules as big as soccer-ball-like fullerene C_{60} [1-3]. In all these cases are can observe the same kind of interference pattern. Moreover, the interference is observed even if the particles are shot one at a time through the slit. Again, if the double slit apparatus is modified to determine precisely which slit each particle posses through, the interference pattern disappears. The double slit experiment according to Richard Feynman [4], has in it the heard of quantum mechanics. The concept of quantum interference states that elementary particles such as photon can not only be in more than one place at any given time (through superposition) but that an individual particle like photon can cross its own trajectory and interfere with the direction of its path. We expect that in the double slit experiment a single photon will go through one slit or the other and will end up in one of the two possible light line areas on the screen. But that is not what actually happens. As Feynman concluded each photon not only goes through slits, but simultaneously traverses every possible trajectory on route to the screen, not just in theory, but in fact. Although the implications of Young's double slit experiment are somewhat difficult to accept, they have produced reliable proof of quantum interference through repeated trials. The topic of quantum interference has led to many counter intuitive phenomena like coherent trapping and lasing without inversion [5-7]. The main idea of lasing without inversion is the cancellation of absorption which leads to amplification (and laser) even if population inversion is not present. Such a situation can be realized in a three level atomic system. When two coherent atomic transitions interfere destructively, cancellation of absorption takes place. In the present work we describe the salient features of the analogy between double slit experiment and the three level atomic configurations leading to LWI. The root of the analogy is the fact that both double slit experiment and three level atomic schemes are ideally interference experiments. We organize the paper inthe following manner. In section 2 the classical interference of light waves and quantum interference is briefly described to emphasize the characteristic features the basic physics in Λ and V schemes of three level atoms leading to lasing without inversion.

In section 4 a comparative analysis of double slit and three level schemes is presented. In section 5 we present an outlook of the comparative work along with concluding remarks. Summary of the comparison between Young's double slit and Λ-type atomic configuration has also been given in tabular from.

Classical Interference of light waves

For convenience let us consider the interference of light waves as shown in Fig 1 in a standard double slit experiment in which monochromatic plane light waves are normally incident on two narrow parallel slits which are separated by a distance. The light from the two slits is projected onto a screen a distance D behind them where $D \gg d$.

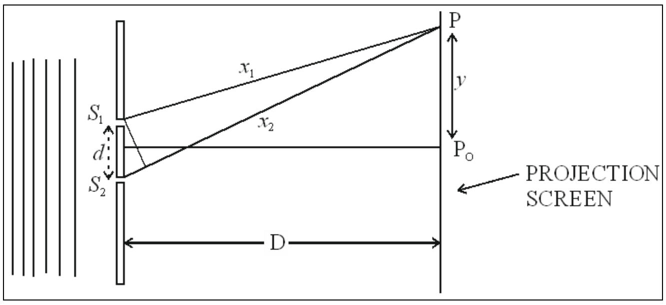

Fig 1: Classical double slit interference experiment of light

Monochromatic light from first slit travels a distance x_1 to reach the point P on the screen located at a distance Y from the central line and light from the second slit travels a slightly longer distance x_2 to reach this point. It is easily shown that

$$x_2 - x_1 = \frac{d}{D} Y \quad \text{.... (2.1)}$$

Provided $d \ll D$, and the well-known fact that light waves are superposible allows us to write the wave function at a point to be written as

$$\psi(y,t) \approx \psi_1 e^{ikx_1} + \psi_2 e^{ikx_2} \quad \text{... (2.2)}$$

where ψ_1 and ψ_2 are the wave functions at the first and second slits respectively, since two slits are assumed to be illuminated by light waves which are in phase and of equal amplitudes.

$$\psi_1 = \psi_2 \quad \text{... (2.3)}$$

It is worthy of remark that we are ignoring the difference in amplitude of the waves from the two slits at the screen, due to the slight difference between x_1 and x_2 compared to the difference in their phases. Thus is a reasonable assumption provided $D \gg \lambda$. It follows that the intensity on the screen at a distance from the central line is

$$I(y) = |\psi(y,t)|^2 \quad \text{... (2.4)}$$

Using equations (1) to equations (4), we have

$$I(y) = \cos^2(k\Delta x/2) \approx \cos^2(kdy/2D) \quad \text{... (2.5)}$$

Fig 2 (a, b) shows the double slit interference pattern which corresponds to the expression (2.5). The pattern consists of equally spaced bright and dark bands of characteristic width

$$\Delta y = \frac{Dy}{d} \quad \text{... (2.6)}$$

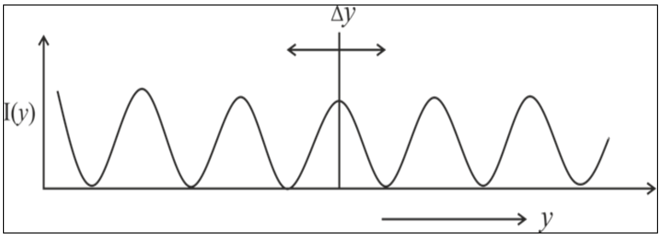

Fig 2 (a) Interference fringes produced by a double slit using the arrangement shown in Fig 1, (b) Interference pattern corresponding to expression (2.5)

From what has been described above we get a familiar view of the double slit experiment which is well known in every introductory book of optics and quantum mechanics is undergraduate level.

Let us now consider the phenomenon of quantum interference in the double slit. Quantum interference is one of the most challenging principles of quantum theory. According to Feynman the essentials of quantum mechanics could be understood from an explanation of the double slit experiment. What is actually happening in the double slit experiment is clearly described by Feynman [4]. In the double slit if one slit is covered the pattern is what would be expected, a single line of light which corresponds to the image of the slit and aligned with whatever slit is open. One would expect that if both slits are open, the pattern of light would reflected the fact, the two lines of light, aligned with the slits. In fact however, what happens is that the photographic film or recorder as the projection screen is entirely separated into multiple lines with alternate bright and dark intensities which we call interference fringes. This is what is also known as interference taking place between waves or particles going through the slits, in what apparently should be two non-crossing trajectories. It is worthwhile to remark here that Feynman [4] must have kept into consideration the assumptions in the double slit that $d \ll D$, $D \gg \lambda$ and presumably d should of the order of millimeters.

We should also expect that if the beam of photons is slowed down enough to ensure that individual photons (or so called antibunched photons) are hitting the photographic film, similar to the experiment of Taylor [10] performed in the 1909, there would be no interference and the pattern of light would be two lines of light, aligned with the slits. In fact, however the resulting pattern still indicates interference which means that single photons are interfering with themselves. Many believe that Dirac's famous remark [11-12] that "each photon interferes only with itself, interference between two photon never occurs" originated from Taylor's experiment. In any case this seems impossible because we expect that a single photon will go through one slit or other and end up in one of the two possible light line areas on the screen. But this is not what is happening. According to Feynman each photon not only goes through both the slits simultaneously but traverses every possible trajectory on way to the target, just not in theory but in fact. In order to see how this might possibly occur, experiments have been performed to track the paths of individual photons. However in these cases what happens is that measurements in some way disturb the photon trajectories and somehow the result of experiments becomes what would be predicted by classical physics. Sudarshan and Rahman [13] re-examined the two slit interferometer and they showed that the standard explanation of two slit interferometer experiment is incorrect because it treats the interference as arising from the photon wave function ψ whereas the interference is really between the coherent states of the field which do not corresponded to the single states. According to Sudarshan and Rahman a close examination of the standard exposition of the two slit experiment reveals several ambiguities with conceptual errors and requires a field theoretic approach. According to them a coherent state is constructed in much the same way as state vectors are assembled in quantum mechanics.

$$|\phi(r)\rangle = \sum_{k} U_k\,|Z_k\rangle$$

when the coherent state $|\phi\rangle$ propagates, the more functions propagates to new mode functions u, since each mode function behaves independently of the other ϕ itself propagates as if were classical as it did in one mode case. The case of many sources-in phases and out of phases is treated in the same way because any state of illumination can be obtained by a suitable weighted average of coherent state. It the sources are intransient phase, they must show transient interference, S Sudarhan specifically indicates this to remind us that many are under the spell of Dirac's famous statement which state that each photon interferes only with itself and interference between two photons never occurs. That this is not true can be verified by anyone who has turned on a car raids to listen to a jammed BBC in Eastern Europe. If one regards the interference as taking place between constitutes a photon disappears. Sudarshan and Rahman further remarks that an experiment will never produce an electric field that corresponds to a single photon and the single photons produced from decays or atomic transitions do not have an electric field associated with them. It is thus reasonable to believe that antibunched photon has no electric field. They also remarked that entire discussion is concerned with for field intensity pattern. If the screen were placed immediately behind the slits one merely records two localized intensities that is no way distinguish between particles and waves. If the screen were placed very close to the slits one finds only two localized uniform lines of light with a very small or no interference pattern between them. In this connection it is worthwhile to discuss the relevant paragraph regarding photon in the text of Sargent, Scully and Lamb In [14-15]. They write "photons are quanta of single (monochromatic) mode of the radiation field and are not localized at any particular position and line within the cavity like fuzzy balls; rather they are spread out over the entire cavity. In fact no satisfactory quantum theory of photons has ever been given. As regards Diracs statement they further add that there is an apparent contradiction in Dirac's statement when it is well known that two separated radius transmitters can produce interference effect and two laser beams can as well produce interference. The difficultly disappears when one remembers that both transmitters are coupled to the modes of the universal radiation field. A photon is simply a particular Eigen state of one such radiation mode. The fields encountered in most problems are not single $|n\rangle$ states but superpositions.

$$|\psi\rangle = \sum_n Cn\,|n\rangle$$

In fact the state vector most nearly corresponding to a classical field in such a superposition and is called the coherent state $|\alpha\rangle$. This has a poisoining distribution among the $|n\rangle$ states that is there exists a number of photons n_p which is most probable and other number are increasingly less probable the more they differ from n_p. It is worthwhile to remark here that Sudarsan's remark on Dirac's statement supports the view expressed in the text of Sargent *et al.* and some other workers [16-17].

Quantum Interference and Lasing without Interference

Lasing without interference (LWI) results from quantum interference. The main idea of LWI is that the absorption is cancelled and the process leads to absorption (and laser) even if the population inversion is not there. Such a situation can be realized in three level system, when two coherent atomic transitions interfere destructively and leads to cancellation of absorption. How this is achieved is shown in following three level atomic configurations as shown in Fig-3.

This configuration is known as Λ (lamda) scheme.

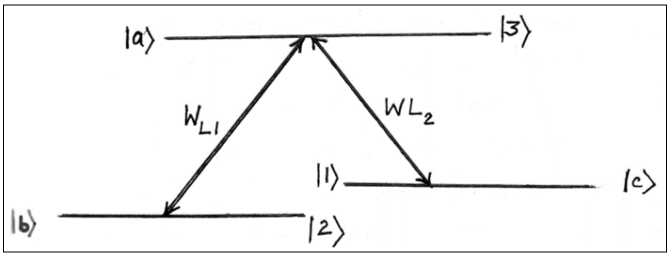

Fig 3: Configuration of a three-level atom interacting with two fields.

The so called Λ configuration is formed by an upper level $|a\rangle$ which is connected to two closely lying lower levels $|b\rangle$ and $|c\rangle$ through interaction with electromagnetic fields E_1 and E_2 respectively, in such a way that only atomic transitions $|a\rangle$-$|c\rangle$and $|a\rangle$-$|b\rangle$ are allowed. The physical reason for canceling absorption is this case is the uncertainty in atomic transition $|c\rangle$-$|a\rangle$and $|b\rangle$-$|a\rangle$which results is destructive interference between them. Since both transitions are directed to the same atomic state $|a\rangle$ it is impossible to find out along which path $|c\rangle$-$|a\rangle$ or $|b\rangle$-$|a\rangle$ such a transition is made. The situation is similar to Young's double slit interferometer, where the interference as a consequence of uncertainty in finding through which of the two slits the photon passed [12]. The absorption probability will be equal to the squared sum of the probability amplitudes corresponding to $|c\rangle$-$|a\rangle$ and $|b\rangle$-$|a\rangle$ transitions. When there is a correlation between these probability amplitudes, it will lead to an interference term which, under appropriate phase relation, can make the total absorption probability equal to zero. The emission probability is equal to the sum of the transition probabilities $|a\rangle$-$|c\rangle$and $|a\rangle$-$|b\rangle$ and is independent of their mutual correlation. This results from the different final states, $|b\rangle$ and $|c\rangle$. In this case there is no uncertainty is atomic transition and therefore there is no interference between these transitions. It may be asserted that the asymmetry between the absorption and emission transition leads to amplification of the system. To work out the absorption and emission probability amplitudes we adopt the method given by Scully and Zubairy in their text Quantum Optics [18]. The method is semi classical. We write the Hamiltonian for the three level atomic systems is written as in notating wave approximation.

$$H = H_0 + H_1 \qquad (3.1)$$

Where,

$$H_0 = \hbar\omega_a |a\rangle\langle a| + \hbar\omega_b |b\rangle\langle b| + \hbar\omega_b |c\rangle c| \qquad (3.2)$$

$$H_1 = \frac{\hbar}{2}(\Omega_{R1} e^{-i\varphi_1} e^{-i\omega_{t1} t} |a\rangle\langle b| + \Omega_{R2} e^{-i\varphi_2} e^{-i\omega_{t2} t} |a\rangle\langle c| + \text{H.C} \qquad (3.3)$$

H_o and H_1 represent the unperturbed and interaction part of the Hamiltonian respectively and

$$\Omega_{R1} e^{-i\varphi_1} = \frac{\wp E_1}{\hbar}, \Omega_{R2} e^{-i\varphi_2} = \frac{\wp E_2}{\hbar} \qquad (3.4)$$

Here $\Omega_{R1} e^{-i\varphi_1}$ and $\Omega_{R2} e^{-i\varphi_2}$ are the Rabi frequencies associated with the interaction of the elecro magnatic fields E_1 and E_2 of frequencies W_{L1} and W_{L2} with atomic transitions $|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$ respectively. The matrix elements of the electric dipole moment corresponding to this transition

$|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$ are given by

$$\wp_{ba} = e\langle b|r|a\rangle; \ \wp_{ba} = e\langle b|r|a\rangle \qquad (3.5)$$

The wave function of this atomic configuration is,

$$|\psi\rangle = C_a(t)|a\rangle + C_b(t)|b\rangle + C_c(t)|c\rangle \qquad (3.6)$$

Let us solve the Schrodinger equation

$$i\hbar|\psi(t)\rangle = H|\psi(t)\rangle \qquad (3.7)$$

Which will give the probability amplitudes C_a, C_b and C_c. Let us consider the initial as $C_a(t=0)$, $C_b(t=0)$, $C_c(t=0)$. Let us express the initial state of the Λ-type three level atomic system as.

$$C_a(0)=0,\ C_b(0)=\frac{1}{\sqrt{2}},\ Cb(0)=e^{(-i\psi)}/\sqrt{2} \qquad (3.8)$$

This means that the population of the lower two levels which are closely lying with fixed phases between them.

Thus, the solution of Schrodinger's equation for the set of initial conditions (3-6) gives the following result for the probability amplitude of the upper level |a>.

$$C_n(t) = i^{x = \frac{t}{2\sqrt{2}}} (\Omega_{R1} e^{(-i\varphi_1)} + \Omega_{R2} e^{-(i\varphi_2+\psi))} \qquad (3.9)$$

In this equation the first and second terms of the sum represent the probability amplitudes corresponding to the transition from |b> to |a> and |c> to |a> respectively. The absorption probability in this case.

$$C_a(t)|^2 = P_a = t\,^2\Omega_R^2[1+\text{Cos}(\varphi_1-\varphi_2-\psi)]/4 \qquad (3.10)$$

Where we have taken $\Omega_{R1}=\Omega_{R2}=\Omega_R$. From equation (3.10) we find that the absorption probability Pa=0, when the phase condition is such that $\varphi_1-\varphi_2-\psi = \pm\pi$. Under such conditions the atomic system will stay at lower energy levels |b> and |c>at all times since there are no transitions to higher energy levels this system will exhibit no absorption at least for the specific condition $\theta = \varphi_1-\varphi_2-\psi = \pm\pi$.

Now let us find the emission probability. Let us suppose that initially the population is in the upper state, i.e, $C_a(0)=1$, $C_b(0)=0$ and $C_c(0)=0$. The solution of Schrodinger equation (3-7) for these initial conditions and assuming.

$$(\Omega_{R1}+\Omega_{R)}{}^2\, t = \Omega_1 << 1$$

Gives the following result approximately

$$C_b(t) = i\Omega_{R1}{}^* t/2, \quad C_b(t) = i\Omega R_2{}^* t/2, \qquad (3.11)$$

The emission probability is equal to the sum of the squared probability amplitudes related to the atomic states |b> and |c>

$$P_{\text{emission}} = |C_b(t)|^2 + |C_c(t)|^2 = \frac{\Omega 2\ t2}{4} \qquad (3.12)$$

Thus, we observe that the emission probability is independent of the relative phase between atomic states between |b> and |c> because in this case first the probability amplitudes are squared and then summed. But for the case of absorption first the probability amplitudes are squared and then summed. That is why the absorption probability is mathematically depended on the relative phase between atomic transitions. One can identify from Eqn (3-12) that the emission probability is always non zero for t > 0. Therefore, if the atomic system is prepared with the phase condition asdescribed above, it is possible to have net gain even when there is no population inversion. This leads to the process of lasing without inversion which have been demonstrated experimentally.

The Analogy and Comparison

From what has been described in the preceding sections we have the materials in hand requires to make a comparison between the two situations present in double slit and Λtype atomic configuration. Before placing the comparison in a tabular from let us discuss the salient features, consider the basic topic of slit width. A simple demonstration of young's experiment can be made by constructing a double slit in an exposed film by drawing the paint of sharp needle across the film guided by a straight edge. A source of light such as a bulb is now viewed by holding the double slit close to the eye and looking at the source, if the slits are close together, for example 0.2 mm apart they give widely spaced fringes, whereas slits are apart, for example 1mm, give very narrow fringes, when the double slit separation is aboard 1.5 mm, one can observe slit narrower fringes. In this the retina of the eye acts as the projection screen. A piece of red glass placed adjacent to and above another of green glass in front of the lamp source will show that the red waves produce wider fringes than the green which is due to their greater wavelengths. This situation is also described by Klauder and Sudarshan [14] in another manner according to them, on the basis of standard classical wave theory a single wave incident on the two slits undergoes to complete destructive interference at various points of observation of the projection screen. The possibility of such destructive interference is closely related to the existence of a definite phase relationship between the constituent signals and under these circumstances we may say that these two beams are coherent. But this picture of complete destructive interference may not be in full agreement with the experiment results of observation for usual thermal source as the angle subtended by the source at the projection screen is not too small. When such source is moved inward toward the screen, thus increasing the angle subtended, it usually occurs that the interference pattern gradually washes out and under these circumstances the maximum and minimum intensities become less pronounced. As a quantitative measure of this aspect Sudorshan has introduced, following Michelson the parameter visibility defined as.

V= (I max-Imin)/(Imax+Imin)

Where I_{max} and I_{min} are the intensities at the maxima and minima fringe pattern. The more slowly V decreases with increasing path difference the sharper the line. With the cadmium line it dropped to 0.5 at a path difference of 10 cm or at d=5 cm. with certain lines the visibility does not decrease uniformly but fluctuates more or less regularly which indicates that the line has a fine structure.

Let us now try to analyze what has been discussed above are compatible with the observations in a three level (Λ-type) atomic system. Let us consider the slit width. In case of double slit experiment using light in the visible sector of the spectrum the slit width about 0.2mm to 1 mm. But the energy level separation between two closely lying levels in on actual Λ-system such as the hyperfine energy level separation in Na atom is 1770 MH$_Z$ as shown in Fig 4. In this figure the relevant energy levels of Na atom showing hyperfine structure in a weak magnetic field are indicated. From Fig4 it may be inferred that the separation of the energy levels corresponding to the separation of the Young's double slit experiment is very small. As shown in Fig 4 the separation 1770 MH$_Z$ (or 000004454 eV) is identical to the well-known Lamb shift separation of 1057 MH$_Z$ (or 0.000004372 eV) of hydrogen atom corresponding to the separation of the levels $^2S_{1/2}$-$2p_{1/2}$. It will of sufficient interest to investigate whether such separation may be used to produce quantum interference in hydrogen atom provided a Λ-configuration is created. In the case of Young's double slit we have the projection screen where interference pattern appears. As an analogous situation in the Λ-configuration we may reasonably infer that the projection screen is at the position of the energy level.

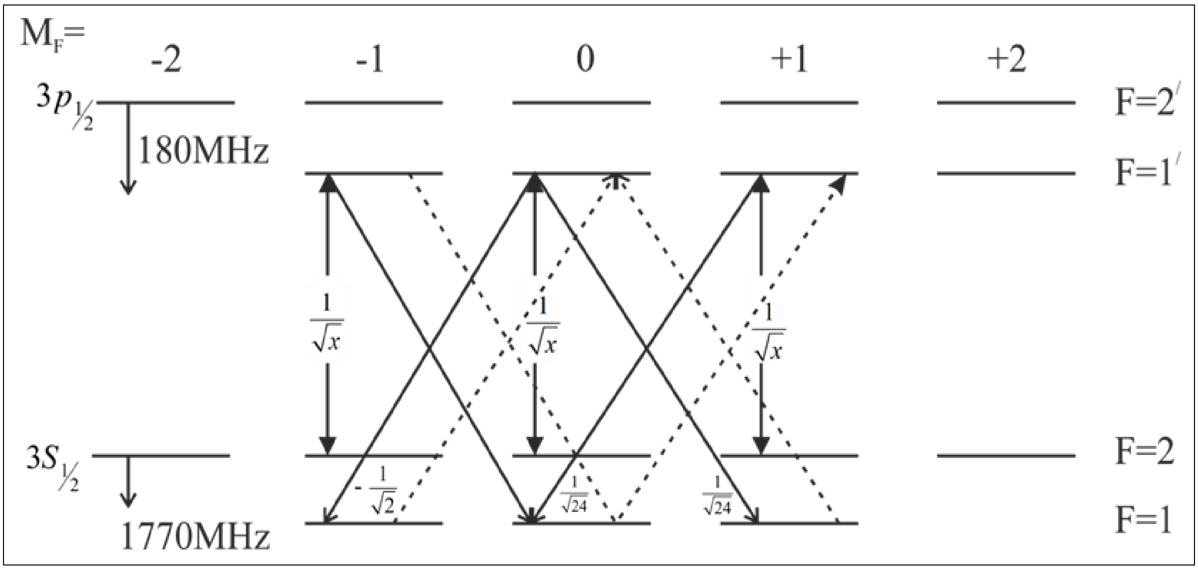

Fig 4: Relevant energy levels of Na atoms showing\g hyperfine structure in a weak magnetic field, appropriate levels of F=1, F=2 and F=1 would each correspond to levels |1>, |2> and |3> of the Λ-system

|a> which is at a distance |a> comes closer and closer to the levels |b> or |c> quantum interference will be less pronounced and when |a> is very close to the low lying levels quantum interference is completely destroyed. In fact, the Λ-configuration vanishes. This is identical to the parameter V which is referred toas visibility of fringes by Sudarshan (following Michelson). Similarly for a fixed position of the level |a> if the separation of |b> and |c> increases quantum interference gradually washes out. In the double slit experiment, it is observed that as the slit width increases interference fringe become narrower and narrower. From this discussion it is reasonable to infer that quantum interference is an atomic

Configuration such as Λ or V schemes may be used to manipulate spontaneous emission. We have observed earlier that Eqn (3-10) shows that the probability of absorption $|Ca(t)|^2$ when $\varphi_1-\varphi_2-\psi = \pm\pi$. Under this condition population is trapped in the lower states |b> and |c> and there is no absorption even in the presence of the field. This is what is known as dark states. Independently of these speculation Alzetta *et al.* [15] reported the experimental evidence of atomic interference is agreement with the model discussed in section 3. It is worthwhile to remark here that the Eqn(3-10) can be put in the form of the Eqn(2-5) of the double slit experiment. But all the information carried by the Eqn (3-10) cannot be obtained in Eqn (2-5). Cosine term indicates interference in both the cases. In the case of double slit we make distinction between interference and diffraction and also compare with single slit pattern. Such distinctions cannot be made in the atomic configuration.

We now proceed to sum up the results which have emerged in this section in a tabular form.

Table formula

Table 1: Summary of the comparison between young's double slit and type atomic configuration.

Quantity	Young's double slit	-Configuration
System	Two straight narrow opening placed in front of a source with a projection screen at a suitable distance.	Three level atomic system with two lower levels close together
Slit Width(d)	≈2	1770MHz (.000004454 eV)
Intensity	$I(y)=\mathrm{Cos}^2(kd/2D)=\mathrm{Cos}^2\delta/2$	$\lvert C_b(t)\rvert^2=(\Omega^2 t^2)/4[1+\mathrm{Cos}(\varphi_1-\varphi_2-\psi)]=(\Omega^2 t^2)/4[\mathrm{Cos}^2(\varphi_1-\varphi_2-\psi)]$
Visibility	$V_f=\frac{I\,max-Imin}{Imax+Imin}$, V_f is visibility of fringes	$V_q=(Imax-Imin)/(Imax+Imin)$ V_q is magnitude of quantum interference
Phase	$I(y)=0$, for $\delta=\pi,3\pi,\ldots$.	$\lvert C_a(t)\rvert^2=0$ for $\varphi 1-\varphi 2-\psi=\pm\pi$.
Energy conservation	No violation of the law of conservation of energy is involved in the interference experiment	No violation of the law of conservation of energy

Conclusion

We appropriately conclude this work with some comments on the comparison between the double slit and Λ-configuration which are analogous. It is responsible to believe that such comparison of analogous situations will help us to understand better the atomic coherence effects so that they may lead to practical applications. It may also be used purpose of science education.

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