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Optical 1-soliton solutions of Lakshmanan-Porsezian-daniel equation with power-law nonlinearity by the trial equation method

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Abstract
This paper obtains some optical soliton solutions to Lakshmanan-Porsezian-Daniel equation with Higher Order Dispersion, Spatio-temporal Dispersion and full-power law nonlinearity. The equation is usually used in modeling the dynamics of optical solution propagations through optical fibers. The Trial Equation Method is being applied in finding some optical solution solutions of the aforementioned equation and validity conditions for the existence of the obtained solutions are stated therein.

Keywords: Solation, Power-law nonlinearity, trial equation method

Introduction
In the last a few decades, study of the dynamics of optical solitons is becoming a core area of research in nonlinear fiber optics. The large variety of present-day communication means and social media like Cell-phone, Internet, Electronic-Mail (e-mail), Facebook, Instagram, Twitter, Whatsapp etc. cannot be well-studied without knowing the dynamics of optical solitons which are data carriers in trans-continental and trans-oceanic information transmission. Optical soliton solutions to Lakshmanan-Porsezian-Daniel (LPD) Equation with full power nonlinearity [1, 2] are being obtained in this paper via the Trial Equation Method [3-8]. The LPD equation was first proposed by three Indian physicists namely Muthusamy Lakshmanan, Kuppuswamy Porsezian and Muthiah Daniel in the year 1988 in the context of Heisenberg spin chain [9, 10]. Afterwards, many researchers began to widely study the equation in the context of nonlinear optics including Fiber Optics. This equation is often studied with different types of nonlinearity like Kerr-law nonlinearity, parabolic-law nonlinearity, qubic-quintic-law nonlinearity and full-power law nonlinearity. In this paper, some solutions of LPD equation with full-power law nonlinearity are obtained via the Trial Equation method stating their validity conditions.

2. Governing Equation
The LPD equation with Higher-Order-Dispersion (HOD) and Spatial-Temporal-Dispersion (STD) is generally written in the form

$$iq_t + aq_{xx} + bq_{xt} + cF(|q|^2)q = \sigma q_{xxxx} + \alpha(q_x)^2 q^* + \beta|q_x|^2 q + \gamma|q|^2 q_{xx} + \lambda q^2 q^*_{xx} + \delta|q|^4 q \qquad (1)$$

In Eq.(1), $i = \sqrt{-1}$ is the imaginary number, the independent variables x and t represent the spatial co-ordinate and time respectively of a dependent variable $q(x,t)$, which is a complex-valued wave function giving the profile of the optical pulse. The first term on the left hand side represents a temporal evolution of the pulse, the second term represents the GVD term where the letter '*a*' is the GVD coefficient, the third term represents the STD term where the letter '*b*' is the STD coefficient, the fourth term is the term showing the type of nonlinearity where the letter '*c*' represents a constant and *F* is a functional in the form of a real-valued nonlinear algebraic function that stems from the refractive index of the fiber and it dictates the type of nonlinearity.

If we treat a complex plane C as a two dimensional linear space R^2, the functional $F(|q|^2)q$ is *r* times differentiable such that

$$F(|q|^2)q \in \cup_{m,n=1}^{\infty} C^r((-n,n) \times (-m,m);\, R^2). \tag{2}$$

On the right hand side of Eq. (1), σ is the fourth-order dispersion (4OD) coefficient while δ is a constant that accounts for a quintic nonlinearity, other nonlinear terms with nonlinear forms of dispersion are indicated by the coefficients of $\alpha, \beta, \gamma, \lambda$. For Kerr-law nonlinearity, the functional *F* has the form $F(s) = s$ and for power-law nonlinearity, *F* has the form

$$F(s) = s^n \tag{3}$$

where *n* represents the power-law nonlinearity parameter that dictates the strength of nonlinearity. For pulse stability, we must have the condition $0 < n < 2$ and also in order to eliminate the condition of self- focusing singularity, it is essential to have the condition $n \neq 2$.

Using Eq. (3) into Eq. (1), we write

$$iq_t + aq_{xx} + bq_{xt} + c\,|q|^{2n}q \tag{4}$$

$$= \sigma q_{xxxx} + \alpha(q_x)^2q^* + \beta|q_x|^2\, q + \gamma|q|^2q_{xx} + \lambda q^2q_{xx}^* + \delta|q|^4q \tag{5}$$

Now, let us introduce the transformation

$$q(x,t) = U(\xi)\, e^{i\eta} \tag{5}$$

with $\xi = x - vt$ and $\eta = -kx + \omega t + \varepsilon$ (6)

where $U(\xi)$ is a function that can give a pulse profile, *v* is the constant speed of wave propagation, *k* is the frequency, ω is the wave number and ε is a phase constant.
Substituting Eq. (5) and its relevant derivatives into Eq. (4), separation of the resulting equation into the real and the imaginary parts yields the *Real Part* as

$$\sigma U'''' - \{(a - bv + 6\sigma k^2) - (\gamma + \lambda)U^2\}\, U'' - (bk\omega - \omega - ak^2 - \sigma k^4)\, U$$

$$-(\alpha - \beta + \gamma + \lambda)k^2U^3 - cU^{2n+1} + (\alpha + \beta)U(U')^2 + \delta U^5 = 0 \tag{7}$$

and the *Imaginary Part* as

$$\{(1 - bk)v + (2ak + 4\sigma k^3 - b\omega)\}U' - 2(\alpha + \gamma - \lambda)kU^2U' - 4\sigma kU''' = 0. \tag{8}$$

In Eqs. (7) and (8), use has been made of the following notations

$$U' = \frac{dU}{d\xi}, \quad U'' = \frac{d^2U}{d\xi^2}, \quad U''' = \frac{d^3U}{d\xi^3}, \quad U'''' = \frac{d^4U}{d\xi^4}.$$

If we set the coefficients of linearly independent functions in Eq. (8) to zero, we will obtain

$$\sigma = 0,$$

$$\alpha + \gamma = \lambda, \tag{10}$$

$$v = \frac{b\omega - 2ak}{1 - bk} \quad (bk \neq 1). \tag{11}$$

Applying the constraint conditions given in Eqs. (9) to (11), we reduce Eq. (7) to the form

$$(a - bv)\, U'' - (\alpha + \beta)U\,(U')^2 - (\gamma + \lambda)U^2U'' + (bk\omega - \omega - ak^2)U$$

$$+(2\lambda-\beta)k^2U^3-\delta U^5+cU^{2n+1}=0. \quad (12)$$

Let us put

$$U(\xi)=V^{\frac{1}{m}}(\xi) \quad (13)$$

where *m* is to be determined from balancing principle.
From the terms containing U^2U'' and U^{2n+1} , the balancing principle yields

$$2m+m+2=\ m(2n+1) \text{ giving } m=\frac{1}{n-1}. \quad (14)$$

Then Eq. (13) becomes

$$U(\xi)=[V(\xi)]^{\frac{1}{n-1}}. \quad (15)$$

Substituting Eq.(15) into Eq. (12) and equating the linearly independent functions to zero, we obtain

$$\frac{a-bv}{n-1}\left\{V''V^{-1}+\frac{2-n}{n-1}(V')^2V^{-2}\right\}+(bk\omega-\omega-ak^2)=0, \quad (16)$$

$$-\frac{\alpha+\beta}{(n-1)^2}(V')^2V^{-2}-\frac{\gamma+\lambda}{n-1}\left\{V''V^{-1}+\frac{2-n}{n-1}(V')^2V^{-2}\right\}+(2\lambda-\beta)k^2+cV^2=0 \quad (17)$$

and $-\delta V^{\frac{5}{n-1}}=0$ giving $\delta=0.$ (18)

Multiplying both sides of Eqs. (16) and (17) by V^2, we re-write them respectively as

$$\frac{a-bv}{n-1}\left\{V''V+\frac{2-n}{n-1}(V')^2\right\}+(bk\omega-\omega-ak^2)V^2=0 \quad (19)$$

and

$$\frac{\alpha+\beta}{(n-1)^2}(V')^2+\frac{\gamma+\lambda}{n-1}\left\{V''V+\frac{2-n}{n-1}(V')^2\right\}-(2\lambda-\beta)k^2V^2-cV^4=0. \quad (20)$$

Using Eq. (19) in Eq. (20), we write

$$\frac{\alpha+\beta}{(n-1)^2}(V')^2+\left\{\frac{(\gamma+\lambda)(ak^2+\omega-bk\omega)}{a-bv}+(\beta-2\lambda)k^2\right\}V^2-cV^4=0. \quad (21)$$

Differentiating both sides of this equation with respect to ξ, we obtain

$$\frac{\alpha+\beta}{(n-1)^2}V''+\left\{\frac{(\gamma+\lambda)(ak^2+\omega-bk\omega)}{a-bv}+(\beta-2\lambda)k^2\right\}V-2cV^3=0. \quad (22)$$

Now, solving Eq.(22) and then using Eqs.(5) and (6), we can obtain solutions of Eq.(1).

3. Outlines of trial equation method

Here, the Trial Equation Method is outlined as in the following. The method consists of the following main steps.

Step I:
A given nonlinear partial differential equation of the form

$$P(u,\ u_x,u_t,u_{xx},u_{xt},\ u_{tt},\ \ldots)=0 \quad (23)$$

where *P* is a polynomial in *u* (which is a function of two independent variables *x* and *t*) and its partial derivatives, can be transformed into a nonlinear ordinary differential equation of the form

$$Q(V,V',V'',V''',\ldots)=0 \quad (24)$$

through the transformation, $u(x,t) = V(\xi),\ \xi = x - vt$ where *v* is a constant, generally the constant speed of propagation of the corresponding wave motion. Here, *Q* is also a polynomial in *V* and its derivatives $V' = \frac{dV}{d\xi}$, $V'' = \frac{d^2V}{d\xi^2}$, *etc.* If all terms of Eq. (24) contain derivatives, then the equation is to be integrated once or twice or any number of times, as the case may be, until at least one term does not contain derivative and all the integration constants should be taken as zero.

Step II:
We then consider a trial equation as

$$(V')^2 = F(V) = \sum_{j=0}^{m} a_j V^j \tag{25}$$

where $a_j\ (j = 0, 1, 2, 3,\ \ldots\ m)$ are constants to be determined latter.
Substituting Eq. (25) and other derivatives of *V* such as V', V'' *etc.* into Eq. (24), we will obtain a polynomial $G(V)$ of *V*. Then, the value of *m* can be determined from the balancing principle. Setting the various coefficients in the polynomial $G(V)$ separately to zero, we will obtain a system of algebraic equations. Solving such system of equations, we can determine the values of $a_j\,(j = 0,1,2,3,\ldots m)$ and the constant wave-speed *v*.

Step III:
In this step, we re-write Eq.(25) as

$$\pm(\xi - \xi_0) = \int \frac{dV}{\sqrt{F(V)}} \tag{26}$$

where ξ_0 is a constant.
Classifying the roots of the function $F(V)$ and evaluating the integral in Eq. (26), we can obtain exact solutions of Eq.(23).

4. Application of Trial Equation:
Now, using Eq. (25) and hence the expressions for other relevant derivatives of *V* in Eq. (22), we balance the degrees of V'' and V^3 from the resulting polynomial equation to yield $m = 4$.
Then, we write Eq. (25) as

$$(V')^2 = F(U) = a_0 + a_1V + a_2V^2 + a_3V^3 + a_4V^4. \tag{27}$$

Differentiating both sides of Eq. (27) with respect to ξ and dividing by $2V'$, we arrive at

$$V'' = \frac{1}{2}F'(V) = \frac{1}{2}a_1 + a_2V + \frac{3}{2}a_3V^2 + 2a_4V^3\,. \tag{28}$$

Substitution of Eq.(28) into Eq.(22) yields

$$\frac{\alpha+\beta}{(n-1)^2}\left(\frac{1}{2}a_1 + a_2V + \frac{3}{2}a_3V^2 + 2a_4V^3\right)$$

$$+ \left\{\frac{(\gamma+\lambda)(ak^2+\omega-bk\omega)}{a-bv} + (\beta - 2\lambda)\,k^2\right\}V - 2cV^3 = 0\,. \tag{29}$$

Equating the various coefficients of different powers of V separately to zero, we obtain a system of algebraic equations as shown below.

$$\frac{(\alpha+\beta)}{2(n-1)^2}\,a_1 = 0,$$

$$\frac{(\alpha+\beta)}{(n-1)^2}\,a_2 + \frac{(\gamma+\lambda)(ak^2+\omega-bk\omega)}{a-bv} + (\beta - 2\lambda)\,k^2 = 0.$$

$$\frac{3(\alpha+\beta)}{2(n-1)^2}\,a_3 = 0$$

and

$$\frac{(\alpha+\beta)}{(n-1)^2}\,a_4 - c = 0.$$

The above system of equations yields

$$a_1 = 0, a_2 = \left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)},$$

$a_3 = 0$ and $a_4 = \frac{c\,(n-1)^2}{(\alpha+\beta)}$.

Substitution of the values of these coefficients into Eqns.(26) and (27) leads to

$$\pm(\xi-\xi_0) = \int \frac{dV}{\sqrt{a_0 + \left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}U^2 + \frac{c\,(n-1)^2}{(\alpha+\beta)}U^4}} \qquad (30)$$

First Choice of the Arbitrary Constant a_0

If we set $a_0 = 0$ in Eq.(30), then performing the integration with respect to *V*, we obtain $V(\xi)$ as

$$V(\xi) = \pm\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}$$

$$\times\ \mathrm{sech}\left[\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(\xi-\xi_0)\right] \qquad (31a)$$

and

$$V(\xi) = \pm\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}$$

$$\times\ \mathrm{cosech}\left[\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(\xi-\xi_0)\right] \qquad (31b)$$

where ξ_0 is a constant.

The validity condition for the above two solutions is

$$(\alpha+\beta)(a-bv)\,\{(\gamma+\lambda)(bk\omega-\omega-ak^2) + (a-bv)(2\lambda-\beta)k^2\}(n-1)^2 > 0.$$

For $(\alpha+\beta)(a-bv)\,\{(\gamma+\lambda)(bk\omega-\omega-ak^2) + (a-bv)(2\lambda-\beta)k^2\}(n-1)^2 < 0,$

Eq. (30) will yield the periodic solutions

$$V(\xi) = \pm\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}$$

$$\times\ \sec\left[\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(\xi-\xi_0)\right] \qquad (32a)$$

and

$$V(\xi) = \pm\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}$$

$$\times\ \mathrm{cosec}\left[\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(\xi-\xi_0)\right]. \qquad (32b)$$

Using Eqs. (5), (6), (15) and (31), we obtain soliton solutions of Eq. (4) (hence of Eq. (1)) as

$$q(x,t) = \left[\pm \sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}\,\right]^{\frac{1}{(n-1)}}$$

$$\times \left\{\text{sech}\left[\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(x-vt-x_0)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]} \tag{33}$$

and

$$q(x,t) = \left[\pm \sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}\,\right]^{\frac{1}{(n-1)}}$$

$$\times \left\{\text{cosech}\left[\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(x-vt-x_0)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]} \tag{34}$$

where $x_0 = \xi_0$ is a constant.
The validity conditions of these two soliton solutions are the same as cited above in the cases of Eqs. (31) and the soliton speed is given by Eq.(11).
Periodic solutions of Eq.(4) or Eq.(1) are also obtained as

$$q(x,t) = \left[\pm \sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bc)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}\,\right]^{\frac{1}{(n-1)}}$$

$$\times \left\{\sec\left[\sqrt{\left\{-\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bc)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(x-x_0-vt)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]}. \tag{35}$$

and

$$q(x,t) = \left[\pm \sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{c\,(a-bc)} + \frac{(2\lambda-\beta)k^2}{c}\right\}}\,\right]^{\frac{1}{n-1}}$$

$$\times \left\{\text{cosec}\left[\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bc)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{(\alpha+\beta)}}\,(x-x_0-vt)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]}. \tag{36}$$

The validity conditions of these two periodic solutions are the same as cited above in the cases of Eqs. (32).
Here, Eq. (33) represents a *bright optical 1-soliton solution* of LPD Equation with power-law nonlinearity whereas Eq. (34) represents a *singular optical 1-soliton solution* of that equation. Eqs. (35) and (36) represent periodic solutions of the same equation.

Second Choice of the Arbitrary Constant a_0:
If we choose $a_0 = \left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{a-bv} + (2\lambda-\beta)k^2\right\}^2 \frac{(n-1)^2}{4c\,(\alpha+\beta)}$, performing the integration with respect to V in Eq. (30), using the resulted expression for $V(\xi)$ and then using Eqs. (5), (6), (15) and (31), we obtain soliton solutions of Eq. (4) (hence of Eq. (1)) as

$$q(x,t) = \left[\pm\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{2c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{2c}\right\}}\right]^{\frac{1}{n-1}}$$

$$\times \left\{\tanh\left[\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{2(\alpha+\beta)}}(x-x_0-vt)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]} \tag{37}$$

and

$$q(x,t) = \left[\pm\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{2c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{2c}\right\}}\right]^{\frac{1}{n-1}}$$

$$\times \left\{\coth\left[\sqrt{-\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{2(\alpha+\beta)}}(x-x_0-vt)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]} \tag{38}$$

Here, Eq. (37) represents a *dark optical 1-soliton solution* of LPD Equation with power-law nonlinearity whereas Eq.(38) represents a yet another *singular optical 1-soliton solution* of that equation. The validity condition of these two optical 1-soliton solutions is given as

$$-(\alpha+\beta)(a-bv)\{(\gamma+\lambda)(bk\omega-\omega-ak^2)+(2\lambda-\beta)(a-bv)k^2\}(n-1)^2 > 0.$$

There are also periodic solutions given by

$$q(x,t) = \left[\pm\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{2c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{2c}\right\}}\right]^{\frac{1}{n-1}}$$

$$\times \left\{\tan\left[\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{2(\alpha+\beta)}}(x-x_0-vt)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]} \tag{39}$$

and

$$q(x,t) = \left[\pm\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{2c\,(a-bv)} + \frac{(2\lambda-\beta)k^2}{2c}\right\}}\right]^{\frac{1}{n-1}}$$

$$\times \left\{\cot\left[\sqrt{\left\{\frac{(\gamma+\lambda)(bk\omega-\omega-ak^2)}{(a-bv)} + (2\lambda-\beta)k^2\right\}\frac{(n-1)^2}{2(\alpha+\beta)}}(x-x_0-vt)\right]\right\}^{\frac{1}{n-1}}$$

$$\times\ e^{-i[kx-\omega t-\epsilon]} \tag{40}$$

with the validity condition

$$-(\alpha+\beta)(a-bv)\{(\gamma+\lambda)(bk\omega-\omega-ak^2)+(2\lambda-\beta)(a-bv)k^2\}(n-1)^2 < 0.$$

5. Conclusion

In this paper, bright, dark and singular optical 1-soliton solutions of Lakshmanan-Posezial-Daniel equation with power-law nonlinearity, that often arises in the investigation of the dynamics of optical solution propagation through optical fibers, are obtained through the Trial Equation Method. This method is a powerful and efficient technique for finding exact optical soliton solutions for a wide extent of nonlinear evolution equations (NLEEs) arising in the studies of nonlinear science and engineering.

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