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Chirped travelling and localized wave solutions in the cubic-quintic nonlinear Schrödinger equation with self-steepening and self-frequency shift

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Abstract
A generalized nonlinear Schrödinger equation possessing cubic-quintic nonlinear components can be used to simulate how ultra-short and femtosecond optical pulses propagate through a nonlinear medium with self-frequency shift as well as self-steepening effects. Starting with an extended auxiliary equation technique, we find an elliptic differential condition with a fifth-degree nonlinear component that depicts the development of the wave amplitude in the metamaterials (MMs) by including an intensity-dependent nonlinear chirp anstaz. As a limiting example of the Jacobi elliptic function solutions for the model under consideration and taking into account the self-frequency shift as well as self-steepening effects, we present a highly rich variety of exact chirped solutions, in particular the solitary wave solutions and periodic solutions. The related chirp is governed by the parameters for self-steepening and self-frequency shift and is proportional to the intensity of the field, according to the results of the generalized nonlinear Schrödinger equation. Parametric conditions for the presence of the traveling wave structures as well as the nonlinear chirp associated with each of these solutions are additionally introduced.
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Introduction

In recent years, we have seen a significant increase in interest in optical solitons and wave packet propagation in optical waveguides as of its key applications in telecommunication and effective signal processing systems. Due to a dynamical equilibrium between dispersive and nonlinear effects, they exhibit stable wave forms known as solitons. Solitons only show a phase shift and do not change shape when they interact in any number [1]. The cubic nonlinear Schrödinger (NLS) equation is frequently used to describe the propagation of picosecond pulses in Kerr media [2]. For the propagation of subpicosecond or femtosecond pulses, higher-order effects must be taken into account, and the problem must be stated using various NLS equation modifications [3]. Numerous optical materials, such as semiconductors and semiconductor-doped glasses, exhibit quantic nonlinearity. The simplest correction to the cubic nonlinear term is quantic nonlinearity, which can be observed in many optical materials including semiconductors, semiconductor doped glasses, polydiacetylene toluene sulfonate (PTS), chalcogenide glasses, and various transparent organic materials [4].

In recent times the transmission of nonlinearly chirped solitons in cubic and cubic-quintic materials has attracted a lot of attention. These chirped pulses have a number of uses in pulse compression or amplification, which significantly improves the design of fiber-optic amplifiers, optical pulse compressors, and solitary-wave-based communications connections [5, 6]. However, it is a difficult task to find chirped femtosecond solitons in nonlinear media since they exhibit not only tunable chirp but also chirp-like properties.

Studies of propagating solitons in higher-order Kerr nonlinear NLS models are also significantly more important than studies of the NLS equation's simpler form [7, 8, 9, 10]. It is noteworthy that finding accurate solutions, especially solitons for higher-order nonlinear NLS models, is crucial because they can help explain a number of important phenomena in optical systems.

Theoretical Model

For pulses with at least 100 *fs* in width, 1 *W* in power, and a significant GVD [11], third-order dispersion can be ignored; however, the effects of self-steepening and self-frequency shift terms remain dominant and should be preserved. Under these circumstances, considering the non-linear higher-order NLSE having cubic-quintic form as:

$$iE_z + \beta_1 E_{tt} + \beta_2 |E|^2 E + \beta_3 |E|^4 E + i\beta_4 (|E|^2 E)_t + i\beta_5 E(|E|^2)_t = 0, \tag{1}$$

Where, $t = \frac{cT}{\lambda_p}$ and $z = \frac{z}{\lambda_p}$ are are the plasma wavelength and normalised time, respectively, and E(z, t) is the complex envelope of the electric field. The group-velocity dispersion (GVD) is denoted by the symbol β_1 whilst the cubic, quintic nonlinearity, the self-steepening effect, and the self-frequency shift effect are denoted by the symbols $\beta_2, \beta_3, \beta_4$ and β_5 respectively. Eq. (1) reduces to the standard NLSE, which only includes the GVD and SPM effects, for $\beta_3 = \beta_4 = \beta_5 = 0$. For $\beta_3 = \beta_5 = 0$, Eq. (1) indicates the modified NLSE, which regulates the transmission of NLSE soliton in the presence of Kerr dispersion. When $\beta_2 = \beta_3 = \beta_5 = 0$, Eq. (1) changes into the Kaup-Newell equation, sometimes referred to as the derivative NLSE-I. Additionally, when $\beta_4 = \beta_5 = 0$ reduces to the cubic-quintic NLSE, which depicts the dynamics of waves in a non-Kerr medium with both third and fifth-order susceptibilities. Also, when $\beta_2 = \beta_4 = 0$ Eq. (1) represents the wave propagation in a pure quintic nonlinear medium. Exploring the propagation features of envelope solitons in the presence of self-steepening effect is fascinating since the latter will drastically alter the physical properties of propagating pulses.

We point out that the intensity dependence of group velocity is what causes the self-steepening, also known as the Kerr dispersion [3]. Surprisingly, the higher-order effects cannot be ignored when the soliton pulse width approaches ultrashort 100 *fs*, and the influence of self-steepening on optical solitons becomes a significant concern in the optical fibre communication system [2, 3].

The model in Eq. (1), with $\beta_5 = 0$ was used by Scalora *et al.* [12] to describe pulse propagation in a negative-index material, where the sign of the GVD may be either positive or negative. Several bright and dark forms of restricted special solutions to Eq. (1) have been identified [13, 14].

Outlook of Extended Auxiliary Equation method

Take a nonlinear PDE with the following two independent variables *x*,*t*, and *q*:

$$Q(q, q_x, q_t, q_{tt}, q_{xx}, \dots\dots), \tag{2}$$

Q being polynomial in indeterminate *q* and involving its partial derivatives in which the highest derivatives and the nonlinear terms are involved. The main steps of the extended auxiliary equation method [15, 16] can be summarized as follows:

Step 1: We presume the traveling wave transformation:

$$\mathrm{q}(\mathrm{x},\mathrm{t}) = \mathrm{q}(\xi), \xi = \mathrm{k}(\mathrm{x} - \lambda \mathrm{t}), \tag{3}$$

Where k is an arbitrary constant and λ is the speed wave constant. Using Eq. (3), thus Eq. (2) reduces to the following ODE:

$$R(q', q'', q''', \dots.), \tag{4}$$

Where *R* is a polynomial in $q(\xi)$ and its total derivatives with respect to the wave variable ξ.

Step 2: We suppose that Eq. (4) holds the formal solution:

$$q(\xi) = \sum_{i=0}^{2N} \alpha_i U^i(\xi), \tag{5}$$

Where $U(\xi)$ is satisfied the following first order ODE:

$$U'^2(\xi) = q_0 + q_2 U(\xi)^2 + q_4 U(\xi)^4 + q_6 U(\xi)^6 \tag{6}$$

Where $q_i(i = 0,2,4,6)$ and $\alpha_i(i = 0,1,2,3, \dots, 2N)$ are arbitrary constants that need to be found.

Step 3: In Eq. (5) balance number *N* is obtained by balancing the highest order nonlinear terms and highest order derivatives of $q(\xi)$.

Step 4: To get a system of algebraic equations for $q_j(j = 0,2,4,6)$, $\alpha_i(i = 0,1,2,3, \dots ,2N)$, k and λ, we replace Eq. (4) with Eq. (6), gather all the coefficients of $U_j(U')^r(j = 0,1,2,)$ and $(r = 0,1)$ and equate them to zero.

Step 5: Using Maple, we solve the system of algebraic equations we acquired in Step 4 to get the values of $q_j(j = 0,2,4,6)$, $\alpha_i(i = 0,1,2,3, \dots ,2N)$, k and λ.

Step 6: It is well-known [13, 14] that Eq. (6) has the solutions:

$$U(\xi) = \frac{1}{2}[-\frac{q_4}{q_6}\left(1 \pm f_i(\xi)\right)]^{\frac{1}{2}} \tag{7}$$

Where the function $f_i(\xi)(\mathrm{i} = 1,2,3, \dots 12)$ could be expressed through the Jacobi elliptic function $sn(\xi, m), cn(\xi, m), dn(\xi, m)$ and so on, where 0 < m < 1 is the modulus of the Jacobi elliptic functions. Using the 12 forms of the functions $f_i(\xi)(\mathrm{i} = 1,2,3, \dots 12)$ given by Eq.(7) in [15, 16, 17], we construct the solution of Eq.(2).

Exact chirped traveling and localised wave solutions

In this case, we're looking for chirped solitonlike solutions to Eq. (1). So, for the complex envelope traveling-wave solutions, we select the form:

$$E(z,t) = A(\xi)e^{i[\eta(\xi)-kz]}, \tag{8}$$

Where $\xi = t - cz$ is the traveling wave coordinate and amplitude $A(\xi)$ and phase modulation parameter $\eta(\xi)$ are real functions of ξ. Here $c = 1/v_g$, is the inverse velocity with v_g the group velocity of the wave packet. The corresponding chirp (instantaneous frequency shift) is given by $\delta w(t,z) = -\frac{\delta}{\delta t}[\eta(\xi) - kz] = -\eta'(\xi)$, where prime represents differentiation with respect to ξ. The real parameter k represents the wave number of oscillatory waves. Substituting Eq. (8) in Eq. (1) and separating out the real and imaginary parts of the equation, the coupled equations in A and η read as:

$$kA + c\eta' A - \beta_1\eta'^2 A + \beta_1 A'' - \beta_4\eta'^{A^3} + \beta_3 A^5 = 0, \tag{9}$$

And

$$-cA' + \beta_1 A\chi'' + 2\beta_1 A'\chi' + (3\beta_4 + 2\beta_5)A^2 A' = 0. \tag{10}$$

Now we adopt an ansatz that depends quadratically on the wave amplitude to solve the above pair of coupled equations as:

$$\eta' = pA^2 + q, \tag{11}$$

Where, respectively p and q stand for the nonlinear and constant chirp parameters. As a result, the chirp that results of the form $\delta w(t,z) = -(pA^2 + q)$. This suggests that the chirp associated with propagating pulses depends on intensity, with $I = |E|^2 = A^2$ and the chirp that results from linear and nonlinear contributions contains both of them. The relations of the chirp parameters q and p are further provided by putting the ansatz (11) into Eq. (10) as

$$p = -\frac{3\beta_4 + 2\beta_5}{4\beta_1}, q = \frac{c}{2\beta_1}. \tag{12}$$

As a result, the nonlinear chirp parameter's value is mostly determined by the GVD, as well as by the self-steepening and self-frequency shift coefficients. This suggests that choosing these coefficients can alter the chirp amplitude. As a result, we may say that higher-order nonlinear processes like the self-steepening effect are where the nonlinear chirp originates.

Now, using Eqs. (11) and (12) into Eq. (9), one obtains

$$A'' + a_1 A + a_2 A^3 + a_3 A^5 = 0, \tag{13}$$

Where

$$a_1 = \frac{4k\beta_1 + c^2}{4\beta_1^2}, a_2 = \frac{(2\beta_1\beta_2 - c\beta_4)}{2\beta_1^2}, a_3 = \frac{16\beta_1\beta_3 - (3\beta_4 + 2\beta_5)(2\beta_5 - \beta_4)}{16\beta_1^2}. \tag{14}$$

A differential equation of the elliptic type, Eq. (13), describes how the wave amplitude changes over time in the metamaterial. Intriguingly, Eq. (13) becomes a cubic nonlinear equation that allows for both dark and bright solitons if $a_1 = 0$. It can be solved for localised solutions via a fractional transformation in the case, When $a_2 = 0$.

We demonstrate that the equation has a solution of the Lorentzian type for $a_3 = 0$. The ϕ^6 field equation, which is known to permit a number of solutions, including bright soliton, dark soliton, kink, double kink, and Weierstrass function solutions of Eq. (1), can be converted into Eq. (13) in the most general case, when all the coefficients have nonzero values. However, finding new, precise soliton solutions to this equation remains a vital task in mathematical physics.

The diversity of dynamics in nonlinear MMs regulated by the model under discussion is illustrated in this work by the range of traveling and localised solutions, present for various parameter settings. We also shown the chirping associated with these structures.

In this section, we provide precise analytical chirped soliton solutions for the NLSE (1). The solutions nontrivial phase chirping, which varies as a function of intensity due to the Kerr dispersion factor, will be demonstrated.

In Eq. (13) we derive the balance number $N = \frac{1}{2}$ by balancing the variables A'' and A^5. We consider the transformation since the balance number is not an integer as:

$$A(\xi) = [\theta(\xi)]^{\frac{1}{2}}. \tag{15}$$

Substituting Eq.(15) into (13), we have the new equation:

$$a_1\theta^2 + a_2\theta^3 + a_3\theta^4 - \frac{1}{4}[(\theta')^2 - 2\theta\theta''] = 0 \tag{16}$$

By balancing $\theta\theta''$ with θ^4 in Eq.(16), we have $N = 1$. From Eq.(5), the formal solution of Eq.(16) has the form:

$$\theta(\xi) = \alpha_0 + \alpha_1 U(\xi) + \alpha_2 U(\xi)^2, \tag{17}$$

Where $U(\xi)$ satisfies Eq.(6), while $\alpha_i (i = 0,1,2)$ are arbitrary constants to be determined. Consequently, we get

$$\theta'^2(\xi) = [\alpha_1 + 2\alpha_2 U(\xi)^2][q_0 + q_2 U(\xi)^2 + q_4 U(\xi)^4 + q_6 U(\xi)^6], \tag{18}$$

$$\theta''(\xi) = \alpha_1[q_2 U(\xi) + 2q_4 U(\xi)^3 + 3q_6 U(\xi)^5] + 2\alpha_2[q_0 + 2q_2 U(\xi)^2 + 3q_4 U(\xi)^4\ 4q_6 U(\xi)^6] \tag{19}$$

Substituting Eqs.(17)-(19) into Eq.(16), we are with a system of algebraic equations through considering the coefficients of each exponent of of $U_i (i = 0,1,\ldots,8)$ and equating them to zero.

Solving thissystem of algebraic equations with the aid of Maple, we obtain the following result:

$$q_0 = q_0, q_2 = -a_1, q_4 = -\frac{\alpha_2 a_2}{2}, q_6 = -\frac{{\alpha_2}^2 a_3}{3}, \alpha_0 = 0, \alpha_1 = 0, \alpha_2 = \alpha_3 \tag{20}$$

Substituting Eq. (20) into Eq.(17) along with Eqs.(15) and (7), we get the following solutions of Eq.(13):

$$A(\xi) = [-\frac{3a_2}{8a_3}(1 \pm f_i(\xi))]^{\frac{1}{2}}, \tag{21}$$

where 0 < m < 1 is the modulus of the Jacobi elliptic functions, $sn(\xi,m), cn(\xi,m), dn(\xi,m)$ and so on may be used to create the function $f_i(\xi), i = 1,2,\ldots,12$, trigonometric and hyperbolic functions are the degenerate forms of the Jacobi elliptic functions, respectively.

The Jacobi elliptic functions for Eq. (13) are given by

Type 1: If $q_0 = \frac{q_4^2(m^2-1)}{32\, q_6^2 m^2}, q_2 = \frac{q_4^2(5m^2-1)}{16 q_6 m^2}, q_6 > 0$, then

$$E_1(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm sn\left(\sqrt{\frac{-3a_2^2}{16m^2 a_3}}(t - cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi) - kz]} \tag{22}$$

$$E_2(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{1}{m\, sn\left(\sqrt{\frac{-3a_2^2}{16m^2a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{23}$$

If m = 1, then $sn(\xi) = \tanh(\xi)$, and hence Eq. (13) has the hyperbolic function solutions

$$E_1(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm tanh\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{24}$$

$$E_2(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm coth\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{25}$$

and the corresponding chirping terms are given by

$$\delta w_1(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm tanh\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right] + q\right) \tag{26}$$

$$\delta w_2(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm coth\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right] + q\right) \tag{27}$$

Type 2: If $q_0 = \frac{q_4^2(1-m^2)}{32q_6^2}, q_2 = \frac{q_4^2(5-m^2)}{16q_6}, q_6 > 0$, then

$$E_3(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm m\, sn\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{28}$$

$$E_4(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{1}{sn\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{29}$$

If m = 0, then $sn(\xi) = \sin(\xi)$, and hence Eq. (13) has rational and the periodic wave solutions

$$E_3(z,t) = \left[-\frac{3a_2}{8a_3}\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{30}$$

$$E_4(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm csc\left(\sqrt{\frac{-3a_2^2}{16m^2a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{31}$$

and the corresponding chirping terms are given by

$$\delta w_3(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\right]^{\frac{1}{2}} + q\right) \quad (32)$$

$$\delta w_4(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm csc\left(\sqrt{\frac{-3a_2^2}{16m^2a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} + q\right) \quad (33)$$

If m =1, then $sn(\xi) = \tanh(\xi)$, and hence we have the same hyperbolic function solutions (24) and (25).

Type 3: If $q_0 = \frac{q_4^2}{32q_6^2m^2}, q_2 = \frac{q_4^2(4m^2+1)}{16q_6m^2}, q_6 < 0$, then

$$E_5(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm cn\left(\sqrt{\frac{3a_2^2}{16m^2a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (34)$$

$$E_6(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{\sqrt{1-m^2}\, sn\left(\sqrt{\frac{3a_2^2}{16m^2a_3}}(t-cz)\right)}{dn\left(\sqrt{\frac{3a_2^2}{16m^2a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (35)$$

If m = 1, then $cn(\xi) = \operatorname{sech}(\xi)$, and hence Eq. (13) has the hyperbolic function solutions

$$E_5(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm sech\left(\sqrt{\frac{3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (36)$$

$$E_6(z,t) = \left[-\frac{3a_2}{8a_3}\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (37)$$

and the corresponding chirping terms are given by

$$\delta w_5(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm sech\left(\sqrt{\frac{3a_2^2}{16a_3}}(t-cz)\right)\right)\right] + q\right) \quad (38)$$

$$\delta w_6(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\right] + q\right) \quad (39)$$

Type 4: If $q_0 = \frac{q_4^2m^2}{32q_6^2(m^2-1)}, q_2 = \frac{q_4^2(5m^2-4)}{16q_6(m^2-1)}, q_6 < 0$, then

$$E_7(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{dn\left(\sqrt{\frac{-3a_2^2}{16(m^2-1)a_3}}(t-cz)\right)}{\sqrt{1-m^2}}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (40)$$

$$E_8(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{1}{dn\left(\sqrt{\frac{-a_2^2}{16(m^2-1)a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{41}$$

If m = 0, then $dn(\xi) = 1$, and hence Eq. (13) has solutions

$$E_7(z,t) = E_8(z,t) = \left[-\frac{3a_2}{4a_3}\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{42}$$

and the corresponding chirping terms are given by

$$\delta w_7(t,z) = \delta w_8(t,z) = -\left(p\left[-\frac{3a_2}{4a_3}\right]^{\frac{1}{2}} + q\right) \tag{43}$$

Type 5: If $q_0 = \frac{q_4^2}{32q_6^2(1-m^2)}, q_2 = \frac{q_4^2(4m^2-5)}{16q_6(m^2-1)}, q_6 > 0$, then

$$E_9(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{1}{cn\left(\sqrt{\frac{a_2^2}{16(m^2-1)a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{44}$$

$$E_{10}(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{dn\left(\sqrt{\frac{a_2^2}{16(m^2-1)a_3}}(t-cz)\right)}{\sqrt{1-m^2}\,sn\left(\sqrt{\frac{a_2^2}{16(m^2-1)a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{45}$$

If m = 0, then $cn(\xi) = \cos(\xi), sn(\xi) = \sin(\xi), dn(\xi) = 1$, and hence Eq. (13) has the periodic solutions

$$E_9(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm sec\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{46}$$

$$E_{10}(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm csc\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \tag{47}$$

and the corresponding chirping terms are given by

$$\delta w_9(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm sec\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right] + q\right) \tag{48}$$

$$\delta w_{10}(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm csc\left(\sqrt{\frac{-3a_2^2}{16a_3}}(t-cz)\right)\right)\right] + q\right) \tag{49}$$

Type 6: If $q_0 = \frac{m^2 q_4^2}{32 q_6^2}, q_2 = \frac{q_4^2(m^2+4)}{16 q_6}, q_6 < 0$, then

$$E_{11}(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm dn\left(\sqrt{\frac{3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (50)$$

$$E_{12}(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm \frac{\sqrt{1-m^2}}{dn\left(\sqrt{\frac{3a_2^2}{16a_3}}(t-cz)\right)}\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (51)$$

If m = 0, then $dn(\xi) = 1$, and hence Eq. (13) has rational solutions

$$E_{11}(z,t) = E_{12}(z,t) = \left[-\frac{3a_2}{4a_3}\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (52)$$

and the corresponding chirping terms are given by

$$\delta w_{11}(t,z) = \delta w_{12}(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\right] + q\right) \quad (53)$$

If m =1, then $dn(\xi) = \text{sech}(\xi)$, and hence we have the same hyperbolic function solutions:

$$E_{11}(z,t) = \left[-\frac{3a_2}{8a_3}\left(1 \pm sech\left(\sqrt{\frac{3a_2^2}{16a_3}}(t-cz)\right)\right)\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (54)$$

$$E_{12}(z,t) = \left[-\frac{3a_2}{8a_3}\right]^{\frac{1}{2}} e^{i[\eta(\xi)-kz]} \quad (55)$$

and the corresponding chirping terms are given by

$$\delta w_{11}(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\left(1 \pm sech\left(\sqrt{\frac{3a_2^2}{16a_3}}(t-cz)\right)\right)\right] + q\right) \quad (56)$$

$$\delta w_{12}(t,z) = -\left(p\left[-\frac{3a_2}{8a_3}\right] + q\right) \quad (57)$$

Results and Discussions

We have shown a few graphs showing the exact solutions in this section. These solutions are periodic, hyperbolic and trigonometric solutions. Different nonlinear waves are described by exact solutions of the results. Specialised types of solitary waves solutions are established accurate solutions with hyperbolic solutions. These solutions have the amazing quality of maintaining their identity as they interact with one another.

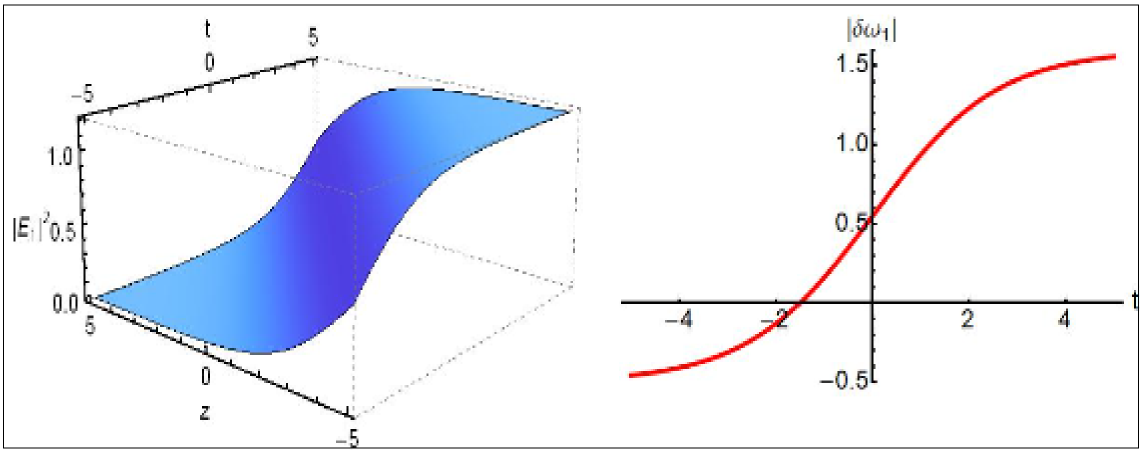

Fig 1: Intensity and corresponding chirp profile of the kink solitary wave solution given by Eq.(24) for the choice of parameter values mentioned in the text.

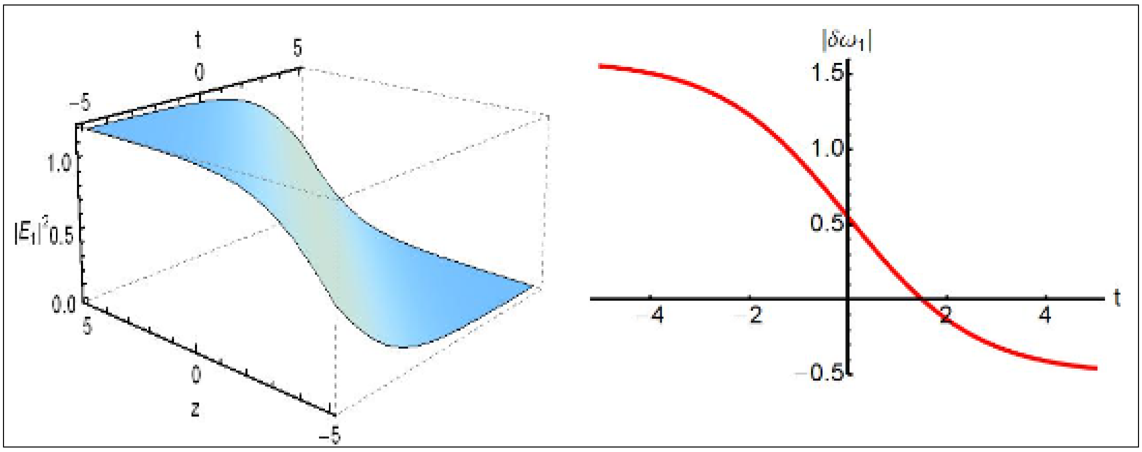

Fig 2: Intensity and corresponding chirp profile of the anti-kink solitary wave solution given by Eq.(24) for the choice of parameter values mentioned in the text

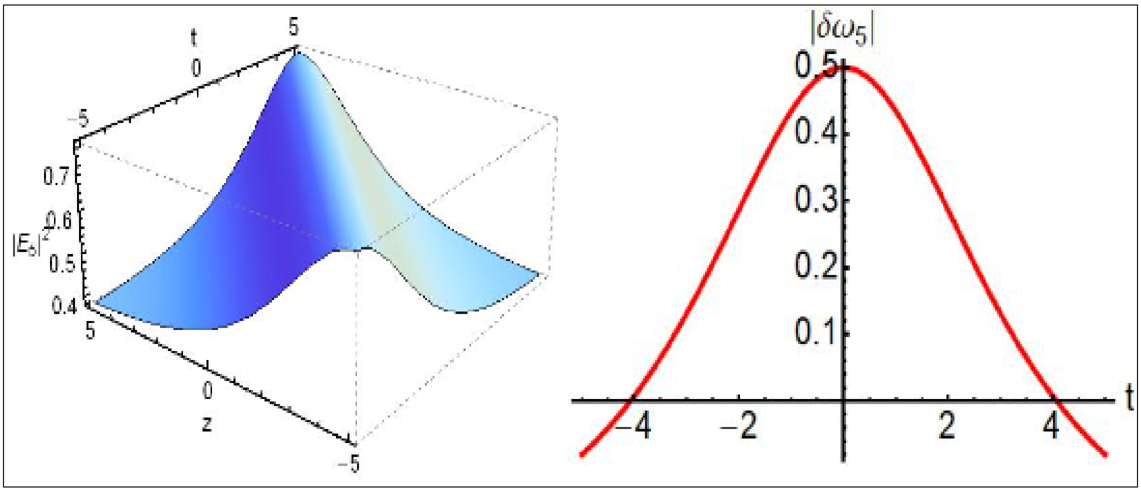

Fig 3: Intensity and corresponding chirp profile of bright soliton Eq. (36) for the choice of parameter values mentioned in the text

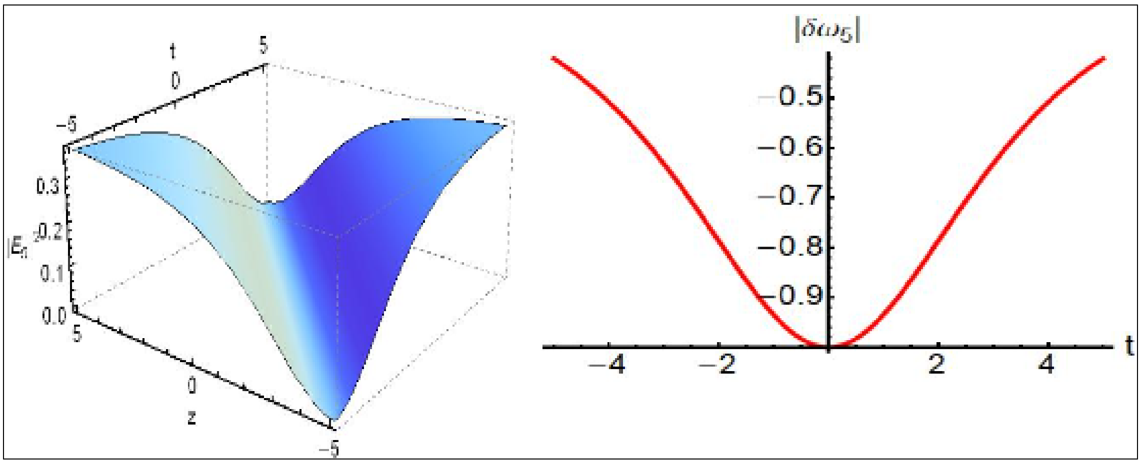

Fig 4: Intensity and corresponding chirp profile of dark soliton Eq. (36) for the choice of parameter values mentioned in the text.

To show the dynamic behaviors of some acquired solutions, we have plotted some of them in Figs. 1-4. The intensity profile of kink (+ sign) or anti-kink(-sign) soliton Eq.(24) is depicted in Figs. 1 and 2 by utilising the following model parameter values $\beta_1 = 1, \beta_2 = 1, \beta_3 = 1, \beta_4 = 1, \beta_5 = 2$ and c=1. The corresponding chirping for kink or anti-kink soliton Eq.(26) is depicted at z=0 for plus and minus sign, respectively.

We noticed from the intensity profiles of the chirped solitary wave solutions that the nonlinear chirp is exactly proportional to the wave's intensity and that the self-steepening and self-frequency shift parameters may be changed to alter the amplitude of the chirped solitary wave solution. The statistics make it very evident that the chirping for the bright soliton is highest near the centre of the pulse, whereas it is smallest for the dark soliton.

Conclusions

These exact solutions are modelled as hyperbolic or trigonometric functions when the modulus of the Jacobi Elliptic function is equal to one or zero, respectively. In this work, we have shown that the competing cubic-quintic nonlinearities can be solved by unique forms of Jacobi elliptic function solutions for the nonlinear cubic-quintic Schrodinger equation with self-steepening and self-frequency shift effects, as well as bright (dark), soliton-like kink (anti-kink), and periodic solitons. We have identified the parameter domains for these optical solitons' existence, which may be useful for long-distance communication networks. Our research is original and has never been published before. Last but not least, we verified using Maple that every single solution discovered in this study fulfils the original equations.

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