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LRS Bianchi type II string magnetized barotropic perfect fluid cosmological model in Rosen's biometric theory of gravitation

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Abstract
We have presented the solution of LRS Bianchi type II space-time with a magnetic field and with string viscous fluid by solving the field equations of Rosen's biometric theory of gravitation. It is observed that the magnetic field could have the cosmological origin of the model. The strong magnetic field ruled out the existence of the universe. Other geometrical and physical behaviour of the model have been studied in the evolution of universe.
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Introduction
The occurrence of magnetic field on galactic scale is a well-established fact today and anisotropic magnetic field models have significant contribution to the evolution of galaxies and stellar objects. Harrison [1] has suggested that magnetic field could have a cosmological origin. Melvin [2] has described that during the evolution of Universe, the matter is in a highly ionized state and due to smooth coupling with field it forms neutral matter as a result of universe expansion. Strong magnetic field can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic field gives rise to anisotropies in the universe. Therefore, the presence of magnetic field in anisotropic string universe is not unrealistic. Asseo and Sol [3] emphasized the importance of the Bianchi Type II universe. Roy and Banerjee [4], Pradhan *et al.* [5] and Bali [6] have investigated the LRS Bianchi type II cosmological models representing the clouds as well as massive strings.
Rosen's [7, 8] biometric theory of gravitation is a theory of gravitation based on two metrics. One is the fundamental metric tensor g_{ij} describes the gravitational potential represents the geometry of curved space-time and the second metric γ_{ij} refers to the flat space–time and describes the inertial forces associated with the acceleration of the frame of reference. This theory agrees with the present observational facts pertaining to general relativity.
The field equations of Rosen's biometric theory of gravitation [8] are

$$N_i^j - \frac{1}{2} N \delta_i^j = -T_i^j \tag{1}$$

Where $N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left(g^{sj} g_{si\,|\,\alpha}\right)_{|\beta}$, $N = g^{ij} N_{ij}$, $k = \sqrt{\frac{g}{\gamma}}$ together with $g = \det\left(g_{ij}\right)$ and $\gamma = \det\left(\gamma_{ij}\right)$. Here the vertical bar $(\,|\,)$ stands for γ-covariant differentiation and T_i^j is the energy–momentum tensor of matter field. The Bimetric theory of gravitation is free from the singularities that occur in general relativity that appeared in the big–bang in cosmological models and therefore several aspects of bimetric theory of gravitation have been studied and investigated many Bianchi-type cosmological models in it by many researchers [9-21]. Locally Rotationally Symmetric (LRS) Bianchi Type II Magnetized String Cosmological Model with Bulk Viscous Fluid have been studied in Rosen's biometric theory of gravitation to see the geometrical and physical behaviour of the model. It is seen that this LRS Bianchi type II space-time with magnetic field and with string viscous fluid by solving the field equations of Rosen's biometric theory of gravitation. Other geometrical and physical behavior of the model have been studied in the evolution of universe.

2. The Metric and Field Equations

We consider the LRS Bianchi Type-II metric in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2(dy - xdz)^2 \quad (2)$$

Where A and B are functions of cosmic time *t* only
The flat metric corresponding to metric (2) is

$$ds^2 = -dt^2 + (dx^2 + dy^2) + (dy - dz)^2 \quad (3)$$

Assume that the energy momentum tensor containing a bulk viscous fluid with one dimensional strings and electromagnetic field.

$$T_i^j = (\rho + p)u_i u^j + p g_i^j - \lambda x_i x^j + E_i^j \quad (4)$$

Where ρ is the rest energy density of the system, λ is the string tension density and p the pressure of the cosmic fluid and θ is the expansion scalar. $u^i = (0,0,0,1)$ Is the four velocity and choosing $x^i = (\frac{1}{A},0,0,0)$ the direction of the string, such that

$$g_{ij}u^i u^j = -g_{ij}x^i x^j = 1 \quad (5)$$

and $u^i x_i = 1$.
Assume that particles are attached with the strings and the energy density for particles is given by

$$\rho_p = \rho - \lambda. \quad (6)$$

E_{ij} is the part of the energy momentum tensor corresponding to the electromagnetic field and is defined as

$$E_{ij} = \frac{1}{4\pi}\left(g^{sp}F_{is}F_{jp} - \frac{1}{4}g_{ij}F_{sp}F^{sp}\right), \quad (7)$$

where F^{sp} is the electromagnetic field tensor. The magnetic field is taken along the x-direction, so that the only non-vanishing component of F^{sp} is F_{23}. Due to assumption of finite electrical conductivity, we have $F_{14} = F_{24} = F_{34} = 0$
From Maxwell's equation, $F_{[ij,k]} = 0$, we write

$$F_{23} = -F_{32} = H = Constant. \quad (8)$$

For LRS Bianchi Type-II metric considered in equation (4), the components of the energy momentum tensor of electromagnetic field are

$$E_{11} = -\frac{H^2 A^2}{8\pi B^4} = -\delta A^2, \quad (9)$$

$$E_{22} = E_{33} = \frac{H^2}{8\pi B^2} = \delta B^2, \quad (10)$$

$$E_{44} = \frac{H^2}{8\pi B^4} = \delta. \quad (11)$$

Equations (4) now yields

$$T_1^1 = (p - \lambda - \delta), \quad (12)$$

$$T_2^2 = T_3^3 = (p + \delta), \quad (13)$$

$$T_4^4 = (\rho + \delta). \quad (14)$$

Rosen's field equations (1), for the metric (2) and (3) with the components of T_i^j, (equations (12) to (14)) becomes

$$\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B} = 16\pi A^2 B(p - \lambda - \delta), \quad (15)$$

$$2\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - 2\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{B^2}{A^2} = 16\pi A^2 B(p + \delta), \quad (16)$$

$$\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{B^2}{A^2} = 16\pi A^2 B(p + \delta), \quad (17)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - 2\frac{\dot{A}^2}{A^2} - \frac{\dot{B}^2}{B^2} = 16\pi A^2 B(\rho + \delta). \quad (18)$$

3. Solution of Rosen's Field Equations

There are four Rosen's field equations (16) – (17) with six unknowns A, B, p, λ and ρ. Therefore, the additional one

constraints relating these parameters are to be taken in order to obtain explicit solutions of the system, of equation (15) – (18).
Hence, we consider that expansion θ is proportional to share σ.

$$A = (B)^n, \text{ for } n > 0 \quad (19)$$

After solving equations (16) and (17), we get,

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right) - \frac{d}{dt}\left(\frac{\dot{B}}{B}\right) = \frac{B^2}{A^2}, \quad (20)$$

and from equations (15) - (18), we write,

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right) - \frac{d}{dt}\left(\frac{\dot{B}}{B}\right) = \frac{B^2}{A^2}, \quad (21)$$

With this equation (21), the differential equations (15), (17), and (18) yield

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}\right)=\frac{B^2}{A^2}. \quad (22)$$

From equations (20) and (22), we get

$$\frac{d}{dt}\left(\frac{\dot{B}}{B}\right)=0, \quad (23)$$

which yield

$$B=e^{(lt+m)}, \quad (24)$$

where $l(>0)$ and m are constants of integration.
From equations (19) and (24), we write

$$A=e^{n(lt+m)}, \quad (25)$$

Hence metric (2) reduces to

$$ds^2=-dt^2+e^{2(lt+m)}[e^n(dx^2+dy^2)+(dy-xdz)^2]. \quad (26)$$

This is the required metric that represents the LRS Bianchi type-II magnetized string cosmological model in the bimetric theory of gravitation, and it is free from a singularity. It is to be noted that the behavior of our model reflects by Volumetric exponential functions.
The expansion scalar θ and the shear scalar σ for the metric given by

$$\theta=u^l\,|_l=(2n+1)l \quad (27)$$

$$\sigma=\frac{1}{\sqrt{3}}[(2n+1)l]^{1/2} \quad (28)$$

The universe is filled with barotropic perfect fluid so that pressure depends only on the density and vice-versa. Hence we have

$$\rho=\gamma\lambda, 0\leq\gamma\leq 1$$

The energy density corresponding to the particles loaded with the string is

$$\rho_p=\rho-\lambda=(\gamma-1)\lambda \quad (29)$$

The electromagnetic field, string tension density λ, the energy density ρ, the isotropic pressure p, particle density ρ_p, the scalar of expansion θ, shear tensor σ, bulk viscosity ξ, spatial volume V, the scale factor R and deceleration parameter q for the model (26) are

$$\delta=\frac{H^2}{8\pi e^{4(lt+m)}} \quad (30)$$

$$p=\frac{1}{e^n}-\frac{H^2}{8\pi e^{4(lt+m)}}+\xi_0 \quad (31)$$

$$\rho=\rho_p=\lambda=0 \quad (32)$$

$$V=e^{(2n+1)(lt+m)} \quad (33)$$

$$R=e^{(2n+1)(lt+m)/3} \quad (34)$$

$$q=e^{n(lt+m)-2} \quad (35)$$

It is realized that the value of δ is infinite, as $t\to 0$ and as $t\to\infty$, $\delta\to 0$. So that the magnetic field is diverges to infinity in the beginning of the model and it is disappeared at late time. The pressure p is infinite when $t\to 0$ and they approaches to finite values, when $t\to\infty$. This shows that the pressure is diverges to infinity, at early stage and they attain constant values at final stage of the model. The quantities ρ, ρ_p, λ are vanishes. The scalar expansion θ is always positive and it is infinite at early stage and admit finite value at final stage. Also the deceleration parameter q is found to be negative. This suggested that the universe is expanding with accelerating expansion which supports the observations of Perlmutter *et al.* [22, 23], Knop *et al.* [24], Tegmark *et al.* [25] and Spergel *et al.* [26, 27] as they observed that the universe is continuously expanding with accelerating expansion. The values $\frac{\sigma}{\theta}\neq 0$, for $l\neq 0$, suggested that the model does not isotropize and for $l=0$, the model is isotropize in all directions. Lastly, the model has volumetric exponential expansion which starts with zero volume and zero scale factors and has infinite volume at late time.

4. Conclusion

We have investigated LRS Bianchi type II the string fluid cosmological model with magnetic field in bimetric theory of gravitation. It is seen that our model has volumetric exponential expansion. It is noticed that our model is exponentially expanding with accelerating expansion, which supports the recent observational data of Perlmutter *et al.* [22, 23], Knop *et al.* [24], Tegmark *et al.* [25] and Spergel *et al.* [26, 27]. Other geometrical and physical aspects of the model have also been studied.

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