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In the Scottish café (Polish: *Kawiarnia Szkocka*) house, the academic scene, a mathematician Stefan Banach and the Scottish book

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Abstract
Mathematicians from the Lwów School of Mathematics collaborated to address research issues, in the paper the academic scene of Scottish Café, particularly in functional analysis, topology and mathematical problems, further how Stefan Banach come across with other mathematicians. This paper also discusses the Scottish Book, a large notebook used by Polish mathematicians at the Lwów School of Mathematics to record issues that they intended to answer. The "Scottish Café," where the journal was stored, inspired its name.

Keywords: Lwow, Stefan Banach, mathematics, Scottish book

1. Introduction
In the 1930s and 1940s, mathematicians from the Lwów School of Mathematics collaborated on research problems, particularly in functional analysis and topology, at the Scottish Café in Lwów, Poland (now Lviv, Ukraine).
According to Stanisaw Ulam, the café's tables had marble tops so that they could take notes while conversing directly on the table. Stefan Banach's wife gave the mathematicians a large notebook, which was used for writing the problems and solutions, and which eventually came to be known as the Scottish Book, in order to prevent the results from being lost and because she got frustrated with their writing directly on the table tops. The book, which was a collection of issues that had been resolved, unresolved issues, and even issues that were unquestionably intractable, was available for checkout by any customers of the café [1].

2. The Academic Scene
Unfortunately, because there is so little history to recount, it is very simple to place the Lwów School of Mathematics which was born at the Scottish Café in the context of Polish mathematics as a whole. Poland had been divided among Austria, Germany, and Russia since 1795. As a result, it mainly missed the sciences rapid development in the 19th century. Literature and poetry were crucial for promoting the idea of independence and even for the language's preservation in the 19th century, according to Zelazko, who writes that “the primary effort of the nation was given to the humanities.” [2] He claims that because the mathematical work was written in Polish, it was lost to the rest of the world.
When World War I came to a conclusion, the partition was resolved, and a revived Poland sprang into vibrant intellectual life. The cities of Warsaw and Lwów were the main forces behind this mathematical renaissance. Under Zygmunt Janiszewski's leadership, the new publication Fundamenta Mathematicae at Warsaw University became the first specialised journal in the subject of mathematics. Sadly, Janiszewski passed away just as issue one was being typeset.
The University and the Polytechnic were the two educational facilities in Lwów that were relevant to mathematics. In 1661, the Jan Kazimierz University, as it was then known, was established. The University had 5,900 students enrolled in its five faculties in 1934–1935, 870 of whom were in the Mathematical Biological Faculty. Roman Catholicism made up 64% of the larger religious organisations, followed by Judaism (21%), and Greek Catholicism (13%). In the beginning, the Polytechnic was founded in 1816. It was known as Lwów Polytechnic School throughout the interwar period and Lwów Polytechnic beginning in 1921 [3].

Both of these institutions the Polytechnic and the University were seen as having equal worth. People appeared to transition between them with ease, adhering to specific professors or ideologies. As a result, it is difficult to distinguish between employees and institutions. People would attend one to study and attend the other to become a professor or instructor. Students would go to either or both lectures.

The literature gives off the strong sense that the Lwów students were genuinely passionate about their studies. Mathematical conversations would erupt outside of the lecture halls and into the cafes and streets. There have been mathematical tea parties. Students would frequently go to each other's homes to discuss mathematically related subjects. Memoirs regularly discuss the ferocious atmosphere that surrounded this most enigmatic of themes, not just in Lwów but also in Warsaw and Krakow.

The Lwów School was largely influenced by a spirited mathematical conversation that took place in Kraków's Planty Park in 1916. In their 20s, Stefan Banach and Otto Nikodym were conversing about the Lebesgue integral while sitting on a park bench. They had a strong commitment to it. There was an uproar. Hugo Steinhaus, a lancer in the Polish army and holder of a doctorate in mathematics from Gottingen, went by in sharp military attire. He stood up and introduced himself as he watched the two argue [4]. Later, Nikodym and Banach travelled to Lwów to enrol in Steinhaus' Lwów Polytechnic. Nikodym went on to have an illustrious career as a Professor of Mathematics in Warsaw and then in the United States of America. Stefan Banach became Stefan Banach

3. Stefan Banach

It took some time before Banach's brilliance in mathematics was acknowledged. His early years had not been very favourable. His mother was unknown, while his father Stefan Greczek worked as a railway official (his surname comes from a laundress, Katarzyna Banach, who played no further role in his life). Years later, Greczek revealed to his renowned son that he was unable to wed his mother due to his financial situation. Stefan was a young man who lived in careful poverty. He began instructing when he was still a student. He didn't ever quit.

We will never know if the Lebesgue integral conversation in Planty Park was actually a tutorial by Banach, but one crucial outcome was that he had a friend in Steinhaus who had gone on to become a professor there when he arrived to study at the Lwów Polytechnic. With Steinhaus' assistance, Banach eventually rose through the University's ranks to become an assistant professor, a professor, and then the Faculty Dean. This was despite the fact that he had no formal education; after struggling to pass his high school exit exams, he only finished two semesters of university study.

However, it made little difference because Banach was manifestly unique. He was given a challenging challenge to tackle by Prof. Steinhaus shortly after his arrival at Lwów, which he accomplished in a single day with ease. Their first joint publication was based on the solution, and they continued to work together after that, co-authoring papers, founding a math journal (Studia Mathematicise), and contributing problems to the Scottish Book. Banach got married to Lucy and continued his career. Life was good.

He truly was an original. The fact that he was tall, blond, and blue-eyed, informal, and perpetually broke (academic salaries were a joke), led him to leave his previous position in the Café Roma and move across the street to the Scottish Café.

The Lwów School of Mathematics at The Scottish Café was centred around Banach. He was employable anywhere. He wasn't bothered by noise. His favoured working position was near to the orchestra in concert halls. He continued thinking with his renowned lucidity. He enjoyed drinking cognac and coffee and conversing about mathematics with nearly anyone while relaxing in the Scottish Café, but you had to be prepared for it since he didn't put up with fools. In contrast to the dapper Prof. Steinhaus, Banach was more casual, to the point where his short-sleeved shirts, lack of a tie, and approachable demeanour with students were viewed as everything but professorial.

However, it was his influence and his relentless pursuit of concepts and answers that fuelled the Lwów School of Mathematics. The first few meetings were sporadic, but after that they began to follow a predictable, daily schedule. Mathematicians crowded inside the Scottish Café. Herman Auerbach, Stanislaw Mazur, and Stanislaw Ulam, three of Banach's prize pupils, were seated at the head table with him every time, but many other people joined them; some of them went on to become well-known around the world. There are almost 30 mathematicians affiliated with the Lwów School of Mathematics, and it is outside the scope of this paper to go into more depth about any one of their lives or bodies of work than is necessary. What were they discussing? In line with Ulam [5]:

The set theory subfields the foundation of set theory, set topology, and later under the influence of Banach and Steinhaus functional analysis with applications to classical analysis were the primary drivers of the original research endeavour. In his lectures at universities, Schauder focused on partial differential equations. His techniques and outcomes are still widely used today. The widely used approach to solving analysis problems by applying geometric methods of function spaces was developed by Banach, Mazur, and Schauder.

It became standard practise to formulate ideas as problems and then share them in what became the American “brainstorm” developed years later as a formal technique at Los Alamos by Ulam, von Neumann, and others [6] as the mathematical conversations in functional analysis, set theory, topology, and probability grew wider and deeper.

Sometimes they started by looking for a problem. Then someone drew a straightforward figure or put characters like $y = f(x - t - y)$ on the tabletop. This heuristic frequently resulted in the formulation of novel issues or the creation of new mathematical paradoxes. Most frequently, Banach, Ulam, or Mazur arrived at the café with a well stated issue. The journey started. The session's attendees made an effort to consider the problem. Their minds were free to stray throughout Banach's rooms. There were extended periods of thought, focus, and tension. They either thought logically or intuitively. Banach was a computer that processed information instantly, but not everyone could keep up. He did not like to walk around the beaten tracks, but he looked for new ways, looked for distant and surprising associations, often looked into each other's eyes, drank coffee or cognac, burned out a lot of cigarettes: they drank too much, they smoked too much, and time seemed to have stopped.

What a beautiful representation of group thinking - this brooding group of men (at the time, there were no female mathematicians on the faculty), focused, muttering the odd comment, engrossed in the issue. After that, [6]:

Often it happened that one of the scholars -sometimes Banach, sometimes Ulam. sometimes Mazur – experienced a revelation… Like a flash of light, there was the initial idea of a solution. If it was an interesting trail, they would write it on

a table top or on napkins. Together, they experienced a feeling of joy and relief, satisfaction and pride. Then they tried to prove the claim using the deductive method. Table tops were densely covered with mathematical signs. When Banach saw that they were beginning to err, he never expressed strong opposition. Rather, he posed new questions, made gentle remarks that often allowed him to find the right path.

Apart from illustrating the method, this quotation points to the very real need for The Scottish Book.

4. The Scottish Book

Armed with soft lead pencils, the mathematicians entered the Scottish Café. These were used to target the marble table surfaces, which were simple to write on and, more importantly, easily erasable. According to one commenter, The Scottish Café was set up in the Viennese style. They found it very helpful to utilise little tables with marble tops as slates that they covered in numbers. Initially, this did not cause the owner to express excessive joy, but over time, Zielinski grew accustomed to this "ruination" of his land. After all, serious university and polytechnic professors were seated at the tables rather than juvenile idiots [6].

After one of Banach's tremendous evening spurts of mathematical brilliance had passed by the next morning, according to Urbanek [7].

When Professor Lomnicki first entered the room, he instructed everyone to "always leave the writing on the table as it is and store the table somewhere till the next day" in case it happened again. The tables that had been marked up were then covered with a cloth and placed aside. Cleaning crews were instructed not to wipe such tables when they arrived in the morning, and around 11 o'clock a student would enter and write down the shambles. In a pinch, this method worked, but nobody was pleased with the results.

In 1933 or 1934, we resolved to give our existing formulation of the issues and conclusions of the discussion a more lasting form, according to Ulam [8]. The decision-making procedure is described in the following way by Bakula. Tomasz Zieliski, the proprietor of the Scottish Café, came up with the notion of how to save all these theories, questions, and difficulties so that they would not perish for ever. He could simultaneously defend the priceless marble tops from being destroyed by the pressure of lead and water. This is the account of how one of the greatest books in mathematics history came to be [9]. Rakhiel clarifies the origin myth as follows: Indelible pencils ruined marble tables, and the cafe owner complained about it to Banach's wife [10].

The words "Ksienga Szkockza" (The Scottish Book) were scratched on the top page by someone, perhaps Banach since it was his notebook and he had given the first problem. After that, pages were filled in with a problem on the left and any solutions (if there were any; many problems are still open today) on the right. The date was entered, along with the names of the problem creator and solution. Small rewards were given out for solutions, including a bottle of wine, five beers, and a live goose, which was given to Per Enfilo of Sweden in 1972 when he submitted a negative answer to problem 153. Mazur. Although the awards imply some light-heartedness, adding issues to the novel was not an impulsive decision. Prior to being considered for "formal" inclusion into the "Book," Ulam stated that "the majority of the submitted issues were expected to have had considerable attention given to them." [8] This explains why it has remained relevant and durable [11].

Despite the claim that the book has 193 numbered problems, there were a few more. Some had no numbers, and others were advertised as second or third instalments of earlier issues when they were actually brand-new. There are 198 problems in all, according to Duda [12]. Thirty mathematicians contributed, with Banach, Mazur, Ulam, and Orlicz dominating the area in the early years and Schreier, Auerbach, and Steinhaus also making significant contributions. The first few years are concluded by Kuratowski, Schauder, Ruziewicz, and Lomnicki. Later authors include well-known foreigners such American John von Neumann and Cyril Offord, the first mathematics professor at the London School of Economics.

Divergent views exist regarding the caretaker of the Book in the Café and the location of its hiding place while not in use. The headwaiter is mentioned as the guardian who "would, upon demand, fetch it out of some secure hiding spot, leave it at the table, and when the guests had left, restore it to its secret location." [8] According to Ulam, This notebook was stored at the Cafe and was brought by the waiter upon request. We filled out the problem, and the waiter dutifully returned it to the location where it was hidden [10]. Hugo Steinhaus asserts that the Scottish Book was kept by a hired employee rather than the cloakroom attendant, a waiter, or the owner [4], although Bogdan Mis maintains that the book was "stored in a cloakroom and provided to mathematicians on request: anyone may post a solution to a problem." [13] The Book was presented to the proprietor, Tomasz Zielinski, after the cafe had closed, who "handled it extremely carefully. He obviously appreciated its historical significance. [10].

As the years passed, the mathematicians at the Scottish Café kept up their mental gymnastics and added the results of their talks and thought experiments to the book. Many of the right-hand pages included answers, whereas the left-hand ones mostly contained questions. In the cited literature, a thorough examination of the book and its multiple authors has been done.

The book thus proven to be beneficial for both the discipline of mathematics and Mr. Zielinski's marble tables. The book still has unquestionable significance as a store and source of unresolved problems, and the findings from this work have influenced a significant portion of current mathematics.

5. Conclusions

This paper focuses on a mathematician Stefan Banach story and his conversations with other great mathematicians in the Scottish Café in the years. The mathematicians initially relates the practical applications of mathematics in subsequent years and mathematicians discussed research problems, particularly in functional analysis and topology from the Lwow School of mathematics. Also, describes the suggestion on "The Scottish Book".

6. References

1. Roman K. Ann Kostant and Wojbor Woyczyński (ed.). Through a reporter's eyes: The life of Stefan Banach. Birkhäuser; c1996. ISBN 0-8176-3772-9. MR 1392949
2. Zelazko W. A short history of Polish mathematics, Mathematical Institute, Polish Academy of Sciences, Warsaw; c2017. p. 1.
3. https://en.wikipedia.org/wiki/University_of_Lvivlast accessed 2018/11/21.
4. Steinhaus H, Banach S. Studia Mathematica, Seria Specjalna. 1963;1:7-15
5. Ulam S. Annals of the Polish Mathematical Society

Series II: Wiadomosci matematyczne/Messiah Announcements. 1969;12:49-58.
6. Jakimowicz E, Miranowicz A, Szkockiej AW. *Burzew Muzgu* (*Brainstorm in the Scottish Café*); c2017.
7. Urbanek M, Genialni. Lwowska szkola matematyczna. Wydawnictwo Iskru, Warsaw; c2014. p. 111.
8. Ulam SM. University of California Press, Oakland; c1991. p. 33. ISBN 0-520-07154-9.
9. Bakula B. Comparison as evidence. Polish-Ukrainian cultural, literary and historical relations. Wyd. Naukowe UAM, Poznań; c2001. p. 207-219.
10. Rakhiel Y. Scottish Book. Lviv's mathematical relic – About the role of Ukrainian scientists in the formation of new mathematics; c2010. http://day.kyiv.ua/en/article/society/scottish-book-lvivs-mathematical-relic.
11. Ulam S. Mathematics from the Scottish Café, Ed by Mauldin, R.D., Birkhäuser, Boston; c1981. ISBN 3-7643-3045-7
12. Duda R, Matematyczna LS. 2nd Ed Uniwersytet Wrocławski, Wroclaw; c2014. ISBN: 978-83-229-3211-7 (2014) (Also available in English as Duda, R. Pearls from a Lost City: The Lvov School of Mathematics. (translated from Polish by Daniel Davies). American Mathematical Society, Providence, USA. ISBN 978-1470410766)
13. Miś B. Tales of the Scottish Book. Perspectives. 1969;12:17-19.