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Automorphisms of non-abelian metabelian groups of order upto 24

[1]Muniya and [2]Harsha Arora
[1]Shri Jagdishprasad Jhabarmal Tibrewala University, Rajasthan, India
[2]Goverment College for Women, Hisar, Haryana, India

Abstract
In this paper, G be the non-abelian metabelian group and $Aut(G)$ denotes the automorphism group of group G. A group is metabelian group if its commutator group is abelian or if it has an abelian normal subgroupin which the factor group is also abelian. There are 35 non-Abelian metabelian groups of order less than or equal to 24. In this study, we investigate the automorphism groups of all non-abelian metabelian groups of order less than or equal to 24 and the verification has been made through GAP (Groups Algorithm Programming) software.

Keywords: metabelian groups, automorphism, order

Introduction
Let G be a finite metabelian group, Z_n denotes the cyclic group of order n, S_n denotes the permutation group of degree n, D_n denotes the dihedral group of order $2n$, Q_n denotes quatornion group. A group G is said to be metabelian if G', the derived subgroup of G is abelian. Much has been investigated about the properties of metabelian groups in the literature. In paper [3], structure of metabelian groups of order upto 24 has been described. In paper [4], the authors studied about the conjugacy classes of metabelian groups of order less than 24. In paper [1, 2], automorphisms of some non-abelian groups of order p^4 are computed. In the present paper, we shall find the automorphisms of metabelian groups of order less than equal to 24.
In [3], Rehman Abdul described the metabelian groups of order less than or equal to 24.

(1) $D_3 \cong S_3 \cong < a, b; a^3 = b^2 = 1, b\,a\,b = a^{-1} >$.

(2) $D_4 \cong < a, b; a^4 = b^2 = 1, b\,a\,b = a^{-1} >$.

(3) $Q_3 \cong < a, b: a^4 = 1, b^2 = a^2, a\,b\,a = b >$.

(4) $D_5 \cong < a, b: a^5 = b^2 = 1, b\,a\,b = a^{-1} >$.

(5) $Z_3 \rtimes Z_4 \cong < a, b: a^3 = b^4 = 1, b^{-1} a\,b = a^2 >$.

(6) $A_4 \cong < a, b, c: a^2 = b^2 = c^3 = 1, b\,a = a\,b, c\,a = a\,b\,c, c\,b = a\,c >$.

(7) $D_6 \cong < a, b: a^6 = b^2 = 1, b\,a\,b = a^{-1} >$.

(8) $D_7 \cong < a, b: a^7 = b^2 = 1, b\,a\,b = a^{-1} >$.

(9) $D_8 \cong < a, b: a^8 = b^2 = 1, b\,a\,b = a^{-1} >$.

(10) $G \cong < a, b: a^8 = b^2 = 1, b\,a\,b = a^3 >$.

(11) $Q_{16} \cong < a, b: a^8 = 1, \quad a^4 = b^2, \quad a\,b\,a = b >$.

(12) $D_4 \times Z_2 \cong < a, b, c: a^4 = b^2 = c^2 = 1, a\,c = c\,a, b\,c = c\,b, b\,a\,b = a^{-1} >$.

(13) $Q_3 \times Z_2 \cong < a, b, c: a^4 = b^4 = c^2 = 1, b^2 = a^2, b\,a = a^3 b, a\,c = c\,a, b\,c = c\,b >$.

$(14) Modular - 16 = G \cong < a, b: a^8 = b^2 = 1, a\ b = b\ a^5 >.$

$(15) B \cong < a, b: a^4 = b^4 = 1, a\ b = b\ a^3 >.$

$(16) K \cong < a, b, c: a^4 = b^2 = c^2 = 1, a\ b = b\ a, a\ c = c\ a, c\ b = a^2 b\ c >.$

$(17) G_{4,4} \cong < a, b: a^4 = b^4 = (a\ b)^2 = 1, a\ b^3 = b\ a^3 >.$
$(18) D_9 \cong < a, b: a^9 = b^2 = 1, b\ a\ b = a^{-1} >.$

$(19) S_3 \times Z_3 \cong < a, b, c: a^3 = b^2 = c^3 = 1, b\ a = a^{-1} b, a\ c = c\ a, b\ c = c\ b >.$

$(20) (Z_3 \times Z_3) \rtimes Z_2 \cong < a, b, c: a^2 = b^3 = c^3 = 1, b\ c = c\ b, b\ a\ b = a, c\ a\ c = a >.$

$(21) D_{10} \cong < a, b: a^{10} = b^2 = 1, b\ a\ b = a^{-1} >.$

$(22) Fr_{20} \cong Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, b\ a = a\ b^2 >.$

$(23) Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, b\ a\ b = a >.$

$(24) Fr_{21} \cong Z_7 \rtimes Z_3 \cong < a, b: a^3 = b^7 = 1, b\ a = a\ b^2 >.$

$(25) D_{11} \cong < a, b: a^{11} = b^2 = 1, b\ a\ b = a^{-1} >.$

$(26) S_3 \times Z_4 \cong < a, b, c: a^3 = b^2 = c^4 = 1, a^b = a^{-1}, a\ c = c\ a, b\ c = c\ b >.$

$$(27) S_3 \times Z_2 \times Z_2 \cong < a, b, c, d: a^3 = b^2 = c^2 = d^2 = 1, a^b = a^{-1}, a\ c = c\ a, a\ d = d\ a, b\ c = c\ b, b\ d = d\ b, c\ d = d\ c >.$$

$(28) D_4 \times Z_3 \cong < a, b, c: a^3 = b^4 = c^2 = 1, b^c = b^{-1}, a\ b = b\ a, a\ c = c\ a >.$

$(29) Q \times Z_3 \cong < a, b, c: a^4 = c^3 = 1, a^2 = b^2, a^b = a^{-1}, b\ c = c\ b, a\ c = c\ a >.$

$$(30) A_4 \times Z_2 \cong < a, b, c, d: a^2 = b^2 = c^3 = d^2 = 1, a\ b = b\ a, c\ a = a\ b\ c, a\ d = d\ a, a\ c = c\ b, b\ d = d\ b, c\ d = d\ c >.$$

$(31) Q_{12} \cong < a, b: a^{12} = 1, a^6 = b^2, b^{-1} a\ b = a^{-1} >.$

$(32) D_{12} \cong < a, b: a^{12} = b^2 = 1, b\ a\ b = a^{-1} >.$

$(33) Z_6 \rtimes Z_4 \cong < a, b: a^4 = b^6 = 1, b\ a\ b = a >.$

$(34)\ Z_3 \rtimes Z_8 \cong < a, b: a^3 = b^8 = 1, b\ a\ b^{-1} = a^{-1} >.$

$(35)\ Z_3 \rtimes Q \cong < a, b, c: a^2 = b^6 = c^2 = 1, ab = ba, \quad ac = ca, (cb)^2 = 1 >.$

Automorphisms of all non-abelian metabelian groups of order less than equal to 24
Automorphisms of groups of order 6
Automorphisms of D_3

$D_3 \cong S_3 \cong < a, b; a^3 = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_3)$ sends the generators of D_3 as
$a \to a^i; (i, 3) = 1,$
$b \to a^i b; 1 \le i \le 3.$
Thus,
$|Aut(D_3)| = 6.$

Automorphisms of Q_3
Automorphisms of groups of order 8
Automorphisms of D_4

$D_4 \cong < a, b; a^4 = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_4)$ sends the generators of D_4 as

$a \to a^i; (i, 2) = 1, 1 \leq i \leq 4,$

$b \to a^j b; 1 \leq i \leq 4.$

Hence

$|Aut(D_4)| = 8.$

$Q_3 \cong < a, b: a^4 = 1, b^2 = a^2, aba = b >$

Any automorphism $\psi \in Aut(Q_3)$ sends the generators of Q_3 as

$a \to a^i b^j; \quad i \in \{0,1,3\}, \ j \in \{0,1,3\}, \ \ i + j \neq 0,$

$b \to a^i b^j; \quad i \in \{0,1,3\}, \ j \in \{0,1,3\}, \ \ i + j \neq 0.$

Therefore,

$|Aut(Q_3)| = 24.$

Automorphisms of groups of order 10
Automorphisms of D_5

$D_5 \cong < a, b: a^5 = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_5)$ sends the generators of D_5 as

$a \to a^i; (i, 5) = 1,$

$b \to a^j b; 1 \leq j \leq 5.$

Thus,

$|Aut(D_5)| = 20.$

Automorphisms of groups of order 12
Automorphisms of $Z_3 \rtimes Z_4$

$Z_3 \rtimes Z_4 \cong < a, b: a^3 = b^4 = 1, b^{-1}ab = a^2 >$

Any automorphism $\psi \in Aut(Z_3 \rtimes Z_4)$ sends the generators of $Z_3 \rtimes Z_4$ as

$a \to a^i; 1 \leq i < 3,$

$b \to a^i b^j; 2 \nmid j, 0 \leq i \leq 2, 1 \leq j \leq 3.$

Thus,

$|Aut(Z_3 \rtimes Z_4)| = 12.$

Automorphisms of A_4

$A_4 \cong < a, b, c : a^2 = b^2 = c^3 = 1, ba = ab, ca = abc, cb = ac >$

Any automorphism $\psi \in Aut(A_4)$ sends the generators of A_4 as

$a \to a^i b^j ; 0 \leq i, j \leq 1, \quad i + j \neq 0,$

$b \to a^i b^j ; 0 \leq i, j \leq 1, \quad i + j \neq 0,$

$c \to a^i b^j c^k ; 0 \leq i, j \leq 1, \quad k \in \{1,2\}.$

So

$|Aut(A_4)| = 24.$

Automorphisms of D_6

$D_6 \cong < a, b : a^6 = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_6)$ sends the generators as

$a \to a^i ; \quad i \in \{1,5\},$

$b \to a^i b ; 0 \leq i \leq 5.$

So

$|Aut(D_6)| = 12.$

Automorphisms of groups of order 14
Automorphisms of D_7

$D_7 =< a, b : a^7 = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_7)$ sends the generators of D_7 as

$a \to a^i ; 1 \leq i \leq 6,$

$b \to a^j b ; 0 \leq j \leq 6.$

So

$|Aut(D_7)| = 42.$

Automorphisms of groups of order 16
Automorphisms of D_8
Any automorphism $\psi \in Aut(D_8)$ sends the generators of D_8 as

$a \to a^i ; (i, 2) = 1, 1 \leq i < 8,$

$b \to a^j b ; 0 \leq i \leq 7.$
So

$|Aut(D_8)| = 32.$

Automorphisms of G

$G \cong< a, b: a^8 = b^2 = 1, bab = a^3 >$

Any automorphism $\psi \in Aut(G)$ sends the generators of G as

$a \to a^i; (2, i) = 1, 1 \le i \le 8,$

$b \to a^i b; 2/i, 0 \le i < 8.$

Thus,

$|Aut(G)| = 16.$

Automorphisms of Q_{16}

$Q_{16} \cong< a, b: a^8 = 1, a^4 = b^2, \quad aba = b >$

Any automorphism $\psi \in Aut(Q_{16})$ sends the generators of Q_{16} as

$a \to a^i; (2, i) = 1, 1 \le i \le 8,$

$b \to a^i b^j; 0 \le i \le 3, \quad j \in \{1,3\}.$

Thus,

$|Aut(Q_{16})| = 32.$

Automorphisms of $D_4 \times Z_2$

$D_4 \times Z_2 \cong< a, b, c: a^4 = b^2 = c^2 = 1, ac = ca, bc = cb, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_4 \times Z_2)$ sends the generators of $D_4 \times Z_2$ as

$a \to a^i c^j; \quad i \in \{1,3\}, \quad j \in \{0,1\},$

$b \to a^k bc^l; 0 \le k \le 3, \quad l \in \{0,1\},$

$c \to a^m c; \quad m \in \{0,2\}.$

Hence
$|Aut(D_4 \times Z_2)| = 64.$

Automorphisms of $Q_3 \times Z_2$

$Q_3 \times Z_2 \cong< a, b, c: a^4 = b^4 = c^2 = 1, b^2 = a^2, ba = a^3 b, ac = ca, bc = cb >$

Any automorphism $\psi \in Aut(Q_3 \times Z_2)$ sends the generators of $Q_3 \times Z_2$ as

$a \to a^i b^j c^k; \quad i, j \in \{0,1,3\}, \quad k \in \{0,1\}, \quad i + j \ne 0,$

$b \to a^l b^m c^n; \quad l, m \in \{0,1,3\}, \quad n \in \{0,1\}, \quad i + j \ne 0,$

$c \to a^q c; \quad q \in \{0,2\}.$
So

$|Aut(Q_3 \times Z_2)| = 96.$

Automorphisms of $Modular-16$

$Modular-16 = G \cong < a, b: a^8 = b^2 = 1, ab = ba^5 >$

Any automorphism $\psi \in Aut(G)$ sends the generators of G as

$a \to a^i b^j; 2 \nmid i, 1 \leq i \leq 8, \quad j \in \{0,1\},$

$b \to a^r b; \quad r \in \{0,4\}.$

So

$|Aut(G)| = 16.$

Automorphisms of B

$B \cong < a, b: a^4 = b^4 = 1, ab = ba^3 >$

Any automorphism $\psi \in Aut(B)$ sends the generators of B as

$a \to a^i b^j; \quad i \in \{1,3\}, \quad j \in \{0,2\},$

$b \to a^l b^m; 0 \leq l \leq 3, \quad m \in \{1,3\}.$

Thus,

$|Aut(B)| = 32.$

Automorphisms of K

$K \cong < a, b, c: a^4 = b^2 = c^2 = 1, ab = ba, ac = ca, cb = a^2bc >$

Any automorphism $\psi \in Aut(K)$ sends the generators of K as $a \to a^i; \quad i \in \{1,3\},$

$b \to b, c, a^2b, a^2c, abc, a^3bc,$

$c \to b, c, a^2b, a^2c, abc, a^3bc.$

So

$|Aut(K)| = 48.$

Automorphisms of $G_{4,4}$

$G_{4,4} \cong < a, b: a^4 = b^4 = (ab)^2 = 1, ab^3 = ba^3 >$

Any automorphism $\psi \in Aut(G_{4,4})$ sends the generators of $G_{4,4}$ as

$a \to a, a^3, b, b^3, ab^2, a^2b, a^2b^3, a^3b^2,$

$b \to a, a^3, b, b^3, ab^2, a^2b, a^2b^3, a^3b^2.$

Thus,

$|Aut(G_{4,4})| = 32.$

Automorphisms of groups of order 18
Automorphisms of D_9

$D_9 \cong < a, b: a^9 = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_9)$ sends the generators as

$a \to a^i; 3 \nmid i, 1 \le i \le 8,$

$b \to a^j b; 0 \le j \le 8,$

So

$|Aut(D_9)| = 54.$

Automorphisms of $S_3 \times Z_3$

$S_3 \times Z_3 \cong < a, b, c: a^3 = b^2 = c^3 = 1, ba = a^{-1}b, ac = ca, bc = cb >$

Any automorphism $\psi \in Aut(S_3 \times Z_3)$ sends the generators of $S_3 \times Z_3$ as

$a \to a^i; 1 \le i \le 2,$

$b \to a^j b; 0 \le j \le 2,$

$c \to c^k; 1 \le k \le 2.$

So

$|Aut(S_3 \times Z_3)| = 12.$

Automorphisms of $(Z_3 \times Z_3) \rtimes Z_2$

$(Z_3 \times Z_3) \rtimes Z_2 \cong < a, b, c: a^2 = b^3 = c^3 = 1, bc = cb, bab = a, cac = a >$

Any automorphism $\psi \in Aut((Z_3 \times Z_3) \rtimes Z_2)$ sends the generators of $(Z_3 \times Z_3) \rtimes Z_2$ as

$a \to ab^i c^j; 0 \le i, j \le 2,$

$b \to b^k c^l; 0 \le k, l \le 2, \quad k + l \ne 0,$

$c \to b^m c^n; 0 \le m, n \le 2, \quad m + n \ne 0.$

So

$|Aut((Z_3 \times Z_3) \rtimes Z_2)| = 432.$

Automorphisms of groups of order 20
Automorphisms of D_{10}

$D_{10} \cong < a, b: a^{10} = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_{10})$ sends the generators as

$a \to a^i; 2 \nmid i, 5 \nmid i, 1 \le i < 10,$

$b \to a^j b; 1 \le j \le 10.$
So

$|Aut(D_{10})| = 40.$

Automorphisms of Fr_{20}

$Fr_{20} \cong Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, ba = ab^2 >$

Any automorphism $\psi \in Aut(Fr_{20})$ sends the generators of Fr_{20} as

$a \to ab^i; 0 \le i \le 4,$

$b \to b^j; 1 \le j \le 4.$

Thus,

$|Aut(Fr_{20})| = 20.$

Automorphisms of $Z_5 \rtimes Z_4$

$Z_5 \rtimes Z_4 \cong < a, b: a^4 = b^5 = 1, bab = a >$

Any automorphism $\psi \in Aut(Z_5 \rtimes Z_4)$ sends the generators of $Z_5 \rtimes Z_4$ as

$a \to a^i b^j; \quad i \in \{1,3\} 0 \le j \le 4,$

$b \to b^k; 1 \le k \le 4.$

Thus,

$|Aut(Z_5 \rtimes Z_4)| = 40.$

Automorphisms of groups of order 21
Automorphisms of Fr_{21}

$Fr_{21} \cong Z_7 \rtimes Z_3 \cong < a, b: a^3 = b^7 = 1, ba = ab^2 >$

Any automorphism $\psi \in Aut(Fr_{21})$ sends the generators of Fr_{21} as

$a \to ab^i; 0 \le i \le 6,$

$b \to b^j; 1 \le j \le 6.$

Thus,

$|Aut(Fr_{21})| = 42.$

Automorphisms of groups of order 22
Automorphisms of D_{11}

$D_{11} = < a, b: a^{11} = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_{11})$ sends the generators of D_{11} as

$a \to a^i; 1 \le i \le 10,$

$b \to a^j b; 0 \le j \le 10.$
So

$|Aut(D_{11})| = 110.$

Automorphisms of groups of order 24
Automorphisms of $S_3 \times Z_4$

$S_3 \times Z_4 \cong < x, y, z: x^3 = y^2 = z^4 = 1, x^y = x^{-1}, xz = zx, yz = zy >$
Any automorphism $\psi \in Aut(S_3 \times Z_4)$ sends the generators of $S_3 \times Z_4$ as

$x \to x^i; 1 \le i \le 2,$

$y \to x^j yz^k; 0 \le j \le 2, \quad k \in \{0,2\},$

$z \to z^l; \quad l \in \{1,3\}.$

So

$|Aut(S_3 \times Z_4)| = 24.$

Automorphisms of $S_3 \times Z_2 \times Z_2$

$$S_3 \times Z_2 \times Z_2 \cong < x, y, z, w: x^3 = y^2 = z^2 = w^2 = 1, x^y = x^{-1}, xz = zx, xw = wx, yz = zy, yw = wy, zw = wz >$$

Any automorphism $\psi \in Aut(S_3 \times Z_2 \times Z_2)$ sends the generators of $S_3 \times Z_2 \times Z_2$ as

$x \to x^i; 1 \le i \le 2,$

$y \to x^j yz^k w^l; 0 \le j \le 2, 0 \le k, l \le 1,$

$z \to z^u w^v; 0 \le u, v \le 1, \quad u + v \ne 0.$

So

$|Aut(S_3 \times Z_2 \times Z_2)| = 144.$

Automorphisms of $D_4 \times Z_3$

$D_4 \times Z_3 \cong < x, y, z: x^3 = y^4 = z^2 = 1, y^z = y^{-1}, xy = yx, xz = zx >$

Any automorphism $\psi \in Aut(D_4 \times Z_3)$ sends the generators of $D_4 \times Z_3$ as

$x \to x^i; 1 \le i \le 2,$

$y \to y^j; \quad j \in \{1,3\},$

$z \to y^k z; 0 \le k \le 3.$

So

$|Aut(D_4 \times Z_3)| = 16.$

Automorphisms of $Q \times Z_3$:

$Q \times Z_3 \cong < x, y, z: x^4 = z^3 = 1, x^2 = y^2, x^y = x^{-1}, yz = zy, xz = zx >$

Any automorphism $\psi \in Aut(Q \times Z_3)$ sends the generators of $Q \times Z_3$ as

$x \rightarrow x^i y^j; 0 \leq i \leq 3, 0 \leq j \leq 1,$

$y \rightarrow x^l y^m; 0 \leq l \leq 3, 0 \leq m \leq 1,$

$z \rightarrow z^k; 1 \leq k \leq 2.$

So

$|Aut(Q \times Z_3)| = 48.$

Automorphisms of

$A_4 \times Z_2$:

$$A_4 \times Z_2 \cong < x, y, z, w: x^2 = y^2 = z^3 = w^2 = 1, xy = yx, zx = xyz, xw = wx, xz = zy, yw = wy, zw = wz >$$

Any automorphism $\psi \in Aut(A_4 \times Z_2)$ sends the generators of $A_4 \times Z_2$ as

$x \rightarrow x^i y^j; 0 \leq i, j \leq 1, \quad i + j \neq 0,$

$y \rightarrow x^k y^l; 0 \leq k, l \leq 1, k + l \neq 0,$

$z \rightarrow x^l y^m z^n; 0 \leq l, m \leq 1, 1 \leq n \leq 2,$

$w \rightarrow w.$

So

$|Aut(A_4 \times Z_2)| = 24.$

Automorphisms of D_{12}

$D_{12} =< a, b: a^{12} = b^2 = 1, bab = a^{-1} >$

Any automorphism $\psi \in Aut(D_{12})$ sends the generators of D_{12} as

$a \rightarrow a^i; 1 \leq i \leq 12, (i, 12) = 1,$

$b \rightarrow a^j b; 0 \leq j \leq 11.$

So

$|Aut(D_{12})| = 48.$

Automorphisms of Q_{12}

$Q_{12} =< a, b: a^{12} = 1, a^6 = b^2, b^{-1}ab = a^{-1} >$

Any automorphism $\psi \in Aut(Q_{12})$ sends the generators of Q_{12} as

$a \rightarrow a^i; 1 \leq i \leq 12, (i, 12) = 1$

$b \to a^j b; 0 \le j \le 11$

So

$|Aut(Q_{12})| = 48.$

Automorphisms of $Z_6 \rtimes Z_4$

$Z_6 \rtimes Z_4 \cong < x, y: x^4 = y^6 = 1, yxy = x >$

Any automorphism $\psi \in Aut(Z_6 \rtimes Z_4)$ sends the generators of $Z_6 \rtimes Z_4$ as

$x \to x^i y^j; \quad i \in \{1,3\}, 0 \le j \le 5,$

$y \to x^k y^l; \quad k \in \{0,2\}, \quad l \in \{1,5\}.$

So

$|Aut(Z_6 \rtimes Z_4)| = 48.$

Automorphisms of $Z_3 \rtimes Z_8$

$Z_3 \rtimes Z_8 \cong < x, y: x^3 = y^8 = 1, yxy^{-1} = x^{-1} >$

Any automorphism $\psi \in Aut(Z_3 \rtimes Z_8)$ sends the generators of $Z_3 \rtimes Z_8$ as

$x \to x^i; 1 \le i \le 2,$

$y \to x^k y^l; 0 \le k \le 2, (2, l) = 1, 1 \le l \le 8.$

So $|Aut(Z_6 \rtimes Z_4)| = 24.$

Automorphisms of $Z_3 \rtimes Q$

$Z_3 \rtimes Q \cong < x, y, z: x^2 = y^6 = z^2 = 1, xy = yx, xz = zx, (zy)^2 = 1 >$

Any automorphism $\psi \in Aut(Z_3 \rtimes Q)$ sends the generators of $Z_3 \rtimes Q$ as

$x \to x,$

$y \to x^l y^m; 0 \le l \le 1, \quad m \in \{1,5\},$

$z \to x^i y^j z; 0 \le i \le 1, 0 \le j \le 5.$

So

$|Aut(Z_3 \rtimes Q)| = 48.$

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