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A new regression model for kumrsawamy pareto with application

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Abstract
In this research, a new regression model called Kumrsawamy Pareto regression was presented and some regression properties were completed, The parameters of the new regression were estimated by the Maximum likelihood. A simulation was made using the Monto Carlo method whereof (kwp) distribution applied to A set of real data with a view to of illustration is conducted Finally, the simulation study and its results using different R packages are obtained and displayed graphically.

Keywords: kumrsawamy distribution; pareto distribution; residual analysis, deviance component residual, for regression model

Introduction

The Pareto distribution is one of the distributions that has different applications in many fields. This distribution is named after a scientist in the field of economics, including, for example, economics, finance, life tests, and mining, which describes the occurrence of extreme weather.
Assuming that we have a random variable X that follows the Kumaraswamy Pareto distribution, we defined the cumulative distribution function as follows

$$G(x) = 1 - \left(\frac{\theta}{x}\right)^{\beta} \quad x > \theta \tag{1}$$

where $\theta > 0$ is a scale parameter and $\beta > 0$ is a shape parameter. The probability density function for the Kamurswamy Pareto distribution is as follows

$$g(x) = \frac{\beta\ \theta^{\beta}}{x^{\beta+1}} \qquad x > \theta \tag{2}$$

In this paper, we will combine the the generalized Kumaraswamy and the Pareto distribution in a new distribution called the Kumaraswmy Pareto distribution (KWP) The cumulative distribution function is defined as follows

$$F(x) = 1 - [1 - G(X)^{a}]^{b} \tag{3}$$

Where $G(x)$ is The cumulative distribution $a, b > 0$ are shape parameters which
And the probability density function (pdf)of equation No. 3 is defined as follows

$$f(x) = abg(x)G(x)^{a-1}[1 - G(X)^{a}]^{b-1} \tag{4}$$

many studies have dealt with the distribution of kumaraswamy Pareto, including as a study),): the gamma kumarsawmy-G familey distribution (Rana Muhammad Imran Arshad, at all, 2019):, Kumaraswamy log-logistic Weibull distribution (P. Mdlongwaa, at all 2019) [5], The Cakmakyapan, at all (2017) [7], Proposing a New Mixture Statistical Distribution Exponential – Kumaraswamy(Mohammed Qadoury,2018), Negative Binomial Kumaraswamy-G Cure The Kumaraswamy Pareto IV Distribution (M. H. Tahir, atall,2021), A Flexible Extension of Pareto Distribution: Properties and Applications(Alshanbari HM atall,2021) and log kumaraswamy Lindley regression models(samy abdelmoe, 2021) The research is organized as it comes in the second section. We introduce Kumaraswamy Pareto, we show him the probability density function, the cumulative distribution, the `quartile function, and some mathematical expressions that we use in the research in section three a new regression, some properties for example martingale residual, Deviance residual, estimation the parameters for regression by using maximum likelihood, in section 4 simulation study by using Monte Carlo to estimate model parameters, Finally, in section 6 some conclusions are addressed.

The KW-P

Substituting from Equation No. (1) into Equation No. (3), we obtained the cumulative function for the Kumaraswmy Pareto.(KWP)

$$F(x,a,b,\beta,\theta) = 1 - \left[1 - \left(1 - (\tfrac{\theta}{x})^{\beta}\right)^{a}\right]^{b} \quad x > \theta \tag{5}$$

where $\theta > 0$ is a scale parameter and $\beta > 0$ $a > 0$, $b > 0$ are a shape parameter. And the probability density function (pdf)of equation to (5) is

$$f(x,a,b,\beta,\theta) = \frac{ab\beta\,\theta^{\beta}}{x^{\beta+1}}\left(\left(1-\left(\frac{\theta}{x}\right)^{\beta}\right)\right)^{a-1}\left[1-\left(1-\left(\frac{\theta}{x}\right)^{\beta}\right)^{a}\right]^{b-1} \tag{6}$$

Quantile function is defined as an inverse of the distribution function. Consider the identity

$$F(x,a,b,\beta,\theta) = U \rightarrow X = F^{-1}(U)$$

Where U is defined uniform distribution $0 < U < 1$, quintile function is one of the very important functions in generating random data for any distribution. It can also be used to calculate kurtosis and skewness when moments of a random variable cannot be calculated., quintile function of the(KWP)distribution is

$$x = \theta\left[1 - \left[1 - [1-u]^{-\frac{1}{b}}\right]^{\frac{-1}{a}}\right]^{\frac{-1}{\beta}} \tag{7}$$

The three quartiles Q1, Q2 and Q3 can be obtained by using u = 0.25,0.50 and 0.75 in Eq.(7) respectively

Linear representation

After presenting the general form of the Kumaraswamy Pareto distribution in this part, we present the general form of the probability density function, as well as the cumulative distribution function. (KWP) Using series expansion

$$(1-W)^{b} = \sum_{i=0}^{\infty}(-1)^{k}\binom{b}{k}W^{K} \quad |W| < 1 \quad ,b > 0 \tag{8}$$

The cdf in (5) It can be expressed as follows

$$F(x,a,b,\beta,\theta) = 1 - \sum_{i=0}^{\infty} (-1)^{i}\binom{b}{i}\left(1-(\frac{\theta}{x})^{\beta}\right)^{ai}$$

$$F(x,a,b,\beta,\theta) = 1 - \sum_{i=0}^{\infty}\sigma_i\, H(x,a,b,\beta,\theta) \tag{9}$$

Where $\sigma_i = (-1)^{i}\binom{b}{i}$ and $H(x,a,b,\beta,\theta) = \left(1-(\frac{\theta}{x})^{\beta}\right)^{ai}$

The pdf of (KWP) Using the series expansion defined as

$$f(x,a,b,\beta,\theta) = \frac{ab\beta\,\theta^{\beta}}{x^{\beta+1}}\cdot\sum_{j=0}^{\infty}(-1)^{j}\binom{b-1}{j}\left(1-\left(\frac{\theta}{x}\right)^{\beta}\right)^{aj+a-1} \tag{10}$$

KWP Regression Model

After arriving at the general form of the Kumarasumy Pareto distribution, we now put a new regression for this distribution, assuming that we have x random variable that traces the Kamurasumy Pareto (KWP) and y is defined by y = -log(x). and $\beta = 1/\sigma$ and $\mu = -\log(\theta)$ the density function defined as

$$f(x, a, b, \sigma, \mu) = \frac{ab}{\sigma} .exp\left(\frac{y-\mu}{\sigma}\right)\left(1 - exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a-1}\left[1 - \left(1 - exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a}\right]^{b-1} \quad (11)$$

Where $y \in$ R, $\mu \in$ R, $\sigma > 0$, b > 0, a > 0. We refer to Equation (11) as the KWP distribution, defined : KWP(a, b, σ, μ), where a,b are shape parameter

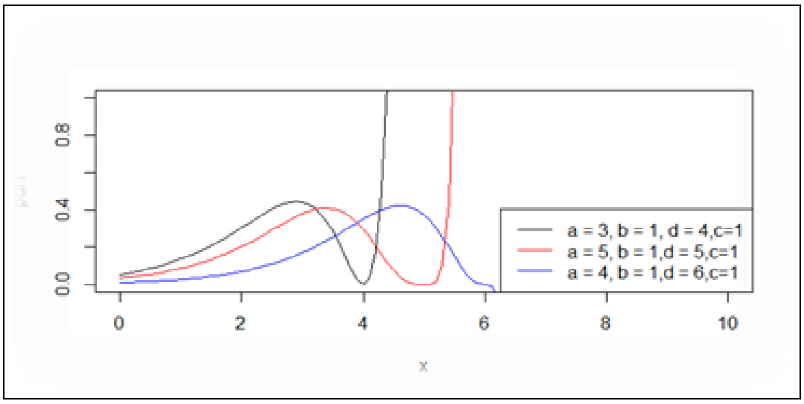

Fig 1: Plots the probability density function at the different values of the parameters

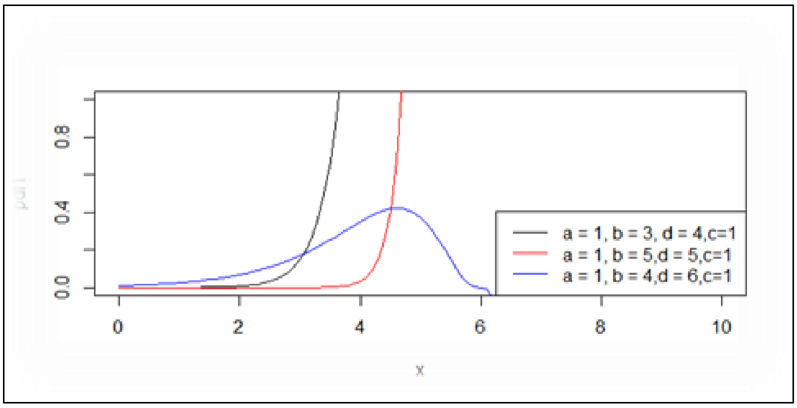

Fig 2: Plots the probability density function at the different values of the parameters the survival function is

$$s(y, \theta, \beta, a, b) = \left[1 - \left(1 - exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a}\right]^{b} \quad (12)$$

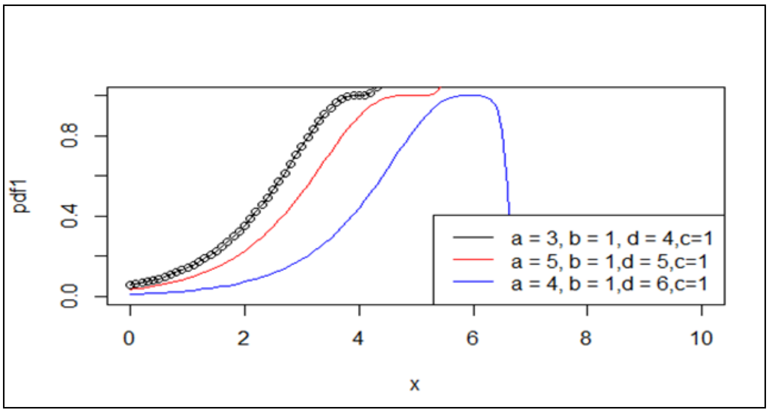

Fig 3: Plots of the survival function at the different values of the parameters. and the hrf is simply $h(y) = f(y)/S(y)$.

$$h(y)=\frac{\frac{ab}{\sigma}.\exp\left(\frac{y-\mu}{\sigma}\right)\left(1-\exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a-1}\left[1-\left(1-\exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a}\right]^{b-1}}{\left[1-\left(1-\exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a}\right]^{b}}$$

the hrf is simply $h(y)$ we obtained

$$h(y)=\frac{\frac{ab}{\sigma}.\exp\left(\frac{y-\mu}{\sigma}\right)\left(1-\exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a-1}}{1-\left(1-\exp\left(\frac{y-\mu}{\sigma}\right)\right)^{a}} \quad (13)$$

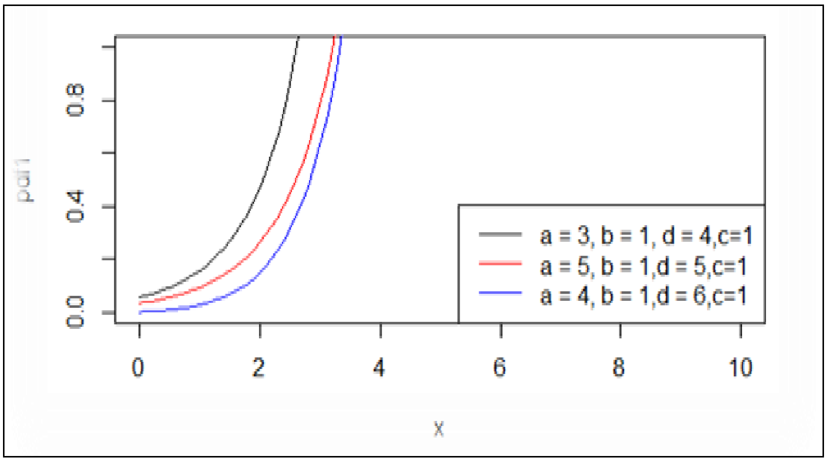

Fig 4: Plots of the hazard rate function at the different values of the parameters.

Regression models are used to estimate univariate survival functions for large-scale controlled data, as the model tends to produce a more accurate estimate of parameters based on the model density, we propose a linear location-scale regression model linking the response variable yi and the explanatory variable vector x_i^T =($x_{i1},\dots\dots..x_{ip}$) defined by $y_i=x_i^T\gamma_i+\sigma z_i \quad i=1,2,\dots.n$ (14)
Where the random error z_i is random error has density function (10), $\gamma_i=(\gamma_1,\dots\dots..\gamma_p)$, $\sigma>0$, b > 0 and a > 0 are unknown parameters, The parameter $\mu_i=x_i^T\gamma_i$ is the location of y_i The location parameter vector $\mu_i=(\mu_1,\dots\dots\dots..\mu_p)^T$, is represented by a linear model $\mu_i=X\gamma_i$ where X= $(x_1,\dots\dots\dots..x_n)^T$ is a known model matrix

Estimation of Parameters' for Regression Model:

Suppose we have a sample of observations (y_1, x_1),…, (y_n, x_n) of n independent, where all responses are random, The log-likelihood function for the parameters $\delta=(a,b,\sigma,\gamma^T)$, We can formulate the Log-likelihood function as:

$$L=\sum_{i=1}^{n}\delta_i\ln f(y_i)+\sum_{i=1}^{n}(1-\delta_i)\ln(S(y_i))\ \delta_i>0 \quad (15)$$

The equation no (10) can expansion as follows

$$f(x,a,b,\beta,\theta)=\frac{ab}{\sigma}.exp\left(\frac{y-\gamma_0-\gamma_1x}{\sigma}\right)\left(1-exp\left(\frac{y-\gamma_0-\gamma_1x}{\sigma}\right)\right)^{a-1}\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1x}{\sigma}\right)\right)^{a}\right]^{b-1} \quad (16)$$

And equation no (11) can expansion as follows

$$s(x,\theta,\beta,a,b)=\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1x}{\sigma}\right)\right)^{a}\right]^{b} \quad (17)$$

Substituting from Equation No (16), (17). 1 into Equation No. (15), we get

$$L=\sum_{i=1}^{n}\delta_i\ln\left\{\frac{ab}{\sigma}.\exp\left(\frac{y-\gamma_0-\gamma_1x_i}{\sigma}\right)\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1x_i}{\sigma}\right)\right)^{a-1}\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1x_i}{\sigma}\right)\right)^{a}\right]^{b-1}\right\}+ \\ +\sum_{i=1}^{n}(1-\delta_i)\ln\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1x_i}{\sigma}\right)\right)^{a}\right]^{b} \quad (18)$$

Simplifying equation number (18), we get the following

$$L(x,a,b,\gamma_0,\gamma_1) = \sum_{i=1}^{n} \delta_i \ln ab - \delta_i \ln\sigma + \frac{y-\gamma_0-\gamma_1 x_i}{\sigma} + (a-1)\sum_{i=1}^{n} \ln\left[(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right] + (b-1)\sum_{i=1}^{n} \ln\left[\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]\right] + \sum_{i=1}^{n}(1-\delta_i)\, b\ln\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right] \quad (19)$$

To estimate the parameters of the regression, we differentiate equation No. (19) with respect to the parameters and set the differential product to zero

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial a} = 0, \frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial b} = 0, \frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial \gamma_0} = 0, \frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial \gamma_1} = 0$$

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial \sigma} = 0$$

$$L(x,a,b,\gamma_0,\gamma_1) = A + B + C + D \quad (20)$$

Where

$C = (b-1)\sum_{i=1}^{n} \ln\left[\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]\right]$, $D = \sum_{i=1}^{n}(1-\delta_i)\, b\ln\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]$

For estimate a

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial a} = \frac{\partial A}{\partial a} + \frac{\partial B}{\partial a} + \frac{\partial C}{\partial a} + \frac{\partial D}{\partial a} = 0 \quad (21)$$

$\frac{\partial A}{\partial a} = \sum_{i=1}^{n} \frac{\delta_i}{a}$ and $\frac{\partial B}{\partial a} = \sum_{i=1}^{n} \ln\left[(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]$

$$\frac{\partial C}{\partial a} = \sum_{i=1}^{n}(-b+1)\frac{\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a \ln\left[1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]}{\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]}$$

$$\frac{\partial D}{\partial a} = \sum_{i=1}^{n}(1-\delta_i)\frac{-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a \ln\left[1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]}{1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x}{\sigma}\right)\right)^a}$$

Substituting in Equation No.(21) for the differentials of parts A, B, C and D. We get the value of the parameter and to get the value of the parameter B

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial b} = \frac{\partial A}{\partial b} + \frac{\partial B}{\partial b} + \frac{\partial C}{\partial b} + \frac{\partial D}{\partial b} = 0 \quad (22)$$

$\frac{\partial A}{\partial b} = Where A = \sum_{i=1}^{n} \delta_i \frac{1}{b}$ and $\frac{\partial B}{\partial b} = 0$ and $\frac{\partial C}{\partial b} = \sum_{i=1}^{n} \ln\left[\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]\right]$

$$\frac{\partial D}{\partial b} = \sum_{i=1}^{n}(1-\delta_i)\ln\left[1-\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]$$

Substituting in Equation No.(22) for the differentials of parts A, B, C and D. We get the value of the parameter and to get the value of the parameter σ

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial\sigma} = \frac{\partial A}{\partial\sigma} + \frac{\partial B}{\partial\sigma} + \frac{\partial C}{\partial\sigma} + \frac{\partial D}{\partial\sigma} = 0 \tag{23}$$

$$\frac{\partial A}{\partial\sigma} = \text{Where} A = \sum_{i=1}^{n} - \delta_i \ \frac{1}{\sigma} - \frac{y-\gamma_0-\gamma_1 x_i}{\sigma^2} \ and \ \frac{\partial B}{\partial\sigma} = \sum_{i=1}^{n}(a-1)\frac{\left[\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]\frac{1}{\sigma^2}}{\left[\left(1-\exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]}$$

$$\frac{\partial C}{\partial\sigma} = (-b+1)\sum_{i=1}^{n}\frac{a\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x}{\sigma}\right)\right)^{a-1} exp\left(\frac{y-\gamma_0-\gamma_1 x}{\sigma}\right)\frac{1}{\sigma^2}}{\left[\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]\right]}$$

$$\frac{\partial D}{\partial\sigma} = \sum_{i=1}^{n}(-1+\delta_i)b\frac{a\left(1-exp\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)^{a-1} exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\frac{1}{\sigma^2}}{\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]}$$

Substituting in equation No. (23), we get the value of the parameter b, and to get the value of the parameter γ_0

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial\gamma_0} = \frac{\partial A}{\partial\gamma_0} + \frac{\partial B}{\partial\gamma_0} + \frac{\partial C}{\partial\gamma_0} + \frac{\partial D}{\partial\gamma_0} = 0 \tag{24}$$

$$\frac{\partial A}{\partial\gamma_0} = 0, \frac{\partial B}{\partial\gamma_0} = \sum_{i=1}^{n}(a-1)\frac{\frac{1}{\sigma}exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)}{\left[\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]}$$

$$\frac{\partial C}{\partial\gamma_0} = (-b+1)\sum_{i=1}^{n}\frac{a\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^{a-1} exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\frac{1}{\sigma}}{\left[\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]\right]}$$

$$\frac{\partial D}{\partial\gamma_0} = \sum_{i=1}^{n}(-1+\delta_i)b\frac{\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^{a-1} exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\frac{1}{\sigma}}{\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]}$$

Substituting in equation No. (24), we get the value of the parameter, anγ_0 d to get the value of the parameter γ_1

$$\frac{\partial L(x,a,b,\gamma_0,\gamma_1)}{\partial\gamma_1} = \frac{\partial A}{\partial\gamma_1} + \frac{\partial B}{\partial\gamma_1} + \frac{\partial C}{\partial\gamma_1} + \frac{\partial D}{\partial\gamma_1} \tag{25}$$

$$\frac{\partial A}{\partial\gamma_1} = 0 \text{ and } \frac{\partial B}{\partial\gamma_1} = \sum_{i=1}^{n}(a-1)\frac{exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\frac{x}{\sigma}}{\left[\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right]}$$

$$\frac{\partial C}{\partial\gamma_1} = (-b+1)\sum_{i=1}^{n}\frac{a\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^{a-1} exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\frac{x_i}{\sigma}}{\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]}$$

$$\frac{\partial D}{\partial\gamma_1} = \sum_{i=1}^{n}(-1+\delta_i)ab\frac{\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^{a-1} exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\frac{x_i}{\sigma}}{\left[1-\left(1-exp\left(\frac{y-\gamma_0-\gamma_1 x_i}{\sigma}\right)\right)^a\right]}$$

Substituting in equation No. (25), we get the value of the parameter, anγ_1 d to get the value of the parameter solve all equation by using package in R program

Residual analysis

After fitted the model, we need some tools to determine what the model was verify the assumptions we defined two types of residuals: the martingale residual and the deviance component residual now we explain all concepts.

The martingale residual It is frequently used in counting operations These residual are asymmetric Where it has a maximum value of one and the smallest value ($-\infty$) we defined as follows

$r_{Mi} = \delta_i - \left(\int_0^y h(u)\,du\right)$ where $\delta_i = 0 \; or \; \delta_i = 1 \int_0^y h(u)\,du = \ln[s(y_i)]$

The martingale residual can be reduces to

$$r_{Mi} = \delta_i + \ln[s(y_i)]$$

The martigal residual for the log kumrsawamy perato model defined as follows

$$r_{Mi} = 1 + \ln\left[1 - \left(1 - exp\left(\frac{y-\mu}{\sigma}\right)\right)^a\right]^b \qquad (26)$$

Deviance component residual:

We proposed the second type of residuals to make it more symmetric around zero. The deviance component for the parametric regression model is defined

$$\hat{r}_{Di} = \text{sign}\,(\hat{r}_{Mi})\,\{-2\,[\hat{r}_{Mi} + \ln\,(1 - \hat{r}_{Mi})]\}^{1/2}$$

Where r_{Mi} is the martingal residual. The deviance component residual for the KWP model is defined by

$$\hat{r}_{Di} = sign\,(\hat{r}_{Mi})\left\{-2\left[\begin{matrix} 1 + ln\left[1 - \left(1 - exp\left(\frac{y-\mu}{\sigma}\right)\right)^a\right]^b \\ +ln\left(-ln\left[1 - \left(1 - exp\left(\frac{y-\mu}{\sigma}\right)\right)^a\right]^b\right)\end{matrix}\right]\right\}^{1/2} \qquad (27)$$

Simulation study

In section 4 simulation study by using Monte carlo to estimate model parameters, The simulation study was conducted to measure the mean (MLE), as well as calculating the mean squared error(MSE) and bias for samples of different sizes 50, 100 and 200, which were obtained from the KWP distribution through the following expression.

$$x = \theta\left[1 - \left[1 - [1-u]^{-\frac{1}{b}}\right]^{\frac{-1}{a}}\right]^{\frac{-1}{\beta}} \qquad (28)$$

Bias and MSE are Evaluated by

$$\text{bais} = \sum_{i=1}^{n} \frac{1}{N}(\hat{a} - \alpha) \qquad \text{and} \qquad MSE = \sum_{i=1}^{n} \frac{1}{N}(\hat{a} - \alpha)^2$$

The simulation study was conducted at different values of parameters and according to The average, mean square error, and bias at different sample sizes. It was noticed that when the sample size increased, the mean square error decreases, as well as the bias amount at the different values of the parameters, It was concluded that MLE performs better to construct a Kamwasumy Pareto(KWP) distribution The following table shows that mean squared error and the mean of the parameters at the different values .

Table 1: Simulation results of the KWP distribution for several values of parameters

a	b	μ	σ	n	bias			
					a	b	μ	σ
				50	0.0043	0.4213	0.6236	0.4287

0.5	2	0	1	100	0.0032	0.2316	0.0423	0.4123
				200	0.0001	-0.0234	0.0001	0.3143
2	0.5	0	1	50	0.5423	-0.1234	0.7231	0.4213
				100	0.4523	-0.4523	0.5321	0.3256
				200	-0.4235	-0.7456	0.0214	0.00123
1.5	2.5	0	1	50	0.4215	0.4253	0.4231	0.4561
				100	0.3251	0.3521	0.3214	0.3512
				200	-0.4253	-0.621	0.0012	0.00123

Table 2: The results of the simulation study at different values of parameters for (KWP)

a	**b**	μ	σ	**n**	**MSE**			
					a	**b**	μ	σ
0.5	2	0	1	50	0.000000391	0.00354987	0.00777753	0.00367567
				100	0. 00000102	0.00053634	0.0000178	0.00169999
				200	0.000000011	0.00000273	0.00000005	0.00049392
2	0.5	0	1	50	0.005889374	0.00030455	0.01045747	0.00197568
				100	0.002045765	0.00204575	0.00283134	0.00177493
				200	0.000896731	0.00277959	0.00000228	0.00053007
1.5	2.5	0	1	50	0.003553242	0.00036176	0.00358027	0.00416054
				100	0.001056900	0.00123974	0.00103297	0.00123341
				200	0.000904400	0.00192820	0.00000072	0.00000007

Application of real data

The real data of the current study represents the relationship between the influence of the Egyptian Stock Exchange, food, health, finance and tourism, where the data were monthly data from 2015 to 2019, where the new regression of the kamarswamy Pareto distribution was applied to the data under study to estimate the parameters of the regression.

Table 3: Estimation of parameters for (Kwp) regression model

Model	**a b** σ γ_0 γ_1	**AIC**	**BIC**
kwp	0.53 0.11 0.23 0.8314 -0.331	874.2	912.51
p	1 1 0.74 2.543 -0.38	942.21	930.12

The following table shows an estimate of the parameters of the Kamraswamy -Pareto regression model for the Egyptian stock exchange data. The comparison is based on the following criteria AIC BIC, where the comparison was made between the two models The Kamraswamy -Pareto regression model and the Pareto regression model and it is concluded from the previous table that the performance of the Kamraswamy -Pareto model is better than the Pareto mode Where the values of AIC, BIC was lower in the case of the Kamraswamy -Pareto regression model than in the Pareto regression model

Conclusions

The conclusion of a new distribution with four parameters called Kamraswamy -Pareto, where it became clear that this distribution is more flexible when applied to the controlled data as well as the uncontrolled data. We also recommend more applications for this new regression, as it turned out that the new regression was better in performance than the rest of the other models

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