International Journal of Physics and Mathematics
www.physicsjournal.net
Online ISSN: 2664-8644, Print ISSN: 2664-8636
Received: 06-02-2022, Accepted: 20-02-2022, Published: 08-03-2022
Volume 4, Issue 1, 2022, Page No. 13-17

The approximate analytical solutions of nonlinear gas dynamics equation within caputo-fabrizo fractional operator

Hussein Gatea Taher
Department of Mathematics, Faculty of Education for Pure Sciences, University of Thi-Qar, Nasiriyah, Iraq

Abstract
The adomian decomposition method (ADM) is applied to solve fractional order nonlinear gas dynamic equation with Caputo-Fabrizo fractional operator in this paper. In this method, illustration example evaluate the accuracy and applicability of the mentioned procedure. The outcomes got by ADM are in acceptable concurrence with the special arrangement of the problem.

Keywords: approximate analytical, nonlinear gas dynamics, caputo-fabrizo

Introduction

The idea of fractionali calculus icoincides with that of iclassical icalculus. Leibniz and L'Hopital ifirst raisedi this issue in 1695 andi in 1730 Euler's iattention was idrawn ito it, followed by Lagrangei in 1772 and Laplace in 1812. The first iconcept of arbitraryi derivationi was iintroduced byi Lacroix and ilater by Fourier, Abel, Liouville, Grunewald, Letnikov iand Riemann. Various ifractional derivatives were thus iintroduced. Grunewald and Krug introducedi the iwork of Riemann and Liouville and introducedi another integral iand derivative called Riemann-Liouville. Caputo iintroduced a new iderivative by rewritingi the Riemann-Liouville iformula [1]. The Caputo and Fabrizioi fractional iderivative (CFFD) and the idifferential equations involvingi the CFFD have ireceived igrowing attention. In recenti years, a considerable iliterature has sprung up in the iframework of CFFDs. The definition of the CFFD and its propertiesi were deeply exploredi and further extendedi in [2], among iwhich the idefinition of the iextended CFFD was first introducedi in [3].

Lately, Most iapproximate and empirical imethodologies have been used to iresolve ordinaryi and partial differential iequations in the Caputo iand Caputo-fractionali sensei in [4-43]. Our aim is to ipresent the ADM, and to used it to isolve the nonlinear FRDE. The remainderi of this work is idivided into the following sections. In Section 2, the fractional ADM analysis method is iimplemented. Applications of fractionali ADM are demonstrated in Section 3. Section 4 contains the iconclusion of this paper.

Analysis of Method

We consider the fractional partial differential equation.

$${}^{CF}_{0}D^{\alpha}_{\tau}\varphi(\mu,\tau) + R\big(\varphi(\mu,\tau)\big) + N\big(\varphi(\mu,\tau)\big) = g(\mu,\tau),\ 0 \le \alpha \le 1, \mu \in \mathbb{R}, \tau > 0 \quad (1)$$

With the initial condition

$$\varphi(\mu,0) = \varphi_0(\mu), \quad (2)$$

Where ${}^{CF}_{0}D^{\alpha}_{\tau}$ is CF operator, R is a linear operator, N is a nonlinear operator and g is a source term.
Taking integrali of Caput- Fabrizio to bothi side, of (1), we get

$${}^{CF}_{0}I^{\alpha}_{\tau}\ {}^{CF}_{0}D^{\alpha}_{\tau}\varphi(\mu,\tau) + {}^{CF}_{0}I^{\alpha}_{\tau}\ R\big(\varphi(\mu,\tau)\big) + {}^{CF}_{0}I^{\alpha}_{\tau}\ N\big(\varphi(\mu,\tau)\big) = {}^{CF}_{0}I^{\alpha}_{\tau}\ g(\mu,\tau). \quad (3)$$

By properties fractional integral, we obtain

$$\varphi(\mu,\tau) - \varphi(\mu,0) = {}^{CF}_{0}I^{\alpha}_{\tau}\ g(\mu,\tau) - {}^{CF}_{0}I^{\alpha}_{\tau}\ R\big(\varphi(\mu,\tau)\big) - {}^{CF}_{0}I^{\alpha}_{\tau}\ N\big(\varphi(\mu,\tau)\big). \quad (4)$$

From (4), we get

$$\varphi(\mu,\tau) = \varphi_0(\mu) + {}^{CF}_{0}I^{\alpha}_{\tau}\, g(\mu,\tau) - {}^{CF}_{0}I^{\alpha}_{\tau}\, R\big(\varphi(\mu,\tau)\big) - {}^{CF}_{0}I^{\alpha}_{\tau}\, N\big(\varphi(\mu,\tau)\big). \tag{5}$$

Let

$$f = \varphi_0(\mu) + {}^{CF}_{0}I^{\alpha}_{\tau}[g(\mu,\tau)]$$

Then, we have

$$\varphi(\mu,\tau) = f - {}^{CF}_{0}I^{\alpha}_{\tau}\, R\big(\varphi(\mu,\tau)\big) - {}^{CF}_{0}I^{\alpha}_{\tau}\, N\big(\varphi(\mu,\tau)\big). \tag{6}$$

Now, we represent solution as an infinite series given below

$$\varphi(\mu,\tau) = \sum_{n=0}^{\infty} \varphi_n(\mu,\tau) \tag{7}$$

and the nonlinear term can be decomposed as

$$N\big(\varphi(\mu,\tau)\big) = \sum_{n=0}^{\infty} A_n(\varphi_0, \varphi_1, \varphi_2) \tag{8}$$

Where

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i\varphi_i\right)\right]_{\lambda=0}, n = 0,1,2,\ldots \tag{9}$$

By substituting (9) and (10) in (8), we have

$$\sum_{n=0}^{\infty}\varphi_n(\mu,\tau) = f - {}^{CF}_{0}I^{\alpha}_{\tau}\left[\left(R\sum_{n=0}^{\infty}\varphi_n + \sum_{n=0}^{\infty}A_n\right)\right] \tag{10}$$

On comparing both sides of the Eq. (12) we get

$$\varphi_0(\mu,\tau) = f$$
$$\varphi_1(\mu,\tau) = -{}^{CF}_{0}I^{\alpha}_{\tau}\big[\big(R(\varphi_0(\mu,\tau) + A_0)\big)\big]$$
$$\varphi_2(\mu,\tau) = -{}^{CF}_{0}I^{\alpha}_{\tau}\big[\big(R(\varphi_1(\mu,\tau) + A_1)\big)\big] \tag{11}$$

In general, the irecursive relation is giveni as

$$\varphi_{n+1}(\mu,\tau) = -{}^{CF}_{0}I^{\alpha}_{\tau}\big[\big(R(\varphi_n(\mu,\tau) + A_n)\big)\big] \tag{12}$$

Finally, we approximate the analytical solution (3) by truncated series:

$$\varphi(\mu,\tau) = \varphi_0(\mu,\tau) + \varphi_1(\mu,\tau) + \varphi_2(\mu,\tau) + \cdots \tag{13}$$

Application

Example: Let us iconsider the following nonlinear PDE with Caputo Fabrizio sense

$${}^{CF}_{0}D^{\alpha}_{t}u(x,t) + \frac{1}{2}(u^2)_x - u + u^2 = 0 \quad , \quad 0 < \alpha \le 1$$

Subject to the initial condition

$u(x,0) = e^{-x}$

Taking ${}^{CF}_{0}I^{\alpha}_{t}$ to both sidesi we get

$u(x,t) = e^{-x} + {}^{CF}_{0}I^{\alpha}_{t}$ [u - $\frac{1}{2}$ (2uu$_x$) – u^2]

$u(x,t) = e^{-x} + {}^{CF}_{0}I^{\alpha}_{t}$ [u – uu$_x$ – u^2]

$$let\ u = \sum_{n=0}^{\infty} u_n, \quad uu_x = \sum_{n=0}^{\infty} A_n\ and \quad u^2 = \sum_{n=0}^{\infty} B_n$$

Then

$u_0(x,t) = e^{-x}$

$u_{n+1}(x,t) = {}^{CF}_{0}I^{\alpha}_{t}\ [u_n - \sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n]$

Now

$A_0 = u_0 u_{0\,x} = -e^{-2x}$

$B_0 = u_0 u_0 = e^{-2x}$

$A_1 = u_0 u_{1x} + u_1 u_{0x}$

$B_1 = u_0u_1 + u_1u_0 = 2\,u_0u_1$

$u_1 = {}^{CF}_{0}I^{\alpha}_{t}$ [$u_0 - A_0 - B_0$]

$= {}^{CF}_{0}I^{\alpha}_{t}\ [e^{-x} + e^{-2x} - e^{-2x}]$

$= {}^{CF}_{0}I^{\alpha}_{t}\ [e^{-x}]$

$= e^{-x}\ [(1-\alpha)1 + \alpha \int_0^t ds]$

$= e^{-x}\ [(1-\alpha) + \alpha[s]_0^t]$

$= e^{-x}\ (1-\alpha+\alpha t)$

$u_2 = {}^{CF}_{0}I^{\alpha}_{t}$ [$u_1 - A_1 - B_1$]

$u_2 = {}^{CF}_{0}I^{\alpha}_{t}\ [e^{-x}\ (1-\alpha+\alpha t) + 2\,e^{-2x}\ (1-\alpha+\alpha t) - 2e^{-2x}\ (1-\alpha+\alpha t)]$

$= {}^{CF}_{0}I^{\alpha}_{t}\ [e^{-x}\ (1-\alpha+\alpha t)]$

$= e^{-x}\ {}^{CF}_{0}I^{\alpha}_{t}\ (1-\alpha+\alpha t)$

$= e^{-x}\ [(1-2\alpha+\alpha^2) + 2(\alpha-\alpha^2)t + \frac{1}{2}\alpha^2 t^2]$

Then, the approximate solution of $u(x,t)$ is given by

$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$

$$= u_0 + u_1 + u_2 + \cdots$$

$$= e^{-x} + e^{-x}(1 - \alpha + \alpha t) + e^{-x}[(1 - 2\alpha + \alpha^2) + 2(\alpha - \alpha^2)t + \tfrac{1}{2}\alpha^2 t^2] + \ldots$$

Therefore, the analytical solution when $\alpha \to 1$ *is given by*

$$u(x,t) = e^{t-x}$$

Conclusion

In this work, the ADM has been iexecuted effectively to solve the ifractional gas dynamics and find the iapproximate solutions of it. The analyticali technique provides a seriesi solution that fast approaches the exacti solution. The achieved results demonstrate that the iproposed method is effective in solving inonlinear fractionali differential equations.

References

1. Alizadeh S, Baleanu D, Rezapour S. Analyzing transient response of the parallel RCL circuit by using the Caputo–Fabrizio fractional derivative, Advances in Difference Equations,2020:55:1-19.
2. J Losada, Nieto JJ. Properties of a new fractional derivative without singular kernel. Prog. Fract. Differ. Appl,2015:2:87–89.
3. Jassim HK. On Local Bifurcations and Chaos of a Three-Dimensional Nonlinear System,. Journal of college of Education for Pure Science,2013:3(2):150–158.
4. H Jafari, Jassim HK. Local Fractional Adomian Decomposition Method for Solving Two Dimensional Heat conduction Equations within Local Fractional Operators, Journal of Advance in Mathematics,2014:9(4):2574-2582.
5. Jafari H, Jassim HK. Local Fractional Laplace Variational Iteration Method for Solving Nonlinear Partial Differential Equations on Cantor Sets within Local Fractional Operators, Journal of Zankoy Sulaimani-Part A,2014:16(4):49-57.
6. Jafari H, Jassim HK. Numerical Solutions of Telegraph and Laplace Equations on Cantor Sets Using Local Fractional Laplace Decomposition Method, International Journal of Advances in Applied Mathematics and Mechanics,2015:2(3):144-151.
7. Jafari H, Jassim HK. A Coupling Method of Local Fractional Variational Iteration Method and Yang-Laplace Transform for Solving Laplace Equation on Cantor Sets, International Journal of pure and Applied Sciences and Technology,2015:26(1):24-33.
8. Jassim HK. Local Fractional Laplace Decomposition Method for Nonhomogeneous Heat Equations Arising in Fractal Heat Flow with Local Fractional Derivative, International Journal of Advances in Applied Mathematics and Mechanics,2015:2(4):1-7.
9. Jassim HK. On the stability and Hopf bifurcation of a Shimizu Morioka model, Scholars Journal of Research in Mathematics and Computer Science,2016:1(1):42- 48.
10. Jassim HK. Some Dynamical Properties of Rössler System, Journal of University of Thi-Qar,2017:3(1):69-76.
11. Jassim HK. A Coupling Method of Regularization and Adomian Decomposition for Solving a Class of the Fredholm Integral Equations within Local Fractional Operators,2017:2(3):95-99.
12. Jassim HK. On Approximate Methods for Fractal Vehicular Traffic Flow, TWMS Journal of Applied and Engineering Mathematics,2017:7(1):58-65.
13. Jassim HK. Extending Application of Adomian Decomposition Method for Solving a Class of Volterra Integro-Differential Equations within Local Fractional Integral Operators, Journal of college of Education for Pure Science,2017:7(1):19-29.
14. Jassim HK. Approximate Methods for Local Fractional Integral Equations, The Journal of Hyperstructures,2016:6(1):40-51.
15. Jassim HK. Solving Poisson Equation within Local Fractional Derivative Operators, Research in Applied Mathematics,2017:1:1-12.
16. Jassim HK. An Efficient Technique for Solving Linear and Nonlinear Wave Equation within Local Fractional Operators, *The Journal of Hyperstructures*,2017:6(2):136-146.
17. Wang SQ, Yang YJ, Jassim HK. Local Fractional Function Decomposition Method for Solving Inhomogeneous Wave Equations with Local Fractional Derivative, Abstract and Applied Analysis, 2014, 1-7.
18. Yan SP, Jafari H, Jassim HK. Local Fractional Adomian Decomposition and Function Decomposition Methods for Solving Laplace Equation within Local Fractional Operators, *Advances in Mathematical Physics*, 2014, 1-7.
19. Jassim HK, Ünlü C, Moshokoa SP, Khalique CM. Local Fractional Laplace Variational Iteration Method for Solving Diffusion and Wave Equations on Cantor Sets within Local Fractional Operators, Mathematical Problems in Engineering, 2015, 1-9.

20. Fan ZP, Jassim HK, Rainna RK, Yang XJ. Adomian Decomposition Method for Three-Dimensional Diffusion Model in Fractal Heat Transfer Involving Local Fractional Derivatives, Thermal Science,2015:19:S137-S141.
21. Xu S, Ling X, Zhao Y, Jassim HK. A Novel Schedule for Solving the Two-Dimensional Diffusion in Fractal Heat Transfer, *Thermal Science*,2015:19:S99-S103.
22. Jafari H, Jassim HK, Mohyuid-Din ST. Local Fractional Laplace Decomposition Method for Solving Linear Partial Differential Equations with Local Fractional Derivative. In *Fractional Dynamics*. C. Cattani, H. M. Srivastava, and X.-J. Yang (Editors), *De Gruyter Open*, Berlin and Warsaw, 2015, 296-316.
23. Jassim HK. New Approaches for Solving Fokker Planck Equation on Cantor Sets within Local Fractional Operators, Journal of Mathematics,2015:1-8:ID 684598.
24. Jafari H, Jassim HK, Moshokoa SP, Ariyan VM, Tchier F. Reduced differential transform method for partial differential equations within local fractional derivative operators, Advances in Mechanical Engineering,2016:8(4):1-6.
25. Jafari H, Jassim HK, Tchier F, Baleanu D. On the Approximate Solutions of Local Fractional Differential Equations with Local Fractional Operator, Entropy,2016:18:1-12.
26. D Baleanu, Jassim HK. Approximate Analytical Solutions of Goursat Problem within Local Fractional Operators, Journal of Nonlinear Science and Applications,2016:9:4829-4837.
27. Jassim HK. The Approximate Solutions of Three-Dimensional Diffusion and Wave Equations within Local Fractional Derivative Operator, Abstract and Applied Analysis,2016:1-5: ID 2913539.
28. Jafari H, Jassim HK. On the Existence and Uniqueness of Solutions for Local differential equations, *Entropy*, 18(2016) 1-9.
29. D. Baleanu, H. K. Jassim, H. Khan, A Modification Fractional Variational Iteration Method for solving Nonlinear Gas Dynamic and Coupled KdV Equations Involving Local Fractional Operators, *Thermal Scienc*e, 2018:22:S165-S175.
30. Jafari H, Jassim HK, Vahidi J. Reduced Differential Transform and Variational Iteration Methods for 3D Diffusion Model in Fractal Heat Transfer within Local Fractional Operators, Thermal Science,2018:22:S301-S307.
31. Jassim HK. *et al*., Fractional variational iteration method to solve one dimensional second order hyperbolic telegraph equations, Journal of Physics: Conference Series,2018:1032(1):1-9.
32. Jassim HK, Baleanu D. A novel approach for Korteweg-de Vries equation of fractional order, Journal of Applied Computational Mechanics,2019:5(2):192-198.
33. Baleanu D, Jassim HK. Approximate Solutions of the Damped Wave Equation and Dissipative Wave Equation in Fractal Strings, Fractal and Fractional,2019:3(26):1-12.
34. Baleanu D, Jassim HK. A Modification Fractional Homotopy Perturbation Method for Solving Helmholtz and Coupled Helmholtz Equations on Cantor Sets, Fractal and Fractional,2019:3(30):1-8.
35. Baleanu D, Jassim HK, Al Qurashi M. Solving Helmholtz Equation with Local Fractional Derivative Operators, Fractal and Fractional,2019:3(43):1-13.
36. Jassim HK. Analytical Approximate Solutions for Local Fractional Wave Equations, Mathematical Methods in the Applied Sciences,2020:43(2):939-947.
37. Singh J, Jassim HK, Kumar D. An efficient computational technique for local fractional Fokker-Planck equation, Physica A: Statistical Mechanics and its Applications,2020:555(124525):1-8.
38. Jassim HK, Vahidi J, Ariyan VM. Solving Laplace Equation within Local Fractional Operators by Using Local Fractional Differential Transform and Laplace Variational Iteration Methods, *Nonlinear Dynamics and Systems Theory*,2020:20(4):388-396.
39. Baleanu D, Jassim HK. Exact Solution of Two-dimensional Fractional Partial Differential Equations, *Fractal Fractional*,2020:4(21):1-9.
40. Jassim HK, Kadhim HA. Fractional Sumudu decomposition method for solving PDEs of fractional order, Journal of Applied and Computational Mechanics,2021:7(1):302-311.
41. Jafari H, Jassim HK, Baleanu D, Chu YM. On the approximate solutions for a system of coupled Korteweg-de Vries equations with local fractional derivative, *Fractals*,2021:29(5):1-7.
42. Jassim HK, Mohammed MG. Natural homotopy perturbation method for solving nonlinear fractional gas dynamics equations, International Journal of Nonlinear Analysis and Applications,2021:12(1):37-44.
43. Alzaki LK, Jassim HK. The approximate analytical solutions of nonlinear fractional ordinary differential equations, International Journal of Nonlinear Analysis and Applications,2021:12(2):527-535.