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Comparing reliability and cost approximation for 1-out-of-2 cold standby systems with exponential distributions

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Abstract
Reliability characteristics/measures of the system are strongly dependent upon the environmental conditions in which the system is exposed off. The present paper emphasizes on the comparative analysis of Reliability measures of two repairable systems of non-identical units subject to different weather conditions. Both systems consist of two units as Main unit and duplicate unit. Both systems are at cold standby redundancy two type of environmental conditions are considered into account – normal and abnormal. The single server is capable to perform all type of repair activities in normal environmental conditions whereas it ceases in abnormal environmental conditions. In Model I, the operation of both units is stopped as precautionary measure, but in Model II both units are allowed to operate in all environmental conditions. Priority is given to one unit over the other for operation only. The repair mechanism is perfect, since both units works as new after repair. The failure rates times and time of rate of change of environmental conditions are constant. They follow negative distribution function. The repair rates follow general distribution functions. All random variables are statistically independent. Switches are perfect. The expression for various reliability measures in derived in steady state. Various reliability measures and system effectiveness measures are determined through Regenerative point technique and Semi-Markov process. The graphical interpretation and comparative analysis are done with respect to normal weather rate function for particular cases.

Keywords: Reliability measures, environmental conditions, random variables, distribution function

Introduction

Reliability is a fundamental attribute for the safe operation of any modern technological system. In reliability theory, it is proved that redundancy plays an important role in enhancing system reliability and to reduce the frequency of failure up to a desirable extent. Scholars including Goel and Sharma (1989) [1] and Singh and Chander (2005) [2] have investigated stochastic Models with different modes of failure but the reliability indices estimated without recognition of weather situations are unrealistic. Later, Malik S and Deswal S (2012 – 2021) [3-6] performed an extensive research on stochastic analysis and evaluation of reliability measures of system of two non-identical units working in different weather state. Also, in real life situations it becomes essential to give priority in operation to one unit over the other.

Thus, it becomes necessary to ascertain whether a system of non-identical units with priority for operation to one unit over the other unit will have a profit if it is allowed to operate in abnormal weather. In view of this, Model I and Model II has been designed to study a repairable system of two non-identical units – one is original (called main unit) and other is a substandard unit (called duplicate unit). Two weather conditions are conditions – normal and abnormal are considered. Initially, the main unit is operative and duplicate unit is kept as spare in cold standby. Priority is given to main unit for operation. The system is considered in upstate if either of the unit is functional. Both units have direct complete failure from normal mode. The set of activities performed by each unit are almost same but their performance measures are different. In Model I, the operation and repair of the units are not allowed in abnormal weather. In Model II, the operation of the units is also allowed to abnormal weather. The single server attends the failed immediately in normal weather. The units work as new after repair.

The failure times of the units and time of change of weather conditions follow negative exponential distributions. The repair times of the units are arbitrarily distributed. All random variables are statistically independent. The switches are perfect. The system is observed at suitable regenerative epochs using semi-MarKov process and regenerative point technique to derive the expressions for some reliability measures such as mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period of the server expected number of visits by the server and profit function. The numerical results of MTSF, availability and profit function have been obtained by considering particular values of the parameters and costs have been obtained to depict their behavior graphically. The MTSF and profit function of the Model I and Model II are compared.

Notations

E	The set of regenerative states
MO/DO	Main/Duplicate unit is good and operative
$\overline{MO}$ / $\overline{DO}$	Main/Duplicate unit is good and operating in abnormal weather
$\overline{MWO}$ / $\overline{DWO}$	Main/Duplicate unit is good and waiting for operation in abnormal weather

DCs	Duplicate unit is in cold standby mode
$\overline{\text{DCs}}$	Duplicate unit is in cold standby mode in abnormal weather
λ / λ_1	Constant failure rate of Main /Duplicate unit
β / β_1	Constant rate of change of weather from normal to abnormal/abnormal to normal weather
MFur/DFur	Main/duplicate unit failed and under repair
MFUR/DFUR	Main/duplicate unit failed and under repair continuously from previous state
MFwr/DFwr	Main/duplicate unit failed and waiting for repair
MFWR/DFWR	Main/duplicate unit failed and waiting for repair continuously from previous state
$\overline{\text{MFwr}}$ / $\overline{\text{DFwr}}$	Main/Duplicate unit failed and waiting for repair due to abnormal weather
$\overline{\text{MFWR}}$ / $\overline{\text{DFWR}}$	Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather
g(t)/G(t)	pdf/cdf of repair time of Main unit
$g_1(t)$/ $G_1(t)$	pdf/cdf of repair time of Duplicate unit
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]
$q_{ij.kr}(t)/ Q_{ij.kr}(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]
$q_{ij.k,(r,s)}{}^{n}(t)/Q_{ij.k,(r,s)}{}^{n}(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.
$M_i(t)$	Probability that the system is up initially in regenerative state Si at time t without visiting to any other regenerative state
$W_i(t)$	Probability that the server is busy in state Siupto time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
m_{ij}	The conditional mean sojourn time in regenerative state Si when system is to make transition in to regenerative state Sj. Mathematically, it can be written as $m_{ij} = E(T_{ij}) = \int_0^\infty t\,d[Q_{ij}(t)] = -q_{ij}^{*\prime}(0)$, where T_{ij} is the transition time from state Si to Sj; Si, Sj ε E.
μ_i	The mean Sojourn time in state Si this is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$, where T_i is the sojourn time in state Si.
Ⓢ/©/©ⁿ	Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution n times
** / *	Symbol for Laplace Steiltjes Transform (L.S.T.)/ Laplace transform (L.T.)
´(desh)	Used to represent alternative result

Methodology

The systems have been analyzed using well known semi-Markov process and regenerative point technique which are briefly described as:

Markov Process

If {X(t), t∈T} is a stochastic process such that, given the value of X(s), the value of X(t), t>s do not depend on the values of X(u), u<s Then the process {X(t), t∈T} is a Markov process.

Semi-Markov Process

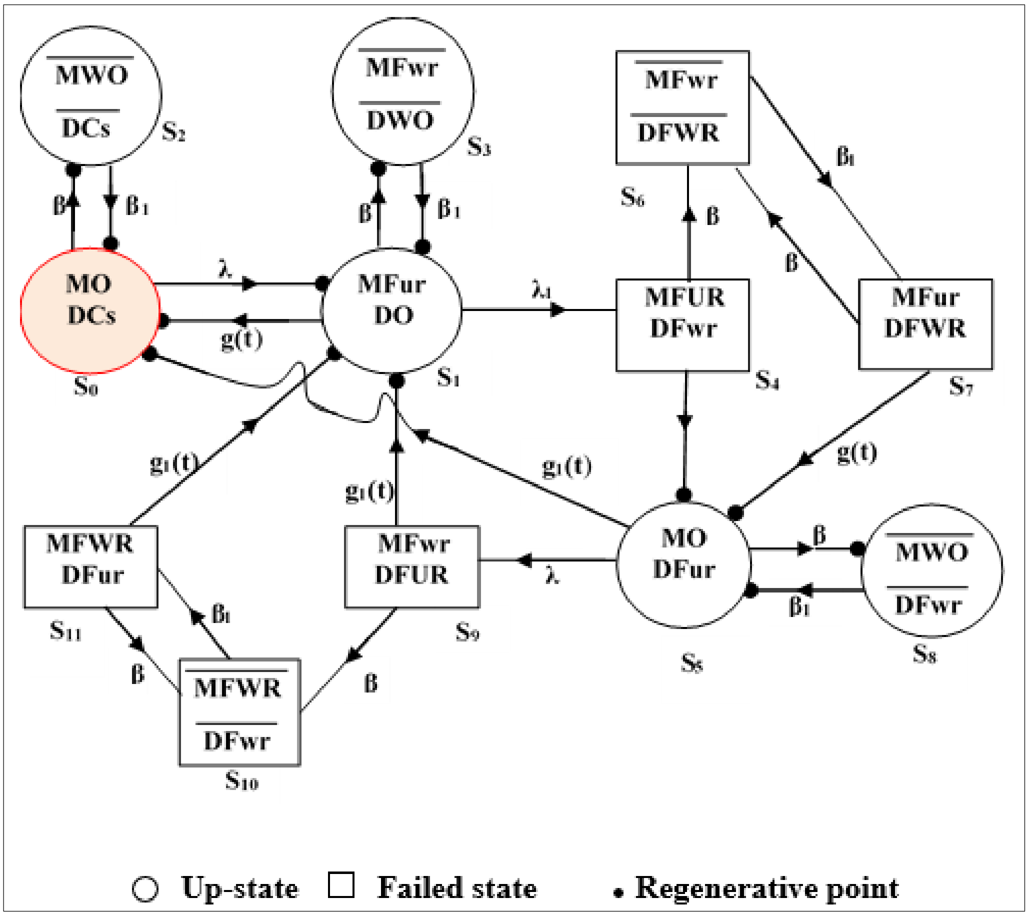
A semi-Markov process is a stochastic process in which changes of state occur according to a Markov chain and in which the time interval between two successive transitions is a random variable, whose distribution may depend on the state from which the transition take place as well as on the state to which the next transition take place.

Regenerative Process

Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex system. In this, we take time points at which the system history prior to the time points is irrelevant to the system conditions. These points are called regenerative points. Let X(t) be the state of the system of epoch. If t1, t2,... are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process {X(t), t = t1, t2.........} is called regenerative process. The state in which regenerative points occur is known as regenerative state.

Model I

The state Transition Diagram of the Model I as provided below –



Model II: The state Transition Diagram of the Model II as provided below –

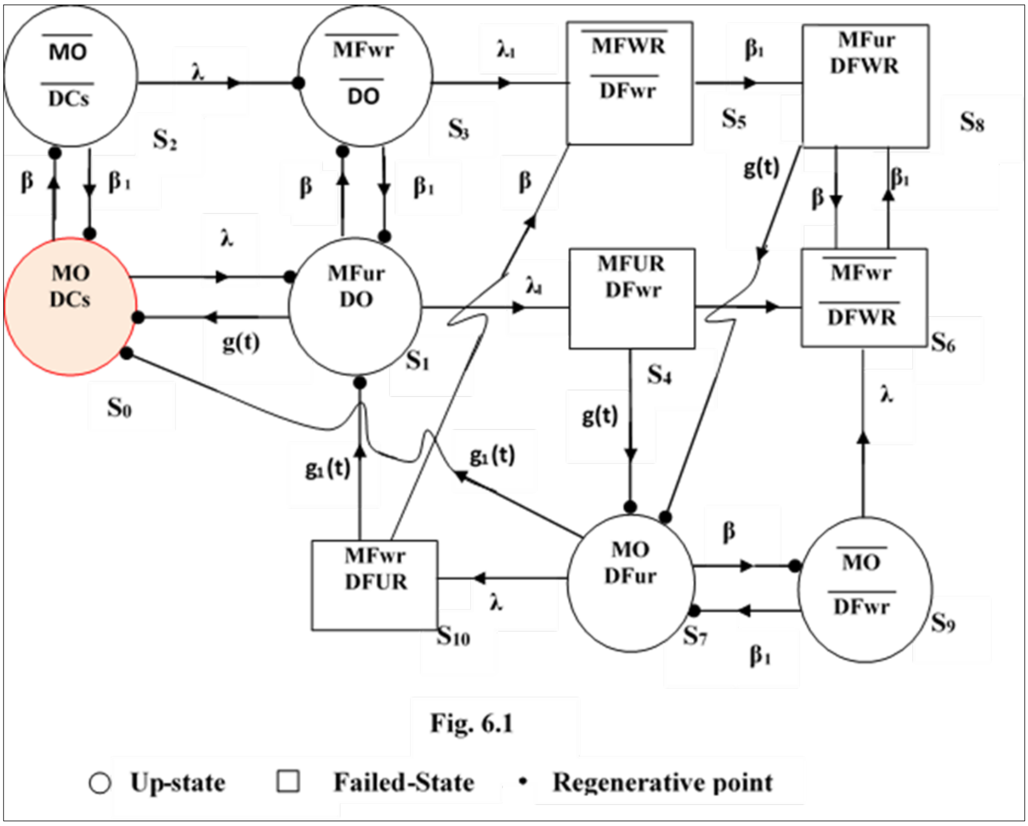


Fig. 6.1

Reliability and mean time to system failure (MTSF)
Let $\emptyset_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding failed state as absorbing state, we have following recursive relations for $\emptyset_i(t)$:

$\emptyset_i(t)$=$\sum_j Q_{i,j}(t)$Ⓢ$\emptyset_j(t) + \sum_k Q_{i,k}(t)$

Where S_j is the un failed regenerative state and $S_k$is the absorbed state

Taking L.S.T. of above relations and solving for $\emptyset_0^{**}(s)$, we get

MTSF = $\lim_{s\to o} \frac{1-\tilde{\phi}_0(s)}{s}$ = $\frac{N_1}{D_1}$

Model I
N_1=$p_{01}(p_{13}\mu_3+\mu_1)$ + $(1-p_{13})(\mu_0+p_{02}\mu_2)$

D_1=$p_{01}p_{14}$

Model II
N_1= $(1-p_{13}p_{31})$ $(p_{02}\mu_2+\mu_0)+p_{01}$ $(\mu_1+p_{13}\mu_1)+p_{02}p_{23}$ $(\mu_1p_{31}+\mu_3)$

D_1=$(1-p_{13}p_{31})(1-p_{02}p_{20})-p_{10}(p_{01}+p_{02}p_{23}p_{31})$

Steady state availability
Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are given as $A_i(t)=M_i(t)+\sum_j q_{i,j}^{(n)}(t)$©$A_j(t)$ where S_j is any successive regenerative state to which regenerative state S_i can transit through n transitions.

Model I:
N_2=$\mu_0((1-p_{13})(1-p_{58})-p_{14}p_{59})+\mu_1p_{01}(1-p_{58})+p_{01}p_{14}\mu_5$

D_2=$(1-p_{58})(p_{01}(\mu'_1+p_{13}\mu_3)+m_{01}p_{10})+p_{14}(p_{01}(\mu'_5+p_{58}\mu_8)+m_{01}p_{50})+(m_{02}+p_{02}\mu_2)((1-p_{58})(1-p_{13})-p_{14}p_{59})$

Model II:
N_2=$(\mu_0+\mu_2p_{02})((1-p_{13}p_{31})p_{70}-p_{71.10}(p_{14}+p_{13}p_{35}))+(\mu_7+p_{79}\mu_9)(p_{35}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{70}(\mu_3(p_{01}p_{13}+p_{02}p_{23})+$
$\mu_1(p_{01}+p_{02}p_{23}p_{31}))$ - $p_{71.10}p_{02}p_{23}(p_{14}\mu_3-p_{35}\mu_1)$

D_2=$(\mu_0+\mu_2p_{02})((1-p_{13}p_{31})p_{70}-p_{71.10}(p_{14}+p_{13}p_{35}))+(\mu_7'+p_{79}\mu_9')(p_{35}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{70}(\mu_3'(p_{01}p_{13}+p_{02}p_{23})+$
$\mu_1'(p_{01}+p_{02}p_{23}p_{31}))$ - $p_{71.10}p_{02}p_{23}(p_{14}\mu_3'-p_{35}\mu_1')$

Busy period analysis of the server
Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state S_i at t=0.The recursive relations for $B_i(t)$ are as follows:

$B_i(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t)$©$B_j(t)$

Model I
N_3=$p_{01}(W_1^*(0)(1-p_{58})+p_{14}W_5^*(0))$

D_2=$(1-p_{58})(p_{01}(\mu'_1+p_{13}\mu_3)+m_{01}p_{10})+p_{14}(p_{01}(\mu'_5+p_{58}\mu_8)+m_{01}p_{50})+(m_{02}+p_{02}\mu_2)((1-p_{58})(1-p_{13})-p_{14}p_{59})$

Model II
N_3=$W_1^*(0)(p_{70}(p_{01}+p_{02}p_{23}p_{31})+p_{02}p_{23}p_{71.10}p_{35})+W_7^*(0)(p_{01}(p_{14}+p_{13}p_{35})+p_{02}p_{23}(p_{14}p_{31}+p_{35}))$

D_2=$(\mu_0+\mu_2p_{02})((1-p_{13}p_{31})p_{70}-p_{71.10}(p_{14}+p_{13}p_{35}))+(\mu_7'+p_{79}\mu_9')(p_{35}(p_{01}p_{13}+p_{02}p_{23})+p_{14}(p_{01}+p_{02}p_{23}p_{31}))+p_{70}(\mu_3'(p_{01}p_{13}+p_{02}p_{23})+$
$\mu_1'(p_{01}+p_{02}p_{23}p_{31}))$ - $p_{71.10}p_{02}p_{23}(p_{14}\mu_3'-p_{35}\mu_1')$

Expected number of visits by the server
Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $N_i(t)$ are given as:

$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledS [\delta_i + N_j(t)]$

The expected numbers of visits per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} sN_0^{**}(s) = \frac{N_4}{D_2}$$

Model I

$N_4 = p_{01}((1+p_{14}p_{46})(1-p_{58})+p_{14}(p_{59}p_{91}-p_{58}))$

$D_2=(1-p_{58})(p_{01}(\mu'_1+p_{13}\mu_3)+m_{01}p_{10})+p_{14}(p_{01}(\mu'_5+p_{58}\mu_8)+m_{01}p_{50})+(m_{02}+p_{02}\mu_2)((1-p_{58})(1-p_{13})-p_{14}p_{59})$

Model II

$N_4=p_{01}(1-p_{13}p_{31}+p_{13}+p_{14}p_{46})+p_{02}p_{23}(1+p_{14}p_{46}p_{31})-p_{71.10}(p_{01}(p_{14}+p_{35}p_{13})-p_{02}p_{23}(p_{35}p_{14}p_{46}-p_{14}))+p_{75.10}(p_{14}(p_{01}+p_{02}p_{23})$ $p_{31}(p_{02}p_{23}+p_{01}p_{13}) - p_{14}p_{46}(p_{02}p_{23}p_{31}+p_{01}) - p_{01}(p_{13}p_{31}))$

$D_2=(\mu_0+\mu_2p_{02})\ ((1-p_{13}p_{31})\ p_{70}-p_{71.10}\ (p_{14}+p_{13}p_{35})) + (\mu_7'+p_{79}\mu_9')\ (p_{35}\ (p_{01}p_{13} + p_{02}p_{23}) + p_{14}(p_{01}+p_{02}p_{23}p_{31})) + p_{70}(\mu_3'(p_{01}p_{13}+p_{02}p_{23}) + \mu_1'(p_{01}+p_{02}p_{23}p_{31}))-p_{71.10}p_{02}p_{23}(p_{14}\mu_3'-p_{35}\mu_1')$

Profit analysis

The profit incurred to the system Model in steady state can be obtained as

$P_i=K_0A_0-K_1B_0-K_2N_0;$

K_0=Revenue per unit up-time of the system

K_1=Cost per unit for which server is busy

K_2= Cost per unit visit by the server and A_0, B_0, N_0 are already defined.

Conclusion

Giving some particular values to the parameters and various costs, the numerical result for MTSF, Availability and Profit function are obtained to depict their graphical behavior with respect to normal weather rate β_1 keeping fixed values of the other parameters.

In Model I: It is observed that MTSF declines with the increase of normal weather rate (β_1) and failure rates (λ, λ_1) of the units. But MTSF increases with the increase of abnormal weather rate (β) and repair rate (α) of the main unit. The availability and profit of the system go on increasing with increase of normal weather rate (β_1) and repair rates (α and α_1) of the units. However, there is a downward trend in the values of these measures as and when values of abnormal weather rate (β) and failure rates (λ and λ_1) increase.

In Model II: It is observed that MTSF keeps on increasing with the increase of normal weather rate (β1) and repair rate (α) of the main unit. However, it decreases with the increase of failure rates (λ and λ1) and abnormal weather rate (β). The availability and profit of the system go on increasing with increase of normal weather rate (β1) and repair rates (α and α1) of the units. But their values decline with increase of abnormal weather (β) and failure rates (λ and λ1).

Comparative study

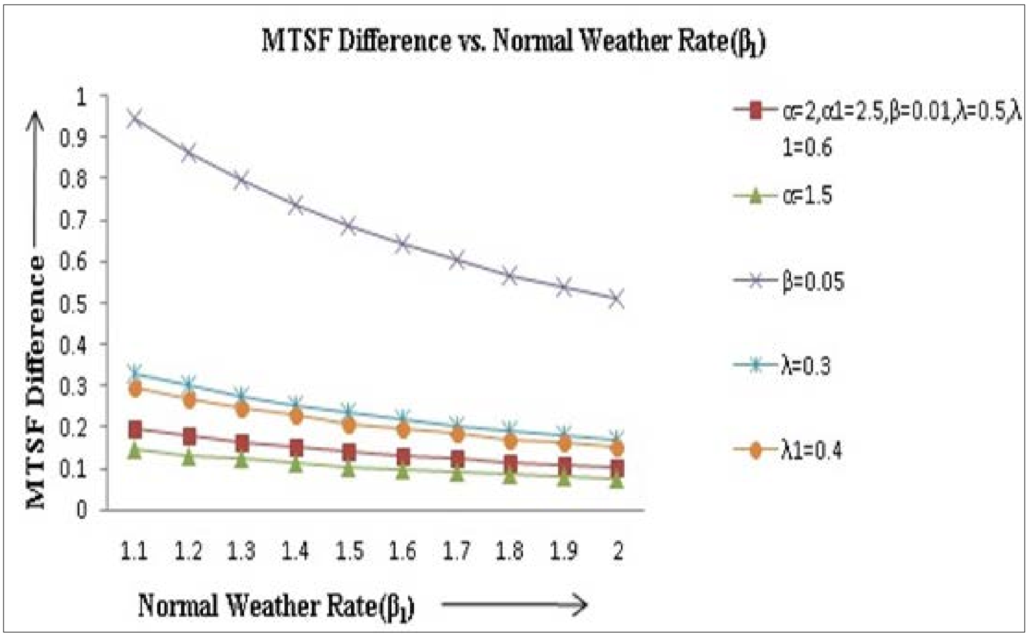
MTSF Comparison between Model I and Model II

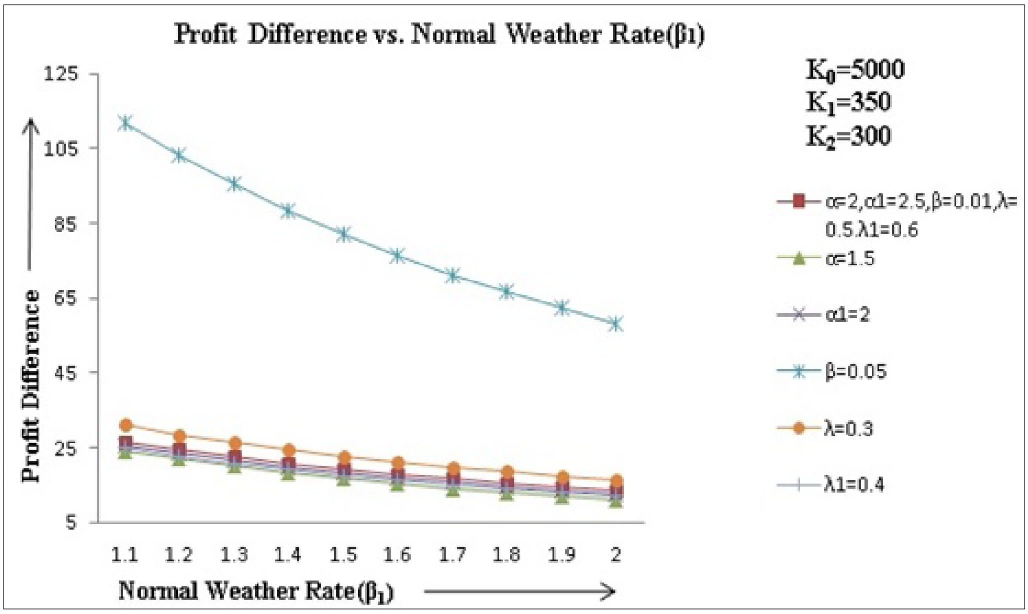
The MTSF difference (Model I - Model II) decreases with increase of normal weather rate (β1) and failure rates (λ and λ1) of the units. But, it increases with the increase of abnormal weather rate (β) and repair rate (α) of the main unit. Thus, the study reveals that the idea to allow operation of the system in abnormal weather is not useful in improving reliability of the system and system Model I has more MTSF as compared to system Model II.

Profit Comparison between Model I and Model II

The profit difference of the models (Model I - Model II) goes on decreasing with the increase of normal weather rate (β1) and failure rates of the units. However, it increases with increase of repair rates of the units and abnormal weather rate (β). The effect of abnormal weather rate (β) on profit difference is much more as compared to other parameters. The results indicate that Model II is profitable over model I. As a whole, it is analyzed that the idea to operate system in abnormal weather with priority for operation to main unit over duplicate unit is helpful in making the system more profitable.

Graphs





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