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Bulk viscous LRS bianchi type-I cosmological model of the inflationary universe in general relativity

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Abstract
We discussed Bianchi type-I bulk viscous cosmological model of inflationary universe in GR. To get deterministic model of the universe, it has been considered a flat region in which potential $V(\phi) = Constant$ where ϕ is Higg's field and also considered coefficient of bulk (ξ) is inversely proportional to the expansion (θ). The physical and geometrical aspects of the model in the context of inflationary scenario are also discussed.

Keywords: LRS Bianchi type-I, inflationary scenario, bulk viscosity

Introduction

The basic idea behind inflation is that a repulsive form of gravity caused the universe to expand. General relativity from its inception predicted the possibility of repulsive gravity; in the context of general relativity you basically need a material with a negative pressure to create repulsive gravity. According to general relativity it's not just matter densities or energy densities that create gravitational fields; it's also pressures. A positive pressure creates a normal attractive gravitational field of the kind that we're accustomed to, but a negative pressure would create a repulsive kind of gravity. It also turns out that according to modern particle theories, materials with a negative pressure are easy to construct out of fields which exist according to these theories. By putting together these two ideas — the fact that particle physics gives us states with negative pressures, and that general relativity tells us that those states cause a gravitational repulsion, we reach the origin of the inflationary theory.

By answering the question of what drove the universe into expansion, the inflationary theory can also answer some questions about that expansion that would otherwise be very mysterious. There are two very important properties of our observed universe that were never really explained by the Big Bang theory; they were just part of one's assumptions about the initial conditions. One of them is the uniformity of the universe — the fact that it looks the same everywhere, no matter which way you look, as long as you average over large enough volumes. It's both isotropic, meaning the same in all directions, and homogeneous, meaning the same in all places. The conventional Big Bang theory never really had an explanation for that; it just had to be assumed from the start. The problem is that, although we know that any set of objects will approach a uniform temperature if they are allowed to sit for a long time, the early universe evolved so quickly that there was not enough time for this to happen. To explain, for example, how the universe could have smoothed itself out to achieve the uniformity of temperature that we observe today in the cosmic background radiation, one finds that in the context of the standard Big Bang theory, it would be necessary for energy and information to be transmitted across the universe at about a hundred times the speed of light.

In the inflationary theory this problem goes away completely, because in contrast to the conventional theory it postulates a period of accelerated expansion while this repulsive gravity is taking place. That means that if we follow our universe backwards in time towards the beginning using inflationary theory, we see that it started from something much smaller than you ever could have imagined in the context of conventional cosmology without inflation. While the region that would evolve to become our universe was incredibly small, there was plenty of time for it to reach a uniform temperature, just like a cup of coffee sitting on the table cools down to room temperature. Once this uniformity is established on this tiny scale by normal thermal-equilibrium processes and I'm talking now about something that's about a billion times smaller than the size of a single proton inflation can take over, and cause this tiny region to expand rapidly, and to become large enough to encompass the entire visible universe. The inflationary theory not only allows the possibility for the universe to be uniform, but also tells us why it's uniform: It's uniform because it came from something that had time to become uniform, and was then stretched by the process of inflation.

In terms of the evolution of the universe, the fact that the universe is at least approximately flat today requires that the early universe was extraordinarily flat. The universe tends to evolve away from flatness, so even given what we knew ten or twenty years ago. we know much better now that the universe is extraordinarily close to flat, we could have extrapolated backwards and discovered that, for example, at one second after the Big Bang the mass density of the universe must have been equal, to an accuracy of 15 decimal places, to the critical density where it counterbalanced the expansion rate to produce a flat universe. The conventional Big Bang theory gave us no reason to believe that there was any mechanism to require that, but it has to have been that way to explain why the universe looks the way it does today. The conventional Big Bang theory without inflation really only worked if you fed into it initial conditions which were highly finely tuned to make it just right to produce a universe like the one we see. Inflationary theory gets around this flatness problem because inflation changes the way the geometry of the universe evolves with time. Even though the universe always evolves away from flatness at all other periods in the history of the universe, during the inflationary period the universe is actually driven towards flatness incredibly quickly. If you had approximately 10^{-34} seconds or so of inflation at the beginning of the universe, that's all you need to be able to start out a factor of 105 or 1010 away from being flat. Inflation would then have driven the universe to be flat closely enough to explain what we see today.

At the present time this inflationary theory, which a few years ago was in significant conflict with observation now works perfectly with our measurements of the mass density and the fluctuations. The evidence for a theory that's either the one that I'm talking about or something very close to it is very, very strong.

Bulk viscous cosmology is also an alternative to gravity modifying theories (Nojiri and Odintsov) [1] in that it alters the right hand side of Einstein's field equations instead of the left hand side. In this situation based on the Eckart theorem (Eckart) [2], the consideration of the DE fluid with viscous is important. The evolution of universe involves sequence of dissipative process. In isotropic and homogeneous model, the process of dissipative is modeled as a bulk viscosity (Ren and Meng [3]; Hu and Meng [4]; Meng and Duo [5]). Brevik *et al.* [6] investigated the overall cases of viscous cosmology in early and late time universe. Norman and Brevik [7] investigated the properties of the characteristic of two different viscous cosmological models for the future universe. Norman and Brevik [8] derived observance of the bulk viscous and approximated the bulk viscosity of the cosmic fluid. The finding of viscosity dominance by late epoch of the universe with accelerated expansion was studied by Padmanabhan and Chitre [9]. Velten *et al.* [10] have investigated phantom DE as an effect of bulk viscosity. It is illustrated by Brevik and Gorbunova [11] that the fluid which lies in the quintessence region can minimize its pressure and cross the barrier, and behaves like a phantom fluid which leads to the inclusion of large bulk viscosity in a sufficient way.

The characterization by shear and bulk viscosities leads to the effect of dissipation at microscopic interactions. As expansion fluid leaves its equilibrium state, the energy density and the pressure decrease. If there is no bulk viscosity, then, the fluid relaxes instantaneously with pressure and density related by. Bulk viscosity slightly met this nature by leading a certain relaxation time scale, but producing a shift between the equation of state pressure and the absolute pressure. Bulk viscosity becomes essential only for such effects where fluid compressibility is essential. Researchers have contributed a necessity role to bulk viscous fluid matter which is different from the traditional case. The effects of both shear and bulk viscosity were explained by Hoogeveen *et al.* [12] using kinetic theory during early time. The detailed implement for the origin of bulk viscosity in the universe is not correctly understood yet.

Based on the hypothetical view, the bulk viscosity can be derived from local thermodynamic equilibrium, the manifestation as an effective pressure to bring back the system to its thermal equilibrium, which was broken when the cosmological fluid expands. The bulk viscosity pressure thus generated ceases as soon as the fluid reaches equilibrium condition. Very recently, the concept of bulk viscosity is introduced into a DE study. It is important to develop a cosmological model. The concept of viscosity has come from fluid mechanics, and it is related to the velocity gradient of the fluid. Misner [13] revealed that during cosmic evolution when neutrinos decouple from the cosmic fluid, bulk viscosity could arise and lead to an effective mechanism of entropy production. The isotropic homogeneous spatially flat cosmological model with bulk viscous fluid was discussed by Murphy [14]. Bulk viscosity related to the grand unified-theory phase transition (Langacher) [15] may lead to explain the cosmic acceleration. The presence of bulk viscosity leads to an inflationary-like solution in Friedmann-Robertson-Walker (FRW) space time obtained by Padmanabhan and Chitre [16].

Guth [17] introduced the idea of early inflationary phase the context of grand unified field theories. Panchapakeshan and Sethi [18] have discussed inflationary scenario and large scale structure of the universe. Bali and Jain [19] discussed inflationary scenario in Bianchi type- I inflationary universe in general relativity. Bali and Singh [20] have investigated Bianchi Type-V viscous fluid string dust cosmological model assuming the condition that the bulk coefficient (ξ) is inversely proportional the expansion (θ) in the model.

In the present work, we have studied Bulk viscous LRS Bianchi Type-I cosmological model of the inflationary universe. The geometrical and physical aspects of the model have been studied. We consider string cosmology far the spherically symmetric is homogeneous anisotropic space-time with metric amsatz.

$$d^2s = -d^2t + a^2 d^2x + b^2\left(d^2y + d^2z\right) \tag{1}$$

Where $a = a(t),\, b = b(t)$

The Lagrangian will be that of gravity minimally coupled to a scalar field $V(\varphi)$ (Stein-Schalers, [21])

$$s = \int -g\left[R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\upsilon \varphi - V(\varphi)\right] d^4x \tag{2}$$

[Notations have their usual meaning and units are taken so that $8\pi G = 1 = C$].

Now the variation of S with respect to the dynamical field, leads to the Einstein field equation in presences of bulk viscosity

$$R_{\mu\vartheta} - \frac{1}{2} R g_{\mu\vartheta} + \xi\theta\left(u_\mu u^\nu + g_{\mu\vartheta}\right) = -T_{\mu\vartheta} \tag{3}$$

Where u^μ the four velocity vector, $\theta = u_{\mu}{}^{\nu}$, is the scalar of expansion and ξ the coefficient of bulk viscosity.

Here

$$T_{\mu\upsilon} = \partial_\mu \varphi \partial_\upsilon \varphi - \left[\frac{1}{2}\partial_\mu \varphi \partial^p \varphi + V(\varphi)\right] g_{\mu\upsilon} \quad ; \quad u_\mu u^\nu = -1 \tag{4}$$

And

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\,\partial^\mu\varphi\right]=-\frac{dV(\varphi)}{d\varphi} \quad (5)$$

The Einstein field equation (3) from the metric (1) are given by,

$$\frac{\dot{b}}{b^2}+2\frac{\ddot{b}}{b}=-\frac{1}{2}\dot{\phi}^2+V(\varphi)+\xi \quad (6)$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{b}}{a\,b}=-\frac{1}{2}\dot{\phi}^2+V(\varphi)+\xi \quad (7)$$

$$2\frac{\dot{a}\dot{b}}{a\,b}+\frac{\dot{b}}{\boldsymbol{b}}=\frac{1}{2}\dot{\phi}^2+V(\varphi) \quad (8)$$

And the equation for the scalar field ϕ leads to

$$\ddot{\phi}+\left(\frac{\dot{a}}{a}+2\frac{\dot{b}}{b}\right)\dot{\phi}+\frac{dV}{d\varphi}=0 \quad (9)$$

Solution of the field equations:

We are interested in inflationary solutions, the flat region is considered where Potential is constant. Also to get determinant model we assume that coefficient of bulk viscosity (ξ) is proportional to expansion θ (Kandalkar *et al.*)[22], thus we have

$$V(\varphi)=\text{constant}=c_0\ (\text{say})\quad \text{and}\quad \xi\theta=\text{constant}=c_1\ (\text{say}) \quad (10)$$

Thus Equation (9) leads to

$$\dot{\phi}=\frac{\alpha}{a^2} \quad (11)$$

Where α is the constant of integration.
From $eq.$ (9) and (7), we have

$$\frac{\ddot{a}}{a}-\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{b}}{a\,b}-\frac{\dot{b}^2}{b^2}=0 \quad (12)$$

Which leads to

$$\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)^{\bullet}+\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)\left(\frac{\dot{a}}{a}-\frac{2\dot{b}}{b}\right)=0 \quad (13)$$

Which on integration leads to

$$\frac{\dot{a}}{a}-\frac{\dot{b}}{b}=\frac{k}{a^2} \quad (14)$$

Now equations $(7)+\frac{1}{2}(6)-\frac{1}{2}(8)$ together with eq. (11) leads to

$$\frac{\ddot{a}}{a}+2\frac{\ddot{b}}{b}=V_0-\frac{\alpha^2}{a^2b^4}\qquad \text{where } c_0+c_1=V_0 \quad (15)$$

$$\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right)^{\bullet}+\frac{\dot{a}}{a^2}+\frac{2\dot{b}}{b^2}=V_0-\frac{\alpha^2}{a^2b^4} \quad (16)$$

Using $ab^2=\eta$ in eq. (16), we get

$$\left(\frac{\dot{\eta}}{\eta}\right)+\left(\frac{\dot{a}^2}{a^2}\right)+2\frac{\dot{b}^2}{b^2}=V_0-\frac{\alpha^2}{\eta^2} \quad (17)$$

Now, $ab^2=\eta$ leads to

$$\frac{\dot{\eta}}{\eta}=\frac{\dot{a}}{a}+2\frac{\dot{b}}{b} \quad (18)$$

From $eq.(14)$, we have

$$\frac{\dot{a}}{a}-\frac{\dot{b}}{b}=\frac{k}{\eta} \quad (19)$$

$$3\frac{\dot{a}}{a}=\frac{\dot{\eta}}{\eta}+\frac{2k}{\eta} \quad (20)$$

And

$$3\frac{\dot{b}}{b}=\frac{\dot{\eta}}{\eta}-\frac{k}{\eta} \quad (21)$$

Using $eq.(20)$ and $eq.(21)$ we have

$$\left(\frac{\dot{\eta}}{\eta}\right)^{\bullet}+\frac{1}{9}\left(\frac{\dot{\eta}}{\eta}+2\frac{k}{\eta}\right)^2+\frac{2}{9}\left(\frac{\dot{\eta}}{\eta}-\frac{k}{\xi}\right)^2=V_0-\frac{\alpha^2}{\eta^2} \quad (22)$$

Which leads to

$$\eta\ddot{\eta}-\frac{2}{3}\dot{\eta}^2-V_0\eta^2+\frac{2}{3}k^2+\alpha^2=0 \quad (23)$$

Using suitable the transformation

$$\dot{\eta}=f(\eta) \quad (24)$$

So that

$$\ddot{\eta}=f\frac{d}{d\eta} \quad (25)$$

$eq.\ (23)$ Now becomes

$$\frac{d^2 f}{d\eta} - \frac{4}{3\eta} f^2 = 2V_0\eta - \frac{\frac{4}{3}k^2 + 2k^2}{\eta} \tag{26}$$

Which leads to

$$f^2\eta^{-4/3} = 3V_0\eta^{2/3} + \left(k^2 + \frac{3}{2}\alpha^2\right)\eta^{-3/4} + m \tag{27}$$

We had four $eq.\ (6)-(9)$ in five unknown $a, b, \varphi\ \xi$ and V, the condition $V = const. = c_0$ and $\xi\theta = const = c_1$, gives us four equations in three unknown which is over determined set. It is easy to verify that these equations are consistent and have a solution when $m = 0$. Thus $eq.(27)$ lead to

$$\dot{\eta} = \sqrt{3V_0\eta^{2/3} + \left(k^2 + \frac{3}{2}\alpha^2\right)} \tag{28}$$

Which leads to

$$\frac{d\eta}{\sqrt{3V_0\eta^2 + \left(k + \frac{3}{2}\alpha^2\right)}} = d \tag{29}$$

And this leads to

$$\eta = m\sinh\sqrt{3V_0}\,(t+\beta) \tag{30}$$

Where,

$$\frac{k^2 + \frac{3}{2}\alpha^2}{3V_0} = m^2 \tag{31}$$

And β is a constant.
Thus from (20) and (21), we have

$$3\frac{\dot{a}}{a} = \frac{\dot{\eta} + 2k}{\eta} = \frac{m\sqrt{3V_0}\cosh\sqrt{3V_0}\,(t+B) + 2k}{m\sinh\sqrt{3V_0}\,(t+B)} \tag{32}$$

And

$$3\frac{\dot{b}}{b} = \frac{\dot{\eta} - k}{\eta} = \frac{m\sqrt{3V_0}\cosh\sqrt{3V_0}\,(t+B) - k}{m.\sinh\sqrt{3V_0}\,(t+B)} \tag{33}$$

$Equation\ (32)\ and\ (33)$, on integration lead to

$$a = \gamma^{1/3}\sinh^{1/3}\sqrt{3V_0}\,(t+B)\left[\tanh\left\{\frac{1}{2}\sqrt{3V_0}\,(t+B)\right\}\right]^{\frac{2k}{3m\sqrt{3V_0}}} \tag{34}$$

$$b = \delta^{1/3}\sinh^{1/3}\sqrt{3V_0}\,(t+B)\left[\tanh\left\{\frac{1}{2}\sqrt{3V_0}\,(t+B)\right\}\right]^{-\frac{k}{3m\sqrt{3V_0}}} \tag{35}$$

Thus,

$$\dot{\varphi} = \frac{\alpha}{a^2 b} = \frac{\alpha}{\eta} = \frac{\alpha}{m\sinh\sqrt{3V_0}\,(t+B)} \tag{36}$$

Using suitable transformation
$t + \beta = T$, $\gamma^{1/3}x = X$, $\delta^{1/3}(y+z) = Y + Z$
The metric (1) leads to

$$ds^2 = -dT^2 + \sinh^{2/3}\left(\sqrt{3V_0}\,T\right)\tanh^{2k/3n\sqrt{3V_0}}\left(\frac{1}{2}\sqrt{3V_0}\,T\right)dX^2 + \sinh^{2/3}\left(\sqrt{3V_0}\,T\right)\tanh^{-k/3n\sqrt{3V_0}}\left(\frac{1}{2}\sqrt{3V_0}\,T\right)(dY^2 + dZ^2) \tag{37}$$

The model (37) represents LRS Bianchi type I bulk viscous inflationary universe in general relativity.

Some Physical and Geometrical Aspects
The scalar of expansion (θ) and the shear (6) for the model (37) are given by

$$\theta = \frac{\dot{a}}{a} + \frac{2\dot{b}}{b} = \frac{\dot{\eta}}{\eta} = \coth\left(\sqrt{3V_0}\,T\right) \tag{38}$$

$$\sigma = \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) = \frac{\sqrt{2}}{\sqrt{3}}\frac{k}{m\sinh\left(\sqrt{3V_0}\,T\right)} \tag{39}$$

The rate of expansion H; (Hubble parameters) in the direction of X, Y, Z are given by

$$H_1 = \frac{m\sqrt{3V_0}\cosh\left(\sqrt{3V_0}\,T\right) + 2k}{3m\sinh\left(\sqrt{3V_0}\,T\right)} \tag{40}$$

And

$$H_2 = H_3 = \frac{m\sqrt{3V_0}\cosh\left(\sqrt{3V_0}\,T\right) - k}{3m\sinh\left(\sqrt{3V_0}\,T\right)} \tag{41}$$

The Higg's field (φ) is given by $Eq^n\ (36)$

$$\dot{\varphi} = \frac{\alpha}{m\sinh\left(\sqrt{3V_0}\,T\right)} \tag{42}$$

The spatial volume V is given by

$$V = \sinh\sqrt{3V_0}\,T \tag{43}$$

The coefficient of bulk viscosity is $\xi \propto \frac{1}{\theta}$

$$\xi = c_o \tanh\left(\sqrt{3V_o}\, T\right) \tag{44}$$

Conclusion

The model (37) has no initial singularity. From equation (31), we have, $\frac{\alpha}{m} = \sqrt{2V_0 - \left(\frac{2k^2}{3m^2}\right)}$, α is constant of integration. Clearly $V_0 > \frac{k^2}{3m^2}$. Thus the quantity $\frac{k}{m}$ has physical relevance. The quantity $\frac{\alpha}{m}$ appears in Higg's field. The model vanish for large T and it diverges when $T = 0$.

The coefficient of bulk viscosity ξ is vanishes for $T = 0$ and tends to constant as $T \to \infty$.

Hence the universe remains anisotropy throughout the evolution of the universe.

The expansion $\theta \to \infty$, when $T \to 0$ and $\theta \to$ *finite quantity when* $T \to \infty$. The $\frac{k}{m}$ measures anisotropy in the model. The shear value of $\sigma \to 0$ when $T \to \infty$ and $\sigma \to \infty$ when $T \to 0$

The spatial volume increases with time. When $T \to \infty$ then spatial volume $V \to \infty$ .Hence inflation is exist in Bulk viscous LRS Bianchi type I model with a mass less scalar field in the potential which has flat space region.

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