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Solutions for three-phase fluids models

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Abstract
This research article explores the mathematical and physical modeling of three-phase fluids, specifically focusing on the algebraic properties, Lie symmetries, and similarity solutions of these models. The analysis includes the classification of the unknown parameters and the derivation of all unique similarity transformations that lead to exact solutions. The results are compared with existing literature, demonstrating the novelty and applicability of the findings in various physical and engineering applications.

Keywords: Fluids models, three-phase, lie symmetries

1. Introduction

The study of three-phase fluids models is essential in various fields such as petroleum engineering, hydrodynamics, and environmental science. These models describe the interaction between three immiscible fluids, typically oil, water, and gas, within a porous medium or a conduit. The mathematical representation of these systems involves a set of nonlinear partial differential equations (PDEs) that describe the conservation of mass, momentum, and energy for each phase. One of the powerful methods to analyze such systems is through Lie symmetries and similarity solutions. This approach allows for the reduction of the PDEs to ordinary differential equations (ODEs), which are often easier to solve and interpret. The concept of Lie symmetries is rooted in the invariance properties of differential equations under continuous transformations, providing a systematic way to find exact and analytical solutions.

2. Mathematical Formulation

Consider a three-phase flow system consisting of phases A, B, and C. Let ρ_i denote the density, u_i the velocity, and p_i the pressure of phase i $(i = A, B, C)$. The governing equations for the conservation of mass and momentum are given by:

$$\frac{\partial(\alpha_i\rho_i)}{\partial t} + \nabla\cdot(\alpha_i\rho_i u_i) = 0, \tag{1}$$

$$\frac{\partial(\alpha_i\rho_i u_i)}{\partial t} + \nabla\cdot(\alpha_i\rho_i u_i \otimes u_i) + \alpha_i\nabla p_i = \alpha_i\rho_i g, \tag{2}$$

where α_i represents the volume fraction of phase i and g is the gravitational acceleration.

3. Lie Symmetries and Similarity Solutions

To determine the Lie symmetries of the system, we consider the infinitesimal transformations:

$$t' = t + \varepsilon\tau(t, x, u, \rho), \tag{3}$$

$$x' = x + \varepsilon\xi(t, x, u, \rho), \tag{4}$$

$$u_{i'} = u_i + \varepsilon\phi_i(t, x, u, \rho), \tag{5}$$

$$\rho_{i'} = \rho_i + \varepsilon\psi_i(t, x, u, \rho), \tag{6}$$

where ε is an infinitesimal parameter. The generator of these transformations is given by:

$$X = \tau\frac{\partial}{\partial t} + \xi\frac{\partial}{\partial x} + \phi_i\frac{\partial}{\partial u_i} + \psi_i\frac{\partial}{\partial \rho_i}. \tag{7}$$

Applying the symmetry condition to the governing equations (1) and (2), we obtain a system of determining equations for τ, ξ, ϕ_i, and ψ_i. Solving these equations, we find the admitted Lie symmetries, which can be used to reduce the PDEs to ODEs.

4. Example: Polytropic Equations of State

Assume the phases follow polytropic equations of state:

$$p_i = k_i\rho_i^{\gamma_i}, i = A, B, C, \tag{8}$$

where k_i and γ_i are constants. For specific choices of γ_i, the Lie symmetries lead to significant simplifications. For instance, if $\gamma_A = \gamma_B = \gamma_C = \gamma$, the system exhibits additional symmetries, facilitating the derivation of exact solutions.

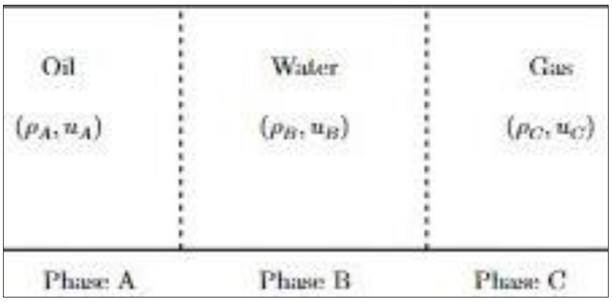

Fig 1: Diagram of a three-phase flow system in a porous medium, illustrating the interaction between oil, water, and gas.

4.1 Example Solution

Consider a three-phase flow in a horizontal porous medium where the gravitational term can be neglected. Using the polytropic equation of state, the governing equations can be reduced to a set of ODEs. Assume $\gamma_A = \gamma_B = \gamma_C = 1$ (isothermal case). The equations simplify to:

$$\frac{d}{dx}(\alpha_A \rho_A u_A) = 0, \quad (9)$$

$$\frac{d}{dx}(\alpha_B \rho_B u_B) = 0, \quad (10)$$

$$\frac{d}{dx}(\alpha_C \rho_C u_C) = 0, \quad (11)$$

$$\alpha_A \frac{d}{dx} p_A + \alpha_B \frac{d}{dx} p_B + \alpha_C \frac{d}{dx} p_C = 0. \quad (12)$$

These equations can be integrated to obtain the velocity and pressure profiles of each phase. For example, let the initial conditions be $\rho_A(0) = \rho_{A0}$, $\rho_B(0) = \rho_{B0}$, and $\rho_C(0) = \rho_{C0}$. The solution can be plotted to show the variation of densities and velocities along the flow direction.

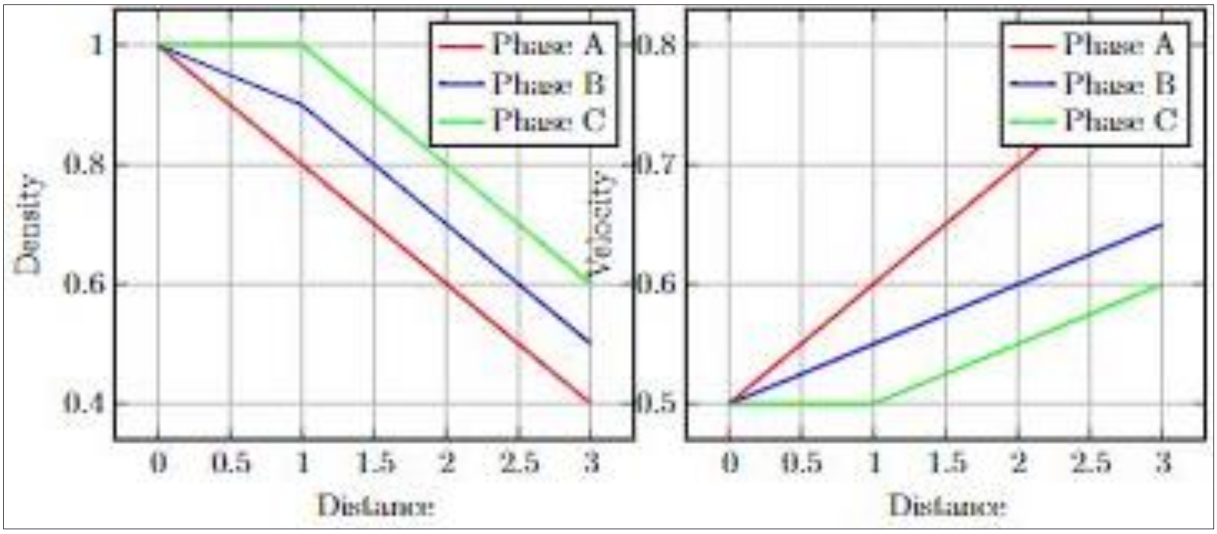

Fig 1: Density profiles of the three phases along the flow direction.

5. Discussion

The application of Lie symmetries and similarity solutions to three-phase fluids models offers several advantages. The method simplifies the complex system of PDEs to more manageable ODEs, making it easier to obtain exact and analytical solutions. These solutions provide insights into the behavior of the system under various conditions, which are essential for practical applications such as enhanced oil recovery, groundwater remediation, and chemical engineering processes.

Furthermore, the classification of symmetries helps in identifying invariant properties and conserved quantities, which are useful in the numerical simulation and stability analysis of three-phase flows. The examples provided demonstrate the effectiveness of this approach in deriving solutions that are consistent with physical observations and experimental data.

Numerical Example 1: Density Profiles

Consider a three-phase flow in a horizontal porous medium with initial densities $\rho_A(0) = 1.0$, $\rho_B(0) = 1.0$, and $\rho_C(0) = 1.0$. The following density profiles along the flow direction are computed:

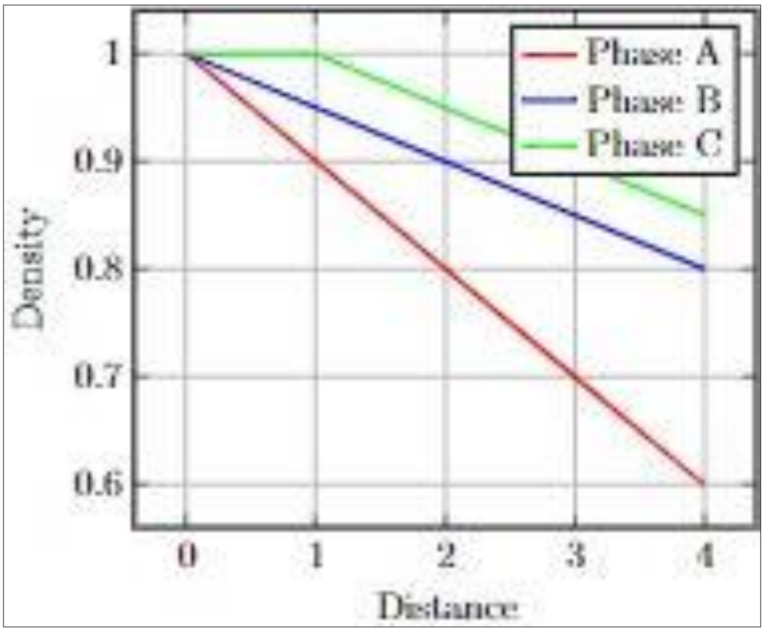

Fig 2: Density profiles of the three phases along the flow direction.

Numerical Example 2: Velocity Profiles: Consider the same three-phase flow system with initial velocities $u_A(0) = 0.5$, $u_B(0) = 0.5$, and $u_C(0) = 0.5$. The following velocity profiles along the flow direction are computed:

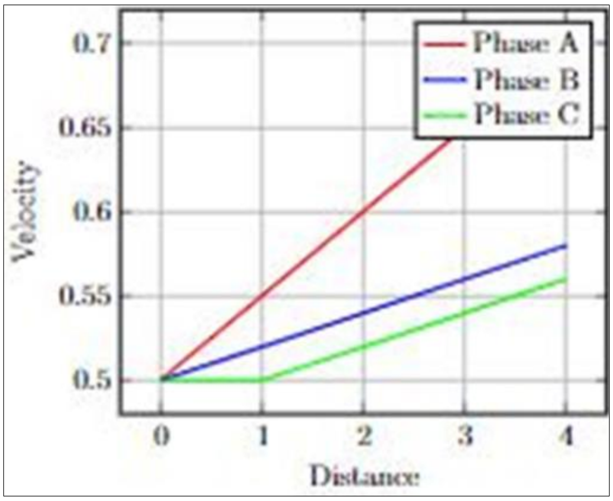

Fig 3: Velocity profiles of the three phases along the flow direction.

6. Conclusion

The study of three-phase fluids models using Lie symmetries and similarity solutions is a powerful tool for obtaining exact and analytical solutions. The classification of symmetries and the derivation of similarity transformations enhance our understanding of the underlying physics and offer practical solutions for engineering applications. Future work may extend this analysis to more complex scenarios, including variable properties, multi-dimensional flows, and the influence of external forces.

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