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Poisson probability distribution of arrivals and departures in a queuing service system

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Abstract
This research paper investigates the use of poisson distribution in the arrival, inter-arrival and departure in a queuing service system which encompasses the total knowledge and application of arrival, waiting and leaving time in the field of operation research. Everyday experience makes people to queue everywhere, even in banks, supermarket, grocery, restaurants, movies hall, pooling booths and a host of others. The queuing model with multiple vacation, closedown, essential and optional repair was also examined. Various terms and systems adopted in queuing operations like first come first served, last come first served, service in random order and priority discipline were all studied.

Keywords: poisson distribution, queuing service system, arrival, departure

Introduction

Queuing theory, also known as Random System Theory, is the body of knowledge about waiting lines and is now an entire discipline within the field of operations research (Nosek and Wilson, 2001; Kavitha and Palaniammal, 2014; and Ramakrishna and Mohamedhusien, 2015) [23, 15]. In fact, queuing theory has become a valuable tool for operations managers (Ramakrishna and Mohamedhusien, 2015) [27]. The authors maintain that waiting has become part of everyday life. For example, queuing system has been employed in our day to day commercial (as well as socio-political) lives (Kavitha and Palaniammal, 2014) [15]. Some scholars maintain that we queue or wait in line to get served in commercial outfits like checkout counters, banks, super markets, fast food restaurants etc. (Kavitha and Palaniammal, 2014) [15], grocery stores, post offices, to waiting on hold for an operator to pick up telephone calls, waiting at an amusement park to go on the newest ride (Mandia, 2009) [20]. Others according to the authors include: waiting in lines at the movies, campus dining rooms, the registrar's office for class registration, at the Division of Motor Vehicles etc. We equally queue in other socio-political settings like queuing to vote, waiting in line to be attended to by a public servant in government offices, etc.

Queuing theory has many applications in human endeavors, some of which include: telephony; manufacturing; inventories; dams; supermarkets; computer and information communication systems and networks; call centers; hospitals, banking, and many others. (SZTRIK, 2010). Nosek and Wilson (2001) [23] confirm that queuing theory has been used extensively by the service industries. The major applications of vacation queuing models are in computer and communication systems, manufacturing systems, service systems, among others. A literature survey on vacation queuing models can be found in Doshi (1986) and Takagi (1991) which include some applications. Lee (1991) developed a systematic procedure to calculate the system size probabilities for a bulk queuing model. Krishna Reddy *et al*. (1998) considered an $M^{\{X\}}/G(a,b)/1$ queuing model with multiple vacations, set-up times and N policy. They derived the steady-state system size distribution, cost model, expected length of idle and busy period. Arumuganathan and Jeyakumar (2005) [2] obtained the probability generating function of queue length distribution at an arbitrary time epoch and a cost model for the $M^{\{X\}}/G(a,b)/1$ queuing model.

However, the queuing theory considers mainly six general characteristics of any queuing processes:

1. Arrival pattern of customers: inter-arrival times most commonly fall into one of the following distribution patterns: A Poisson distribution, a Deterministic distribution, or a General distribution. However, inter-arrival times are most often assumed to be independent and memory-less, which is the attributes of a Poisson distribution.
2. Service pattern: the service time distribution can be constant, exponential, hyper exponential, hypo-exponential or general. The service time is independent of the inter-arrival time
3. Number of servers: the queuing calculations change depends on whether there is a single server or multiple servers for the queue. A single server queue has one server for the queue. This is the situation normally found in a grocery store where there is a line for each cashier. A multiple server queue corresponds to the situation in a bank in which a single line waits for the first of several tellers to become available.
4. Queue Lengths: the queue in a system can be modeled as having infinite or finite queue length.
5. System capacity: the maximum number of customers in a system can be from one up to infinity. This includes the customers waiting in the queue.
6. Queuing discipline: there are several possibilities in terms of the sequence of customers to be served.

- **FCFS:** First Come, First Served. This is the most commonly used discipline applied in the real-world situations, such as check-in counters at the airport.
- **LCFS:** Last Come, First Served**.** This illustrates a reverse order service given to customer versus their arrival.
- **SIRO:** Service in Random Order.
- **PD:** Priority Discipline. Under this discipline, customers will be classified into categories of different priorities.

Research methodology

The random variables of interest are: "in fixed time interval, how many arrivals take place? How many departures take place? What is the distribution of the arrivals? What is the distribution of the departures?"

Distribution of Arrivals

Theorem: If the arrivals are completely random, then the probability distribution of number of arrivals in a fixed interval follows a Poisson distribution.

Proof: Let us define the terms that are commonly used in the development of various queuing model:
Δt = a time interval so small that the probability of more than one customer's arrival is negligible, that is, during any given small interval time Δt only one customer can arrive

$\lambda\Delta t$ = probability that a customer will arrive in the system during time Δt

$1-\lambda\Delta t$ = probability that no customer will arrive in the system during time Δt

Case 1: $n \geq 1 \; and \; t \geq 0$

Sometimes customers arrive and join the queue before the start of the service. They may have presumed that their early arrival would reduce their waiting time. Now the objective is to know the number of customers waiting in line when the service begins. Thus, $P_n(t+\Delta t)$, the probability of n customers in the system at time $t+\Delta t$ can be expressed as the sum of the joint probabilities of the following two mutually exclusive cases:

1. the system contains n customers at time t and there is one arrival during time interval Δt
2. the system contains $(n-1)$ customers at time t and there is one arrival and no departure during time interval Δt

That is,

$P_n(t+\Delta t)$={(probability of n customers in the system at time t) X (probability of no arrival during time Δt)}+{(probability of (n-1)-customers in the system at time t) X (probability of one arrival during time Δt)}

$$= P_n(t)\{1-\lambda\Delta t\} + P_{n-1}(t)\{\lambda\Delta t\} + O(\Delta t) \tag{1}$$

Case I1: $n = 0 \; and \; t \geq 0$

If there is no customer in the system at time $t+\Delta t$, then there will be no arrival during Δt. Thus, the probability of no customer in the system at time $t+\Delta t$ is given by:

$$P_0(t+\Delta t) = (probability \; of \; no \; customer \; at \; time \; t) X (probability \; of \; no \; arrival \; during \; time \; \Delta t)$$

$$= P_0(t)\{1-\lambda\Delta t\} \tag{2}$$

Equations (1) and (2) may be written respectively as:

$$\frac{P_n(t+\Delta t)-P_n(t)}{\Delta t} = -\lambda P_n(t) + \lambda P_{n-1}(t) + \frac{O(\Delta t)}{\Delta t}; \; n \geq 1$$
$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = -\lambda P_0(t) + \frac{O(\Delta t)}{\Delta t}; \; n = 0$$

Letting $\Delta t \to 0$ and taking limits of both sides, the following system of differential difference equations are developed:

$$P_n^1(t) = \frac{d}{dt}\{P_n(t)\} = -\lambda P_n(t) + \lambda P_{n-1}(t); \; n \geq 1 \tag{3}$$

$$P_0^1(t) = \frac{d}{dt}\{P_0(t)\} = -\lambda P_0(t); \; n = 0 \tag{4}$$

Solution of differential difference equations: Equation (4) can be rewritten as:

$$\frac{P_0^1(t)}{P_0(t)} = -\lambda$$

By integrating both sides with respect to t, this result into:

$$log\, P_0(t) = -\lambda t + A \quad (5)$$

Where

A is the constant of integration and its value can be determined by using the following initial conditions:

$$P_0(t) = \begin{cases} 1; \; n = 0, t = 0 \\ 0; \; n \geq 0, t = 0 \end{cases}$$

Substituting $t = 0$ into equation (5), this result into $P_0(0) = 1$. Thus, the value of $A = 0$. Now, equation (5) reduces to the form:

$$log\, P_0(t) = -\lambda t \; or \; P_0(t) = e^{-\lambda t}, t \geq 0 \quad (6)$$

Now, putting $n = 1$ into equation (3) and by using equation (6), this result into:

$$\mathrm{P_1^1(t) = -\lambda P_1(t) + \lambda P_0(t)} \quad (7)$$

$$\mathrm{or\; P_1^1(t) + \lambda P_1(t) = \lambda e^{-\lambda t}} \quad (8)$$

Equation (8) is a linear differential equation of the first order. Therefore, it can be solved by multiplying both sides of it by integrating factor:

$$Integrating\; factor\; (IF) = exp\left(\int \lambda\, dt\right) = exp(\lambda t)$$

Then equation (8) becomes:

$$e^{\lambda t}\{P_1^1(t) + \lambda P_1(t)\} = \lambda \; or \; \frac{d}{dt}\{e^{\lambda t}P_1(t)\} = \lambda$$

On integrating both sides with respect to t, it results into:

$$e^{\lambda t}P_1(t) = \lambda t + B \quad (9)$$

Where

B is the constant of integration and its value can again be obtained by initial conditions. That is, setting $t = 0$ into equation (9), this result in to $P_0(0) = B = 0$, since $P_1(0) = 0$. Thus, equation (9) reduces to the form:

$$e^{\lambda t}P_1(t) = \lambda t \; or \; P_1(t) = \frac{\lambda t}{e^{\lambda t}} = \lambda t e^{-\lambda t} \quad (10)$$

Again, putting $n = 2$ into equation (3) and using the result of equation (10), this result into:

$$P_2^1(t) + \lambda P_2(t) = \lambda\left(\lambda + e^{-\lambda t}\right) \; or \; \frac{d}{dt}\{e^{\lambda t}P_2(t)\} = \lambda(\lambda t)$$

$$or\; e^{\lambda t}P_2(t) = \frac{\lambda(\lambda t)t}{2!} + C = \frac{(\lambda t)^2}{2!} + C$$

where

C is the constant of integration and its value is $C = 0$. For $t = 0$ and$P_2(0) = 0$. Thus:

$$e^{\lambda t}P_2(t) = \frac{(\lambda t)^2}{2!} \; or \; P_2(t) = \frac{(\lambda t)^2}{2!}e^{-\lambda t}$$

In general, this result into:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \, for \, n = 0,1,2 \ldots$$

Thus, general solution for $P_n(t)$ indicates the number of customers in the system at time t before the start of the service facility and it follows Poisson distribution with mean and variance equal to λt. The expected or mean number of customer does not depend on the service time and therefore this solution holds good irrespective of the nature of service that will be provided to the waiting customers in the system.

Remarks: The linear first order differential equation of the form:

$$\frac{d}{dt}\{f(x)\} + \phi(x).f(x) = P(x)$$

Has the solution

$$f(x) = Cexp\left\{-\int \phi(x)\right\}dx + exp\left\{-\int \phi(x)\right\}dx\left[\int exp\left\{-\int \phi(x)\right\}dxP(x)dx\right]$$

Where
C is a constant.

Alternative Method: Equations (3) and (4) can also be solved by using the probability generating function approach. Defining the probability generating function of $P_n(t)$ as follows:

$$G(z,t) = \sum_{n=0}^{\infty} P_n(t)z^n; \, /z/ \leq 1$$

Also,

$$\frac{d}{dt}\{G_n(z,t)\} = \frac{d}{dt}\left\{\sum_{n=0}^{\infty} P_n(t)z^n\right\} = \sum_{n=0}^{\infty} \frac{d}{dt} P_n(t)z^n$$

or

$$G^1(z,t) = \sum_{n=0}^{\infty} {P_n}^1(t)z^n$$

Multiplying both sides of equation (3) by z^n and summing over the appropriate range on n, it results into:

$$\sum_{n=0}^{\infty} {P_n}^1(t)z^n = -\lambda \sum_{n=1}^{\infty} P_n(t)z^n + \lambda \sum_{n=1}^{\infty} P_{n-1}(t)z^n \qquad (11)$$

Adding equations (4) and (11), we obtain:

$$\sum_{n=0}^{\infty} {P_n}^1(t)z^n = -\lambda \sum_{n=0}^{\infty} P_n(t)z^n + \lambda \sum_{n=0}^{\infty} P_{n-1}(t)z^n$$

or $G^1(z,t) = -\lambda G(z,t) + \lambda z G(z,t)$

or $\frac{G^1(z,t)}{G(z,t)} = \lambda(z-1)$

or $\frac{d}{dt}\{\log G(z,t)\} = \lambda(z-1)$

Integrating both sides of this differential equation, we get:

$$\log G(z,t) = \lambda(z-1)t + C \qquad (12)$$

Where
C is the constant of integration and its value can be obtained by using initial condition, that is, set $t = 0$. By so doing, it results into:

$\log G(z,0) = C$ for $t = 0$ But

$$G(z,0) = \sum_{n=0}^{\infty} P_n(0)z^n = P_0(0) + \sum_{n=1}^{\infty} P_n(0)z^n = 1$$

Since $P_n(0), for\ n = 1$. Thus, $C = \log G(z,0) = log1 = 0$. Hence, equation (12) reduces to the form:

$$\log G(z,t) = \lambda(z-1)t \text{ or } G(z,t) = e^{\lambda(z-1)t} \quad (13)$$

From generating function at $z = 0$, this results into:

$$\frac{d^n}{dz^n}\{G(z,t)\} = n!\,P_n(t)$$

or $P_n(t) = \frac{1}{n!}\left\{\frac{d^n}{dz^n}\{G(z,t)\}\right\}$ $or\ P_0(t) = [G(z,t)]_{z=0} = e^{-\lambda t}$

Using the result of equation (13) for $n = 0\ and\ z = 0$. Similarly, at $z = 0$

$$P_1(t) = \left\{\frac{d}{dz}G(z,t)\right\} = \left\{e^{\lambda(z-1)t}(\lambda t)\right\} = \frac{\lambda t e^{-\lambda t}}{1!}$$

$$P_2(t) = \frac{1}{2!}\left\{\frac{d^2}{dz^2}G(z,t)\right\} = \frac{(\lambda t)^2 e^{-\lambda t}}{2!} \vdots P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, n = 0,1,2 \ldots$$

Which is the same as derived earlier

Distribution of Inter-arrival Times
Theorem: If the arrival process follows the Poisson distribution:

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, n = 0,1,2 \ldots \quad (14)$$

then an associated random variable defined as the inter-arrival time T follows the exponential distribution $f(t) = \lambda e^{-\lambda t}$ and vice versa.

Proof: Let T be the interarrival time having a distribution function $F(t)$. If there is no customer in the system at time $t = 0$, then we have:

$$F(t) = Prob(T \leq t), probability\ that\ T\ takes\ on\ a\ value\ \leq t$$

$$= 1 - Prob(T > t) = Prob(no\ customerarrive\ during\ time\ t)$$

$$= 1 - P_0(t) = 1 - e^{-\lambda t}, t \geq 0$$

Differentiating both sides with respect to t, we get $f(t) = F^1(t) = \lambda e^{-\lambda t}$, which is an exponential distribution. Here, $f(t)$ is the probability density function for T. The expected (or mean) time of first arrival is given by:

$$E(t) = \int_0^{\infty} tf(t)dt = \int_0^{\infty} \lambda t e^{-\lambda t}\, dt = \frac{1}{\lambda} \quad (15)$$

Where
λ is the mean arrival rate. Thus, its variance would be $\frac{1}{\lambda^2}$

Corollary: The Markovian property of inter-arrival times states that the probability that a customer, currently in service, is completed at sometime t is independent of how long he has already been in service. That is:

$Prob\ (T \geq t_1\ such\ that\ T \geq t_0) = Prob\ (0 \leq T \leq t_1 - t_0)$

Where
T is the time between successive arrivals.

Proof: Since the left-hand side of Markovian property shows the conditional probability, it can be expressed as:

$$Prob\ (T \geq t_1/T \geq t_0)\ =\ \frac{Prob\ [T \geq t_1)\ \cap (T \geq t_0]}{(Prob\ (T \geq t_0)}$$

$$= \frac{\int_{t_0}^{t_1} \lambda e^{-\lambda t} dt}{\int_{t_0}^{\infty} \lambda e^{-\lambda t} dt} = \frac{-(e^{-\lambda t_1} - e^{-\lambda t_0})}{e^{-\lambda t_0}} = 1 - e^{-\lambda(t_1 - t_0)} \tag{16}$$

This is because of the reason that the inter-arrival times are exponentially distributed. Since,

$$Prob\ (0 \leq T \leq t_1 - t_0) =\ \int_0^{t_1 - t_0} \lambda e^{-\lambda t}\, dt = 1 - e^{-\lambda(t_1 - t_0)} \tag{17}$$

Therefore, it is concluded that $Prob\ (T \geq t_1/T \geq t_0)\ =\ Prob\ (0 \leq T \leq t_1 - t_0)$

Distribution of Departures (Pure Death Process)

Sometimes a situation may arise when no additional customer joins the system while the service is continued for those who are in the queue. Let, at time $t = 0$, (for example closing time), there be $N \geq 1$ customer in the system. It is clear that the service will be provided at the rate of θ. The number of customers in the system at time $t \geq 0$ is equal to N minus total departures up to time t. The distribution of departures can be obtained with the help of the following basic axioms.

1. Probability of departure during time Δt is $\theta \Delta t$
2. Probability of more than one departure between t and $t + \Delta t$ is negligible
3. The number of departures in non-overlapping intervals are statistically significant, that is, the process has independent arrivals

The following terms are defined since that would be helpful in the development of various queuing models.
$\theta \Delta t$ = probability that a customer in service at time t will complete service during time Δt
$1 - \theta \Delta t$ = probability that the customer in service at time t will not complete service during time Δt
For small time interval $\Delta t > 0$, $\theta \Delta t$ gives probability of one departure during time Δt. Using the same argument as the one used in pure birth process case, the term $O(\Delta t)^2 \to 0$ as $\Delta t \to 0$ and hence it can be neglected. Hence, the differential-difference equations for this can be obtained as given below:

$$P_N(t + \Delta t) =\ P_N(t)\{1 - \theta \Delta t\};\ n = N, t \geq 0$$

$$P_n(t + \Delta t) =\ P_n(t)\{1 - \theta \Delta t\} + P_{n+1}(t)\theta \Delta t;\ 1 \leq n \leq N - 1, t \geq 0$$

$$P_0(t + \Delta t) =\ P_0(t) + P_1(t)\theta \Delta t;\ n = 0, t = 0$$

Rearranging the terms and taking the limits as $\Delta t \to 0$, this result into:

$$P_N^1(t) =\ -\theta P_N(t);\ n = N$$

$$P_n^1(t) =\ -\theta P_n(t) +\ \theta P_{n+1}(t);\ 0 \leq n \leq N - 1$$

$$P_0^1(t) =\ \theta P_1(t);\ n = 0$$

The solution of these equations with the initial conditions:

$$P_n(0) = \begin{cases} 1; & n = N \neq 0 \\ o; & n \neq N \end{cases}$$

Can be obtained in the same manner as in the case of pure birth process. The solution to these equations is obtained as:

$$P_n(t) = \begin{cases} \frac{(\theta t)^{N-n} e^{-\theta t}}{(N-n)!}; & 1 \leq n \leq N, t \geq 0 \\ 1 - \sum_{b=1}^{\infty} P_n(t); & n = 0, t \geq 0 \\ 0; & n \geq N+1, t \geq 0 \end{cases}$$

Distribution of Service Times

The probability density function $s(t)$ of service time $T = t$ is given by:

$$s(t) = \frac{d}{dt}\{S(t)\} = \begin{cases} \theta e^{-\theta t}; & 0 \leq t \leq \infty \\ 0; & otherwise \end{cases}$$

The proof for this is exactly the same as earlier discussed in section 3.3 earlier. This shows that service time follows exponential distribution with mean $\frac{1}{\theta}$ and variance$\frac{1}{\theta^2}$.

Conclusion

This research paper has presented that the random variables for arrivals, inter-arrivals and departures follow a Poisson probability distribution in a queuing system.

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