International Journal of Physics and Mathematics
www.physicsjournal.net
Online ISSN: 2664-8644, Print ISSN: 2664-8636
Received: 01-01-2021; Accepted: 13-02-2021; Published: 19-04-2021
Volume 3, Issue 1, 2021, Page No. 06-12

Dark energy $f(T)$ gravity cosmological models in the form of wet dark fluid

SR Bhoyar
Department of Mathematics, Phulsing Naik Mahavidyalaya, Pusad, Yavatmal, Maharashtra, India

Abstract
The paper devotes to dynamical investigations of spatially homogeneous Bianchi type-I space-time filled with wet dark fluid, which is a candidate for dark energy in the framework of $f(T)$ gravity. The equation of state parameter modeled on $p=\omega(\rho-\rho^*)$ in the form of wet dark fluid for the dark energy component of the universe. Solutions to the corresponding field equations are obtained towards power and exponential volumetric law of expansion. The geometrical and physical parameters for both the models are studied in details.
PACS numbers: 98.80.Es; 14.65.q; 04.60.m; 04.50.Kd.

Keywords: bianchi type-i space-time; $f(T)$ gravity; wet dark fluid

Introduction

The present universe is dominated by dark energy which provides the dynamical mechanism of the accelerated expansion of the universe. Further, it was confirmed by the observations from CMBR, LSS, WMAP, etc. The strength of this acceleration is a remarkable question in recent year. Generally, there are two approaches to describe the current acceleration of the universe: one is to modify the energy-momentum tensor in Einstein's field equation. There are many models have been introduced to explain this current acceleration of the universe. The second approach is to modify the geometry of the space-time in Einstein's equations called modified gravity. Modified gravity has become one of the most popular candidates to understand the idea of dark energy. In modified theories, one modifies the laws of gravity so that the late time accelerated expansion of the universe is realized without recourse to an explicit dark energy matter component. One of the simplest modified gravity models is the so-called f(R) gravity. Capozziello *et al*. (2008) [10], Azadi *et al*. (2008) [1], katore and Shaikh (2014) [27], Miranda *et al*. (2009) [31], Sharif and Yousaf (2014) [38], Bhoyar *et al*. (2016) [9], Chirde and Shekh (2016; 2017) [14, 16] along with many authors have discussed some features of $f(R)$ gravity using different space-time. Another one is $f(R,T)$ gravity, some relevant work in $f(R,T)$ is presented by Sharif and Zubair (2012) [39], Katore *et al*. (2012) [25], Katore and Shaikh (2013) [26], Sahoo *et al*. (2014) [35], Chirde and Shekh (2015) [18], Bhoyar *et al*. (2015; 2016a) [8, 7]. Recently, authors show much interest in the modification of general relativity, so-called the $f(T)$ gravity which uses the Weitzenbock connection in place of the Levi-Civita connection having torsion without curvature responsible for the acceleration of the universe. Wu and Yu (2010) [44], Chen *et al*. (2011) [12], Bamba *et al*. (2010) [3], Dent *et al*. (2011) [12] are the authors who have to investigate inflation in $f(T)$ gravity along with Bamba *et al*. (2011) [2] studied the cosmological evolutions of the equation of state for dark energy in the exponential and logarithmic as well as their combination form of $f(T)$ theory and showed that the crossing of the phantom divide line can be realized in the combined form of $f(T)$ theory even though it cannot be in the exponential or logarithmic form. Sharif and Rani (2013) [40] considered different dark energy models in this scenario along with a time-dependent viscous fluid to construct the viscous equation of state parameter for dark energy models. Also, use graphical representation of this parameter to investigate the viscosity effects on the accelerating expansion of the universe and showed that the behavior of the universe depends upon the viscous coefficients which showing the transition of the Universe from decelerating to accelerating phase. It leads to the crossing of the phantom divide line and becomes phantom dominated for specific ranges of these coefficients. Zuber (2015) considered the same theory as an efficient tool to explain the current cosmic acceleration and associate its evolution with the known dark energy models by applying the numerical scheme to reconstruct theory from dark energy model with constant equation of state parameter and holographic dark energy model and set the model parameters as describing the different evolution eras and show the distinctive behavior of each case realized in theory. Chirde and Shekh (2014; 2015; 2015a) [13, 18, 17].
have discussed some cosmological model with different source in the same gravity while recently, Khurshudyan *et al*. (2017) [28] consider a particular example of $f(T)$ theory and study the effects of various interactions on a cosmological model using phase space analysis to have a qualitative understanding of the late-time behavior of the models and found that phenomenological models being in good agreement with the observational data along with phase transition from a decelerated expansion to the accelerated (recent) expansion of the universe.
Bhoyar *et al*. (2017) [6] discussed the stability of the accelerating universe with a linear equation of state in $f(T)$ gravity using hybrid expansion law.

Some Basics Related To $f(T)$ Gravity with Wet Dark Fluid

In this section, we give a brief description of $f(T)$ model and a detailed derivation of its field equations. Let us define the notations of the Latin subscript as these related to the tetrad field and the Greek one related to the space-time coordinates. For a general space-time metric, we can define the line element as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

This line element can be converted to the Minkowski's description of the transformation called tetrad, as Follows

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j, \quad dx^\mu = e_i^\mu \theta^i, \quad \theta^i = e_\mu^i dx^\mu, \tag{2}$$

Where η_{ij} is a metric on Minkowski space-time and $\eta_{ij} = diag[1, -1, -1, -1]$ $e_i^\mu e_\nu^j = \delta_\nu^\mu$ or $e_i^\mu e_\mu^j = \delta_i^j$. The root of the metric determinant is given by $\sqrt{-g} = \det[e_\mu^i] = e$. For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist. The Weizenbock connection components are defined as

$$\Gamma_{\mu\nu}^\alpha = e_i^\alpha \partial_\nu e_\mu^i = -e_\mu^i \partial_\nu e_i^\alpha, \tag{3}$$

which has a zero curvature but nonzero torsion. Through the connection, we can define the components of the torsion tensors of the form

$$T_{\mu\nu}^\alpha = e_i^\alpha (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i). \tag{4}$$

The difference between the Levi-Civita and Weitzenbock connections is a space-time tensor, and is known as the contorsion tensor defined as

$$K_\alpha^{\mu\nu} = (-\frac{1}{2})(T_\alpha^{\mu\nu} + T_\alpha^{\nu\mu} - T_\alpha^{\mu\nu}). \tag{5}$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor S μν α from the components of the torsion and contorsion tensors as

$$S_\alpha^{\mu\nu} = (\frac{1}{2})(K_\alpha^{\mu\nu} + \delta_\alpha^\mu T_\beta^{\beta\nu} - \delta_\alpha^\nu T_\beta^{\beta\mu}). \tag{6}$$

The torsion scalar T is obtained as

$$T = T_{\mu\nu}^\alpha S_\alpha^{\mu\nu}. \tag{7}$$

The action $f(T)$ gravity is defined as

$$S \int [T + f(T) + L_{matter}] e d^4 x. \tag{8}$$

Here, $f(T)$ denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (8) with respect to the tetrads, we get the following equations of motion

$$S_\alpha^{\nu\rho} \partial_\rho T \; f_{TT} + [e^{-1} e_\mu^i \partial_\rho (e e_i^\alpha S_\alpha^{\nu\rho}) + T_{\lambda\mu}^\alpha S_\alpha^{\nu\lambda}](1 + f_T) + \frac{1}{4} \delta_\mu^\nu (T + f) = 4\pi (T_\nu^\mu + \overline{T}_\nu^\mu), \tag{9}$$

Where T_μ^ν is the energy-momentum tensor and $f_T = df(T)/dT$. The field equation [9] is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equation. By setting $f(T) = a_0$ = constant this is dynamically equivalent to the general theory of relativity. The wet dark fluid is a new candidate for dark energy in the script of generalized chaplygin gas, where a physically motivated equation of state is offered with the properties relevant for a dark energy problem. The equation of state for a wet dark fluid is

$$\frac{p_{\omega df}}{\omega} + \rho^* = \rho_{\omega df}. \tag{10}$$

Equation (10) is a good approximation for many fluids, including water. The parameter ω and ρ^* are taken to be positive and we restrict ourselves to $0 \le \omega \le 1$. Note that if c_s denotes the adiabatic sound speed in wet dark fluid, then $c_s^2 = \partial p / \partial \rho \ge 0$. To find the wet dark fluid energy density, we use the energy conservation equation

$$\rho_{\omega df}^* + 3H(p_{\omega df} + \rho_{\omega df}) = 0. \tag{11}$$

Keep in mind the equation of state provided in equation (10) and using $3H = \frac{\dot{V}}{V}$ in equation (11), we get

$$p_{\omega df} = \left(\frac{\omega}{1+\omega}\right) \rho^* + \frac{c}{V^{(1+\omega)}}, \tag{12}$$

Where c is the constant of integration and V is the volume expansion, the wet dark fluid naturally includes these components, a piece that behaves like a cosmological constant as well as a standard fluid with an equation of state $p = \omega\rho$. We can show that if we take $c > 0$ this fluid will not violate the strong energy Condition $p + \rho \ge 0$. Thus, we get

$$p_{\omega df} + \rho_{\omega df} = (1+\omega)\rho_{\omega df} - \omega\rho^* = (1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right) \ge 0. \tag{13}$$

Chaubey (2009) [11] who is the author studied Bianchi type-V universe filled with dark energy in the form of wet dark fluid. Jain *et al*. (2012) [24] premeditated the axially symmetric cosmological model with wet dark fluid in the bimetric theory of gravitation. Samanta (2013) [37] studied the Bianchi type-V universe filled with dark energy from a wet dark fluid in $f(R,T)$ gravity. Sahoo and Mishra (2015) [36].

have investigated the five-dimensional Kaluza–Klein space-time with wet dark fluid, which is a candidate for dark energy, in the framework of $f(R,T)$ gravity. Chirde and Shekh (2016b) [15] inspect the stability of plane symmetric space-time with wet dark fluid which is a candidate for dark energy, in the framework of same gravity using volumetric expansion and get hold the model is consistent with the cosmological observations.

Field Equations and Some Kinematical Quantities

According to the WMAP data, the universe should achieve a slightly anisotropic special geometry in spite of the inflation, contrary to generic inflationary models and this might indicate a non-trivial isotropization history of the universe due to the presence of an anisotropic energy source. Thus, the Bianchi models that remain anisotropic have gained increasing interest in observational cosmology. Hence we consider spatially homogeneous and anisotropic Bianchi type-I space-time of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + dz^2\right], \tag{14}$$

where A and B be the metric potential which is the functions of cosmic time t only. Considering the same model Chirde and Shekh (2015a) [17] investigated cosmic string within the framework of $f(T)$ gravity using numerous scale factors. Bhoyar *et al.* (2017) [6] discussed stability along with the linear equation of state filled with a perfect fluid within the framework of $f(T)$ (linear and quadratic form) gravity with the help of kinematic ansatz and give an idea about the singular model having initially unstable while de-Sitter like expansion at late time. Dawande *et al.* (2017) [22] have studied the same space-time in $f(T)$ gravity using conservation equation and obtained some well-known $f(T)$ models which represent the different phases like matter, radiation and dark energy eras of the universe. Let us consider that the matter content is dark energy in the form of wet dark fluid, the energy momentum tensor T_μ^ν read as

$$T_\mu^\nu = \left(p_{\omega df} + \rho_{\omega df}\right)u^\nu u_\mu - p_{\omega df}\, g_\mu^\nu, \tag{15}$$

Together with commoving coordinates $u^\nu = (0,0,0,,1)$ and $u^\nu u_\nu = 1$, where u^ν is the four-velocity vector of the fluid, p and ρ be the pressure and energy density of the fluid respectively. The components of the equation of motion (9), using Bianchi type-I space-time (14) for the fluid of stress energy tensor (15) can be obtained as

$$(T+f) + 4(1+f_T)\left\{\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} + 4\frac{\dot{B}}{B}\dot{T}\, f_{TT} = k^2\left(-p_{\omega df}\right), \tag{16}$$

$$(T+f) + 2(1+f_T)\left\{\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{T}\, f_{TT} = k^2\left(-p_{\omega df}\right), \tag{17}$$

$$(T+f) + 4(1+f_T)\left\{\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = k^2\left(\rho_{\omega df}\right), \tag{18}$$

Where the overhead dot denotes the differentiation with respect to cosmic time (where time is in Gyr). Finally, here we have three differential equations with five unknowns namely A, B, f, $\rho_{\omega df}$, $p_{\omega df}$. So in order to close the system, we need one more relation that we shall obtain in the following section by formulating4 a special law of variation for Hubble's parameter. Some kinematical quantities of the space-time that describe the kinematics of the Universe are Spatial volume and average scale factor respectively as

$$V = a^3 = AB^2 \tag{19}$$

Another important dimensionless kinematical quantity is the mean deceleration parameter which tells whether the universe exhibits accelerating volumetric expansion or not is

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right), \tag{20}$$

For $-1 \le q < 0, q > 0$ and $q = 0$ the universe exhibit accelerating, decelerating volumetric expansion and expansion with constant rate respectively. The mean Hubble parameter, which expresses the volumetric expansion rate of the universe, given as

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{21}$$

where H_1, H_2, H_3 are the directional Hubble parameter in the direction of x, y and z -axis respectively. Using equations (19) and (21), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\dot{a}}{a}. \tag{22}$$

To discuss whether the universe either approach isotropy or not, an anisotropy parameter of the universe is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{23}$$

The expansion of scalar and shear scalar of the universe is respectively defined as follows

$$\theta = u^\mu_{;\mu} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \quad \sigma^2 = \frac{3}{2}H^2 A_m. \tag{24}$$

Subtracting equation (17) from equation (18), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = 0, \tag{25}$$

Rewriting above equation (25), we obtained

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0, \tag{26}$$

Which on integration gives

$$\frac{A}{B}=k_2\exp\left[k_1\int\frac{dt}{V}\right], \tag{27}$$

Where k_1 and k_2 are constants of integration. In view of equation (19), we write A and B in the explicit form as

$$A=D_1V^{\frac{1}{3}}\exp\left(\chi_1\int\frac{1}{V}dt\right), \tag{28}$$

$$B=D_2V^{\frac{1}{3}}\exp\left(\chi_2\int\frac{1}{V}dt\right), \tag{29}$$

where $D_i(i=1,2)$ and $\chi_i(i=1,2)$ are the constants satisfy the relation $D_1D_2^2=1$ and $\chi_1+2\chi_2=0$. Thus, the metric functions are represented explicitly in terms of the spatial volume. Once we obtain the value of spatial volume, we can find the metric potential. As already described in above section many authors have tried to find the solutions of the quadrature equations (28) - (29) by using different techniques. In the present paper, we solve these equations by using the value of average scale factor by the assuming law of variation of Hubble parameter in the following sec., which has a physical interest to describe the accelerating expansion of the universes.

Solution of Field Equations with Law of Variation of Hubble's Parameter

The most striking discovery of the modern cosmology (type Ia Supernova observations) indicate that the current universe is not only expanding but also accelerating this behavior of the universe is confirmed by various independent observational data, including LSS, CMB radiation and so on. There is a consensus on the conclusion that the universe has entered in a state of accelerating expansion at redshift $z\approx 0.5$. The law of variation for the Hubble's parameter which was initially proposed by Berman for FRW space-time which yields a constant value of deceleration parameter (CDP) (Berman 1983; Berman and Gomide 1988) [5, 4] and is also approximately valid for slowly time-varying deceleration parameter (Singh 2008) [41]. After the discovery of the late time acceleration of the universe, many authors have used CDP to obtain cosmological models in the context of dark energy in general theory of relativity and some other modified theories of gravitation within the framework of spatially isotropic and anisotropic space-times. In literature, Singh and Kumar (2006; 2007; 2007a; 2008) [42, 2943, 41] proposed a similar law of variation for the Hubble parameter in anisotropic space-time metrics that yields a constant value of the deceleration parameter, and generated solutions for Bianchi type-I, II and V space-times in general theory of relativity while Rahaman *et al.* (2005) [33] and Reddy *et al*. (2007) [34]. have well thought-out some cosmological models by considering the same CDP. In the investigations of Cunha and Lima (2008) [20] and Cunha (2009) [21], the transition redshift to the accelerating expansion of the current universe has been examine using the kinematic approach to cosmological data analysis that provides a direct evidence to the present accelerating stage of the universe which does not depend neither on the validity of general theory of relativity, nor on the matter-energy content of the universe. Li *et al.* (2011) [30] have examined the current acceleration of the universe together with Union2, BAO and CMB radiation, conclude that all favor an increase of the present cosmic acceleration with the plot of change of the deceleration parameter q with redshift $z<2$. Motivated from the studies outlined above, Akarsu and Dereliwe (2012) [32] proposed a linearly varying deceleration parameter of the form (which can be used in obtaining accelerating cosmological solutions)

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-kt+m-1, \tag{30}$$

Where $k\geq 0$ and $m\geq 0$ are constants.

As a special case if $k=0$, equation (30) covers the special law of variation for the Hubble parameter, which yields CDP models of the universe, presented by Berman (1983) [5], Berman and Gomide (1988) [4]. Also, it is consistent with the results of Cunha (2009) [21] and exhibits similar behavior to the one obtained by Li *et al*. (2011) [30]. The universe would exhibit decelerating expansion if $q>0$, an expansion with constant rate if $q=0$, accelerating power law expansion if $-1<q<0$, exponential expansion (also known as de-Sitter expansion) if $q=-1$ and super-exponential expansion if $q<-1$.

After solving equation (30) one obtained two different form of solutions for the scale factor related with spatial volume as

$$V=a^3=a_1(mt+m_1)^{\frac{1}{m}}\ \text{For } k=0 \text{ and } m>0, \tag{31}$$

$$V=a^3=a_2e^{3m_2t}\ \text{For } k=0 \text{ and } m>0, \tag{32}$$

Where a_1,a_2,m_1,m_2 are the constants of Integration. Both are these solutions are for constant q and hence correspond to the solutions under CDP ansatz.

Model for $k=0$ and $m>0$

Using (31) in (28) and (29), we get the following expressions for metric potentials in power law expansion

$$A=D_1a_1(mt+m_1)^{\frac{1}{m}}\exp\left(\frac{\chi_1}{a_1^3}\int\frac{1}{(mt+m_1)^{\frac{3}{m}}}dt\right) \tag{33}$$

$$B = D_2 a_1 (mt+m_1)^{\frac{1}{m}} \exp\left(\frac{\chi_2}{a_1^3}\int \frac{1}{(mt+m_1)^{\frac{3}{m}}}\,dt\right) \quad (34)$$

Equations (33) and (34), gives the value of metric potentials. It is seen that both the metric potentials are the product of power and exponential term. At an initial stage when $t \to 0$, both metric potentials are approaches to constant value and with the passage of time A and B increase indefinitely. At large time i.e. $t \to \infty$ then $A, B \to \infty$.

Using metric potentials, the model (14) can be written as

$$ds^2 = dt^2 - (D_1 a_1)^2 (mt+m_1)^{\frac{2}{m}} \exp\left(\frac{2\chi_1 (mt+m_1)^{\frac{m-3}{m}}}{ma_1(m-3)}\right) dx^2 - (D_2 a_1)^2 (mt+m_1)^{\frac{2}{m}} \exp\left(\frac{2\chi_2 (mt+m_1)^{\frac{m-3}{m}}}{ma_1(m-3)}\right)\left[dy^2 + dz^2\right]. \quad (35)$$

From the above model (35), it is observed that at $t=0$, the model is constant show null-like property but at a specific time $t = t_s = \frac{-m_1}{m}$, the metric potential in the model vanishes hence the model represent singular model. Also, there is no such relation between the constants in the model for which the model shows isotropy except the constants D_1, D_2, χ_1 and χ_2 but if $D_1 = D_2$ and $\chi_1 = \chi_2$, the metric potentials are identical. Hence it shows isotropy. The expressions for pressure and energy density of wet dark fluid for the model (35) as

$$\rho_{\omega df} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{a_1^3 (mt+m_1)^{\frac{3(1+\omega)}{m}}}, \quad (36)$$

and

$$p_{\omega df} = \frac{c\omega}{a_1^3 (mt+m_1)^{\frac{3(1+\omega)}{m}}} - \left(\frac{\omega}{1+\omega}\right)\rho^*. \quad (37)$$

The expressions for the deceleration parameter, the average Hubble's parameter, the average anisotropy parameter, scalar of expansion, magnitude of shear for the model (35) are obtained as,

Deceleration parameter, $q = -1 + m$.

The rate of expansion in the direction of x, y and z -axis are

$$H_x = \frac{\chi_1 (mt+m_1)^{\frac{m-6}{3}}}{3a_1} + \frac{1}{(mt+m_1)},$$

$$H_y = H_z = \frac{\chi_2 (mt+m_1)^{\frac{m-6}{3}}}{3a_1} + \frac{1}{(mt+m_1)}.$$

The average Hubble's parameter,

$$H = \frac{1}{(mt+m_1)}.$$

The average anisotropy parameter,

$$A_m = \frac{(\chi_1^2 + 2\chi_2^2)}{27a_1^2}(mt+m_1)^{\frac{2}{3}(m-3)}.$$

Scalar of expansion,

$$\theta = \frac{3}{(mt+m_1)}.$$

Magnitude of shear,

$$\sigma^2 = \frac{(\chi_1^2 + 2\chi_2^2)}{18a_1^2}(mt+m_1)^{\frac{2}{3}(m-6)}.$$

From the above results, it can be seen that the spatial volume is constant at $t=0$ when $m_1 > 0$ and is zero when $m_1 = 0$ and expands with cosmic time. We observe that all the three directional Hubble parameters are constant at $t=0$. In the derived model, the energy density and pressure also have same constant value. Hence initially the model is non-singular. The rate of the expansion, the mean anisotropy parameter for $m < 3$ and shear scalar all are also constant at $t=0$. As $t \to \infty$ the scale factors, A and B tend to infinity. The energy density and pressure becomes zero as $t \to \infty$. The expansion scalar, mean anisotropy parameter and shear scalar all tend to zero as $t \to \infty$ (for $m < 3$). This shows that the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero. At the initial stage of expansion, when pressure and energy density are constant, the Hubble parameter is also constant and with the expansion of the universe Hubble's parameter, expansion scalar decrease. Since $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} = 0$ and $\neq 0$ for $m < 3$ and $m \geq 3$ respectively, hence for $m < 3$ the model approach isotropy for large time and for $m \geq 3$ the models do not approach isotropy. The dynamics of the mean anisotropy parameter depends on the value of m. For $m < 3$, Am has singular state, with infinite energy density. Our model satisfies all conditions of homogeneity and isotropization according to the formal definitions given by Collins and Hawking (1973) [19] when $m < 3$. Also for $k = 0$ and $m > 0$ the model has a singularity at the point $t = \frac{-m_1}{m}$. The universe accelerates for $0 < m < 1$, decelerate for $m > 1$ and expands with constant velocity for $m = 1$. At the singular point for $m > 0$ the volume of the universe vanished.

Thus we can conclude that from the observation the universe starts its expansion from zero volume. The mean Hubble parameter, expansion scalar and shear scalar all are infinitely large at a singular point. Also, the mean anisotropy parameter is diverse for $0 < m < 1$ or for $m > 1$. Again we observed that the expansion scalar and shear scalar is decreased for $m > 0$ with respect to the cosmic time. The isotropy condition $(\sigma^2/\theta^2) \rightarrow$ constant satisfied at infinite expansion, this indicates that after a large time the expansion will stop completely and approaches to isotropy in the singular model.

Model for k = 0 and m = 0:

Using (32) in (28) and (29), we get the following expressions for metric potentials in an exponential expansion

$$A = D_1 a_2 e^{m_2 t} \exp\left(\chi_1 \int \frac{1}{a_2^3 e^{3m_2 t}} dt\right) \tag{38}$$

$$B = D_2 a_2 e^{m_2 t} \exp\left(\chi_2 \int \frac{1}{a_2^3 e^{3m_2 t}} dt\right) \tag{39}$$

Equations (38) and (39), gives the value of metric potentials. It is seen that both the metric potentials are the product only exponential term. At an initial stage when $t \rightarrow 0$, both metric potentials are approaches to constant value and with the passage of time A and B increase indefinitely. At large time i.e.

$t \rightarrow \infty$, $A, B \rightarrow \infty$.

Using metric potentials, the model (14) can be written as

$$ds^2 = dt^2 - (D_1 a_2)^2 \exp\left(2m_2 t - \frac{2}{3}\frac{\chi_1 e^{-3m_2 t}}{a_2^3 m_2}\right) dx^2 - (D_2 a_2)^2 \exp\left(2m_2 t - \frac{2}{3}\frac{\chi_2 e^{-3m_2 t}}{a_2^3 m_2}\right)\left[dy^2 + dz^2\right]. \tag{40}$$

From the above model (40), it is observed that at $t = 0$, the model is constant and non-singular show null-like property. Also, there is no such relation between the constants in the model for which the model shows isotropy except the constants D_1, D_2, χ_1 and χ_2 but if $D_1 = D_2$ and $\chi_1 = \chi_2$, the metric potentials are identical. Hence it shows isotropy.
The expressions for pressure and energy density of wet dark fluid for the model (40) as

$$\rho_{\omega df} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{a_2^3 e^{\left(\frac{3m_2 t(1+\omega)}{m}\right)}}, \tag{41}$$

And

$$p_{\omega df} = \frac{c\omega}{a_2^3 e^{\left(\frac{3m_2 t(1+\omega)}{m}\right)}} - \left(\frac{\omega}{1+\omega}\right)\rho^*. \tag{42}$$

The expressions for the deceleration parameter, the average Hubble's parameter, the average anisotropy parameter, scalar of expansion, magnitude of shear for the model (42) are obtained as Deceleration parameter,

$q = -1.$

The rate of expansion in the direction of x, y and z -axis are

$$H_x = m_2 + \frac{\chi_1}{a_2^3} e^{-3m_2 t},$$

$$H_y = H_z = m_2 + \frac{\chi_2}{a_2^3} e^{-3m_2 t}.$$

The average generalized Hubble's parameter,

$H_x = m_2$.

The average anisotropy parameter,

$$A_m = \frac{\left(\chi_1^2 + 2\chi_2^2\right)}{3m_2^2 a_2^6} e^{-6m_2 t}.$$

Scalar of expansion,

$\theta = 3m_2$.

Magnitude of shear,

$$\sigma^2 = \frac{2\left(\chi_1^2 + 2\chi_2^2\right)}{9a_2^6} e^{-6m_2 t}.$$

In this model, the directional Hubble parameters, H_x, H_y, H_z all are constant at $t = 0$. They deviate from the mean Hubble parameter because of m_2. If time tends to infinity, the directional Hubble parameter tends to a constant value and the universe asymptotically approaches to de-Sitter space.
At large time shear becomes insignificant. As t increases, the anisotropy of the expansion decreases exponentially to null. The value of the anisotropy parameter shows that as time tends to infinity the anisotropy parameter tends to zero that is the universe tends to isotropy. Thus, space approaches isotropy in this model. The expansion of scalar is constant throughout the evolution of the universe. The shear scalar is also constant at $t = 0$ and becomes zero as $t \rightarrow \infty$. The spatial volume is constant at $t = 0$. It expands exponentially as t increases and becomes infinitely large as $t \rightarrow \infty$. The deceleration parameter shows that the universe is purely accelerating. The value of expansion scalar is constant i.e. the rate of expansion of the universe is constant. The

ratio of shear scalar to expansion scalar is non-zero i.e. the universes is anisotropic and as time increases it tends to zero i.e. at the late time the universe tending to isotropy. Wehave obtained the deceleration parameter $q=-1$ and $dH/dt=0$ for this model. Hence, it gives the largest value of the Hubble parameter and the fastest rate of expansion of the universe.

Discussion and Concluding Remarks

In this paper, we have obtained some exact Bianchi type-I space-time in $f(T)$ theory of gravity with wet dark fluid. To find the solution of the field equation, we have put the law of variation for Hubble parameter that yields a constant value of deceleration parameter. This law of variation for Hubble parameter for the model gives two types of cosmologies, (i) first form (for $m \neq 0$) shows the solution for positive as well as negative value of deceleration parameter indicating the power law expansion of the universe whereas (ii) second one (for $m = 0$) shows the solution for negative value of deceleration parameter, which shows the exponential expansion of the universe. Under the law of variation for Hubble's parameter, it has been shown that the two classes of solutions lead to the conclusion that, if $q>0$ the model expands but always decelerate whereas $q<0$ provides the exponential expansion and later accelerates the universe. The evolution of the universe in such a scenario is shown to be consistent with the present observations predicting an accelerated expansion. The power law solution represent the singular model where the scale factors and spatial volume vanish at $t=0$. The energy density and pressure are infinite at initial epoch. As $t \to \infty$, the scale factors diverge and pressure, density both tend to zero. Anisotropy and shear are constant at the initial time but decrease with cosmic time and vanish as $t \to \infty$ hence the model shows the isotropic state in the later time of its evolution.

The exponential solutions represent the singularity-free model of the universe. All the parameters such as spatial volume, scale factor, pressure, density, expansion scalar shear, and anisotropy parameters are constants at $t=0$. The rate of expansion of the universe is uniform throughout the evolution.

References

1. Azadi A, Momeni D, Nouri-Zonoz M. Phys. Lett. B,2008:670:210.
2. Bamba K. *et al*. 2011, JCAP, 1101:021.
3. Bamba K, Geng CQ, Lee CC. 2010 arXiv:1008.4036 [astro-ph.CO].
4. Berman MS, Gomide FM. Gen. Relativ. Gravit,1988:20:191
5. Berman MS. Nuovo Cimento B,1983:74:182.
6. Bhoyar SR, Chirde VR, Shekh SH. Astrophysics,2017:60:259.
7. Bhoyar SR, Chirde VR, Shekh SH. Int. J. of App. Resea,1016a:6(11):398.
8. Bhoyar SR, Chirde VR, Shekh SH. Inter. J. of Adv. Resea,2015:3(9):492.
9. Bhoyar SR, Chirde VR, Shekh SH. Prespacetime J,2016:7:456.
10. Capozziello S, Martin-Moruno P, Rubano C. Phys. Lett. B,2008:664:12.
11. Chaubey R. Astrophys Space Sci,2009:321:241.
12. Chen SH, Dent JB, Dutta S, Saridakis EN. Phys. Rev. D,2011:83:023508.
13. Chirde VR, Shekh SH. Bulgerian journal of physics,2014:41:258.
14. Chirde VR, Shekh SH. Bulgerian journal of physics,2016:43:57.
15. Chirde VR, Shekh SH. J Astrophys Astron,2016b:37:15.
16. Chirde VR, Shekh SH. J. of Theoretical Phy. and Cryptography,2017:13.
17. Chirde VR, Shekh SH. The African Review of Physics,2015a:10:0057:4759.
18. Chirde VR, Shekh SH. The African Review of Physics,2015:10:0020:145.
19. Collins CB, Hawking SW. Astrophys. J,1973:180:317.
20. Cunha JV, Lima JAS. Monthly Notices of the Royal Astro. Society,2008:390:210.
21. Cunha JV. Physical Review D,2009:79:047301.
22. Dawande MV, Adhav KS, Nerkar SS,2017. DOI:10.22606/adap.2017.22006.
23. Dent JB, Dutta S, Saridakis EN,2011. JCAP 1101:009.
24. Jain P, Sahoo P, Mishra B. Int. J. Theor. Phys,2012:51:2546.
25. Katore SD, Chopde BB, Shekh SH. Int. J. Basic and Appl. Res,2012:283.
26. Katore SD, Shaikh AY. Prespacetime Journal,2013:3:11.
27. Katore SD, Shaikh AY. Afr. Rev. Phys,2014:9:0055.
28. Khurshudyan M, Myrzakulov R, Khurshudyan A. Mod. Phys. Lett. A,2017:32:1750097.
29. Kumar S, Singh CP. Astrophys. Space Sci,2007:312:57.
30. Li Z, Wu P, Yu H. Physics Letters B,2011:695:1.
31. Miranda V, Joras S, Waga I, Quartin M. Phys. Rev. Lett,2009:102:221101.
32. Akarsu O, Dereli T. Int. J. of Theor. Phy,2012:51:612.
33. Rahaman F, Begum N, Bag G, Bhui B. Astrophys. Space Sci,2005:299:211.
34. Reddy DRK, Naidoo RL, Adhav KS. Astrophys. Space Sci,2007:307:365.
35. Sahoo P, Mishra B, Chakradhar G, Reddy D. Eur. Phys. J. Plus,2014:129:49.
36. Sahoo P, Mishra B. Can. J. Phys,2015:93:1.
37. Samanta GC. Int J The. Phy,2013:52:2303.
38. Sharif M, Yousaf Z. J. Cosmo. Astropart. Phys,2014:06.
39. Sharif M, Zubair M,2012. JCAP 03028.
40. Sharif M, Shamaila rani. Mod. Phys. Lett. A,2013:28:1350118.
41. Singh CP. *et al*. Astrophys. Space Sci,2008:315:181.
42. Singh CP, Kumar S. Int. J. Mod. Phys. D,2006:15:419.
43. Singh CP. Pramana J. Phys,2007a:68:707.
44. Wu P, Yu HW. Phys. Lett. B,2010:693:415.
45. Zubair M,2015. DOI: 10.1155/2015/292767.