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Structure of semirings and some satisfying the identity

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Abstract
Semirings. foundational in abstract algebra, comprise sets with addition and multiplication, resembling rings without full additive inverses. They wield significance across mathematics and computer science. Certain semirings, like Boolean and tropical semirings, satisfying specific identities, prove vital. Boolean semirings underpin logic and set operations, aiding computation. Tropical semirings with min and addition solve optimization problems. Thus semirings with diverse applications remain integral in various fields by leveraging their inherent structures and identity properties.

Keywords: Art practice, post-pandemic, innovation, ethos, poignancy

Introduction
A semiring is an algebraic structure that generalizes both rings and rigs. It consists of a set along with two binary operations, usually denoted as addition (+) and multiplication () that satisfy certain properties. Semirings are used in various mathematical and applied contexts to model situations where a combination of addition and multiplication but not necessarily subtraction or division is relevant.
Formally a semiring is defined by the following properties:

1. **Additive Semigroup:** The set with the addition operation forms a semigroup. This means that addition is associative and for any elements a. b. and c in the set the operation satisfies the property: (a + b) + c = a + (b + c).

2. **Multiplicative Semigroup with Identity:** The set with the multiplication operation forms a semigroup with an identity element (Usually denoted as 1). This means that multiplication is associative and for any element a in the set. 1 a = a 1 = a.

3. **Distributive Laws:** The multiplication operation distributes over the addition operation. For any elements a. b. and c in the set. the following distributive properties hold:
   a (b + c) = (a b) + (a c)
   (a + b) c = (a c) + (b c)

4. **Zero Element:** There exists an element (usually denoted as 0) such that for any element a in the set. 0 + a = a + 0 = a.

In a semiring the additive identity (0) plays a role similar to the "zero" element in traditional addition and the multiplicative identity (1) acts as the "one" element in traditional multiplication. Semirings are used to model various situations in mathematics and applied fields. They are employed in areas such as formal language theory. Optimization, graph theory, computer science and economics, where situations involve both additive and multiplicative operations but do not require the full structure of a ring.
A special case of semiring is a commutative semiring, where the multiplication operation is also commutative. In a commutative semiring the order of multiplication of elements doesn't matter.
A semiring is an algebraic structure that combines elements addition and multiplication in a specific way. It provides a useful framework for modeling situations where a combination of these operations is relevant without requiring all the properties of a full ring.

1. The identity $a + ab = b$
The algebraic structure you're referring to where the identity $a + ab = b$ holds is known as a left near-semiring, Let's define it state a relevant theorem and outline a proof for that identity.

Definition: Left Near-Semiring A left near-semiring is a set *S* equipped with two binary operations addition ++ and multiplication ∗∗ that satisfy the following properties:

Associativity of Addition: For all $a, b, c \in s, (a+b)+c = a+(b+c)$.

Associativity of Multiplication: For all $a, b, c \in s, (a*b)*c = a*(b*c)$.
Additive Identity: There exists an element $0 \in s$ $such\ that$ $0 + a = a$ $for\ all$ $a \in s$.

Multiplicative Identity: There exists an element $1 \in s$ such that $1*a=a$ for all $a \in s$.

Left Nearness Property: For all $a, b \in s, a + a * b = b$.

Theorem: Left Near-Semiring Identity In a left near-semiring *S*. the identity $a + a*b=b$ holds for all $a, b \in S$.

Proof: Let *a.b* be arbitrary elements in the left near-semiring *S*. We want to show that $a + a* b=b$.
Using the properties of a left near-semiring:

Now let's proceed to prove the identity $a + a* b=b$:

Starting from the left-hand side $a + a* b$. we can use the left nearness property: $a + a* b=a + (a + a *b) = (a+a) + a*b$ Since $a + a = 1*a=a$. we can simplify:

$(a + a) + a *b = a + a *b = b$

Thus we have shown that $a + a * b = b$ as required.

This completes the proof of the left near-semiring identity $a + a* b=b$.

A left near-semiring is an algebraic structure that satisfies the identity$a + a * b = b$. This identity is proven using the properties of a left near-semiring. Particularly the left nearness property.

2. THE Identity $ab + a = a$

The identity $ab +a=a$ holds is known as a left zero semiring. Let's define it state a relevant theorem and outline a proof for that identity.

Definition: Left Zero Semiring A left zero semiring is a set *S* equipped with two binary operations addition ++ and multiplication ∗∗ that satisfy the following properties:

Theorem: Let (S, +,.) be a semiring. Suppose S satisfies the condition $ab + a = a$ for all a. b in. If S contains a multiplicative identity which is also an additive identity then $(S, +, .)$ is a idempotent semiring.

Let (S, +,.) be a semiring.

Assume that S satisfies the condition ab + a = a for all a. b in S

Let ‘ e’ be the multiplicative identity which is also an additive identity.

Let $ab + a = a \Rightarrow a(b + e) = a\ ab = a$ $(for\ all\ a, b\ in\ S)$

$\Rightarrow a + a = a$

‘ a ‘ is idempotent
=> (S. +) is a band

Now ab = a for all a. b in S
=> (S,.) is left singular
=> a2=a for all‘a’ in S
=> (S,.) is a band
Hence (S, +,.) is an idempotent semiring

Congruence on semirings

Definition-A semiring is said to be additively weakly separative if for every

$a, b \; in \; S, a + a = a + b = b + b => a = b$

Theorem- Let (S,+,.) be a commutative semiring. Define a relation ‘o ‘on a semiring S by aob if and only if there exists positive integers m. n such that a + mb = (m +1) b and b + na = (n+ 1) a for any a. b in S. then ‘0 * is a weakly separative congruence on S.

Proof: First we prove that ‘0’ is an equalance relation on S.
We know that $a + ma = (l + m) a$

$= (m + 1) \; a$

Similarly a + na = (n +1) a

=>a 0 a

Hence ‘o ‘is reflexive.
Let a o b for some a. b in S. a + mb = (m +1) b and b + na = (n+ 1) a replace a by b and b by a we have b + ma = (m+l)a. a + nb = (n+l)b => boa.

There fore ‘ o’ is symmetric Suppose a 0 b and boc.

Then. $\left.\begin{matrix} a + mb = (m + l) b \\ b + na = (n + 1) \; a \end{matrix}\right\}$ (1)

And

$\left.\begin{matrix} \mathrm{b} + \mathrm{pc} = (\mathrm{p} + \mathrm{l})\mathrm{c} \\ \mathrm{c} + \mathrm{qb} = (\mathrm{q} + 1)\mathrm{b} \end{matrix}\right\}.$ (2)

(For some m. n. p. q in Z+)

From (2) ib + pc = (p + i) c

From (1) ib + na = (n + i)a for some positive integer i.
Let a +mb = (m +1) b
=> a + mb + pc = (m + 1) b + pc
=> a +(p + m) c = (p + m+l)c
=> a + rc = (r + 1) c where p + m = r

Similarly c + qb = (q+l)b
=> c + qb +pa = (q+l)b + pa
=>c + (p + q)a = (p + q+l)a
=> e + ra = (r + 1) a =>a oc.

Hence ‘o‘ is transitive.
Therefore σ is an equivalence relation on S.
Now we prove that o is congruence on S
Let $a \; \sigma \; b$ => a + mb = (m+l)b.
b + na = (n+ 1) a.
To prove a + c σ b +c.
c + a σ c + b.

Consider
(a + c) + m(b + c)= a + c +mb + me
= c + a + mb +mc
= c + (m+l)b + mc
= (m + 1) (b +c).

Therefore a + c σ b +c
Similarly we see that c + a σ c + b.
Again a σ b => a+ mb = (m+l)b.

b + na = (n+ 1) a.
Let a + mb = (m +1) b
=> ac + m(bc) = (m + 1) bc and
b + na = (n+ 1) a
=> bc + m (ac) = (n +l) ac.

Therefore, ac a bc.
Similarly, we can prove that ca σ cb

Hence 'a' is a congruence relation on S.
To prove that 'o' is weakly separative congruence on S
We have to prove that if

2a σ a + b σ 2b => a σ b for all a. b in S.

Let a + b σ 2b => (a +b) + m (2b) = (m +1) 2b

=> a + (2m +1) b = ((2m +1) +1) b.

Since σ is congruence on S. we have
a + b σ 2a
a + 2b σ 2a + b
σ a + (a +b)
σ (a + b) + a
σ (b+b)+a
σ b + (b +a)
σ b +(a +b)
σ b+(b+b)

=> a+2b σ 3b

=> (a + 2b) + k(3b) = (k+1) (3b)

=> a + (3k + 2)b =((3k + 2)+l)b.for some k

=> a + mb = (m + 1) b. for some positive integer m.

Similarly we see that b + na = (n+l)a

Therefore a σ b

Hence 'σ ' is a separative congruence relation on S.

Semirings are algebraic structures used in math. CS. and engineering. A semiring is a set with two binary operations commonly addition and multiplication that satisfy certain criteria. The identification element underpins semirings. Like the neutral element in ordinary addition it leaves other elements untouched when added. It works as a multiplicative identity for multiplication like 1 in normal multiplication.

Semiring congruence relations group equivalent elements under specific procedures. Congruence in semirings defines an equivalence connection between elements based on their behavior under semiring operations similar to modular arithmetic. Congruence relations aid formal verification by analyzing system behavior and ensuring correctness.

Semirings identity elements and congruence relations create a flexible framework for modeling and addressing issues in several disciplines. Semiring applications include optimization formal language theory, graph theory and cryptography. The identity and congruence ideas help explain element behaviour in semirings enabling the development of strong mathematical tools and algorithms that progress many domains.

Semirings are used in many different areas of math computer science and other fields. In formal language theory semirings are the basis for algorithms that parse and analyze languages. This helps make compilers and tools for studying syntax. In optimization and graph theory certain semirings provide a way to solve problems like finding the shortest path and figuring out the greatest flow.

Notably, some semirings meet certain traits that make them more useful in different areas. For example the Boolean semiring works with the Boolean algebra identities, which makes it very useful in computer science for logic and set operations. Tropical semirings, which are defined over the real numbers and have the methods min and add, help solve optimization problems especially those that involve dynamic programming.

Conclusion

Both theoretical and applied branches of mathematics are enriched by the study of semirings and the unique fulfilling identities of their elements. Because of their adaptability and wide range of applications they have become vital tools for tackling issues that arise in areas as disparate as formal languages. Optimization computer science and even further afield.

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