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Thermal radiation and chemical reaction effects on a porous media flow with heat generation

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Abstract
The present study investigates the effects of thermal radiation and chemical reaction on the unsteady free-convection flow over a vertical infinite porous plate under the action of a transverse magnetic field, suction velocity and heat sources. The incompressible Newtonian fluid model is considered and the convective, diffusive and reactive processes are incorporated to describe the time evolution of fluid temperature and mass concentration within the flow. These led to a coupled system of partial differential equations alongside the relevant conditions. A decoupled finite difference scheme is formulated for the dimensionless model, and the resulting linear system are solved, using the Gauss Seidel algorithm, for the field variables. The stability and convergence of the method are proved, and the results are numerically verified using the method of manufactured solutions. The numerical results are presented and the effects of the various flow parameters on the field variables are discussed. It is observed that an increase in the radiation parameter decreases the temperature, while an increase in the chemical reaction parameter leads to an increase in the concentration

Keywords: porous media, finite difference scheme, thermal radiation, chemical reaction, implicit scheme

1. Introduction
Analysis of free convection processes in porous media has become popular area of study in recent years because of its applications in science and engineering [1]. At high operating temperatures, thermal radiation becomes significant, since many processes in engineering occur at high temperature it becomes pertinent to investigate the resulting effects on the system. Some foreign bodies may give rise to chemical reactions. In many transport processes, heat and mass transfer is a consequence of buoyancy effects caused by diffusing of heat and chemical species. Therefore, the study of combined flow of mass, heat and fluid becomes important for engineering and technological applications

Rajput and Sahu [2] studied the effect of chemical reaction on free convective MHD flow past an exponentially accelerated infinite vertical plate through a porous medium with variable temperature and mass diffusion. Abdulaziz and Hashim [3] investigated free convection flow convection flow of a micropolar fluid flow between porous vertical plate using Homotopy analysis. Putil and Kulkarni [4] considered free convection flow of a polar fluid through porous medium in the presence of chemical reaction and internal heat generation. Jha and co-authors [5] considered natural convection flow heat generation and absorption near a vertical plate using the Laplace transform technique. IN another study, Elbasheshy et. Al [6] investigated the heat transfer over an unsteady porous stretching surface embedded in a porous medium in presence of heat source with variable heat flux. They observed that the rate of heat transfer decreases with increase in porous medium parameter but increases with increase in suction. Aruna et. al. [7] investigated the free convection flow of Jeffery fluid past an infinite vertical porous plate with constant heat flux. Adeniyan and collaborators [8] studied the effects of thermal dissipation on MHD mixed convection flow over a permeable vertical plate embedded in an anisotropic porous medium. Chand et. al. [9] investigated hydromagnetic oscillatory flow through a porous medium bounded by vertical plate with heat source. They observed that Soret has significance influence in the flow.

Mishra et. al. [10] studied the effect of thermal radiation on free convection flow due to heat and mass transfer through a porous medium bounded by two vertical walls. Heat distribution in the soil is investigated in [11] without consideration to an associated fluid flow. Nwaigwe [12] also investigated heat and mass transfer in a vertical channel but without radiation, porosity and suction. Ahmad et al. [13] investigated free convection flow in a porous medium with heat generation in an infinite vertical plate using perturbation technique. They did not put thermal radiation and chemical reaction into consideration and to our knowledge, thermal radiation and chemical reaction has not be investigated in this situation. We extend this work [13] by incorporating the effects of thermal radiation and chemical reaction on the problem of free convection in a porous medium with heat generation.

The paper is organized as follows. The mathematical formulation is presented in section 2. The numerical method is formulated, analysed and verified in section3. The simulation results are presented and discussed in section 4, and the paper is concluded in section 5.

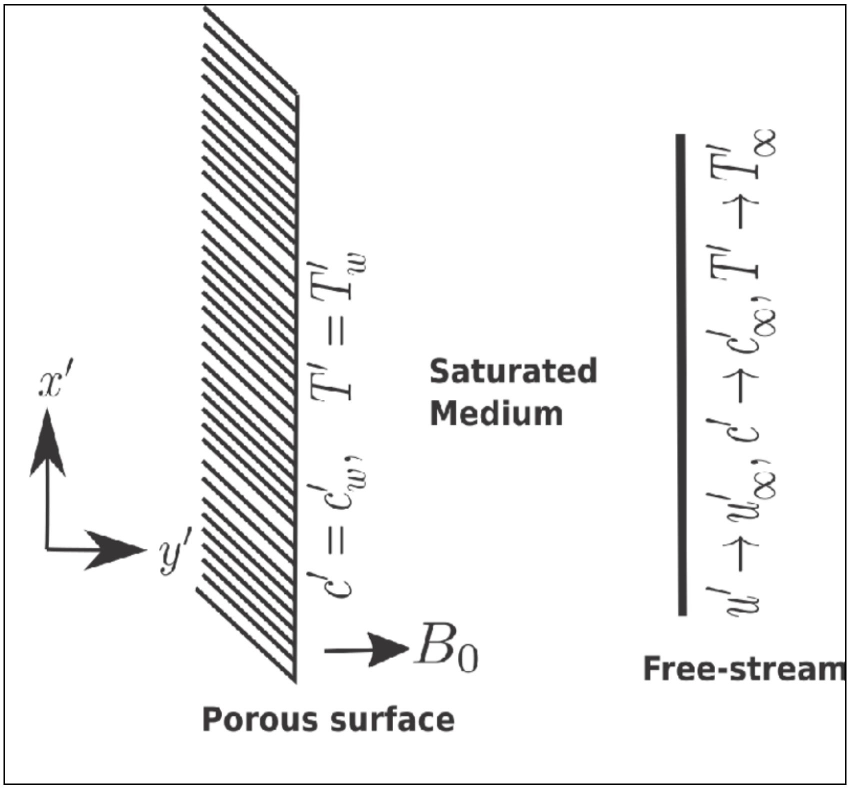

Fig 1: Physical setup of flow in saturated vertical porous channel

2. Problem Formulation

Consider the unsteady free convection flow of a viscous incompressible and electrically-conducting fluid over an infinite vertical permeable plate in a saturated porous medium. A magnetic field of strength B_0 is applied perpendicular to the surface and a constant pressure gradient is maintained along the plate. The x' -axis is taken along the planar surface in the upward direction and the y' -axis taken to be normal to it as shown in Figure 1. Due to the infinite plane surface assumption, we neglect variations along the plate length and take the flow fields to vary spatially with y' only. Hence, to study the flow and transport in the system, we propose the following model [14, 15, 16, 17, 12] for the velocity u', concentration c' and temperature T' :

$$\frac{\partial v'}{\partial y'} = 0, \qquad (1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{\partial P'}{\partial x'} + \upsilon\frac{\partial^2 u'}{(\partial y')^2} - \left(\frac{\upsilon}{k_1} + \frac{\sigma B_0^2}{\rho}\right)u' + g\beta_1(T' - T'_\infty) + g\beta_1(c' - c'_\infty), \qquad (2)$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\alpha}{\rho C_p}\frac{\partial^2 T'}{(\partial y')^2} - \frac{\alpha}{\rho C_p}\frac{\partial q_r}{\partial y'} + Q_0\frac{T' - T'_\infty}{\rho C_p}, \qquad (3)$$

$$\frac{\partial c'}{\partial t'} + v'\frac{\partial c'}{\partial y'} = D\frac{\partial^2 c'}{(\partial y')^2} + \gamma_1(c' - c'_\infty), \qquad (4)$$

We consider constant boundary values on the plate, namely $u'_w = 0, T'_w$ and c'_w for the velocity, temperature and concentration respectively, and their free-stream values $u'_\infty = U, T'_\infty$ and c'_∞ respectively. Hence, we the following boundary conditions:

$$u'(0,t') = 0,\ T'(0,t') = T'_w,\ c'(0,t') = c'_w \quad \forall\ t \ge 0,$$
$$u'(y',t') \to u'_\infty,\ T'(y',t') \to T'_\infty,\ c'(y',t') \to c'_\infty \text{ as } y' \to \infty,\ \forall\ t \ge 0. \tag{5}$$

The problem is completed with the following initial flow fields:

$$u'(y',0) = -\frac{20}{3d^2}(y')^2 + \frac{23}{3d}y',$$
$$T'(y',0) = 1.0 + \sin\left(\frac{y'/d - 1}{2}\pi\right),$$
$$c'(y',0) = \frac{y}{d}e^{3(1-y'/d)}, \tag{6}$$

Where $d = \frac{\upsilon}{u'_w}$ is assumed to be distance of the free-stream (or boundary layer thickness) away from the stationary plate.

2.0.1 Non-dimensional Formulation

Equation (1) given $v' = -v'_0 < 0$ which is some constant suction velocity. Using the following non-dimensional parameters [12, 13]:

$$u = \frac{u'}{L}, y = \frac{Ly'}{\upsilon}, t = \frac{t'L^2}{\upsilon}, T = \frac{T' - T'_w}{T'_\infty - T'_w}, c = \frac{c' - c'_w}{c'_\infty - c'_w}, \tag{7}$$

the model (2)-(6) leads to the following non-dimensional equations:

$$\frac{\partial u}{\partial t} - \lambda\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - (M + K)u + G_r T + G_c c + N, \tag{8}$$
$$\frac{\partial T}{\partial y} - \lambda\frac{\partial c}{\partial y} = \frac{(1 + R)}{P_r}\frac{\partial^2 u}{\partial y^2} + HT, \tag{9}$$
$$\frac{\partial c}{\partial t} - \lambda\frac{\partial c}{\partial y} = \frac{1}{S_c}\frac{\partial^2 u}{\partial y^2} + \gamma c, \tag{10}$$
$$\text{for all } x \in (0,1),\ t > 0,$$

subject to the conditions:

$$u(y,0) = -\frac{20}{3d^2}y^2 + \frac{23}{3d}y,$$
$$T(y,0) = 1.0 + \sin\left(\frac{y-1}{2}\pi\right),$$
$$c(y,0) = ye^{3(1-y)},$$
$$\forall\ x \in [0,1], \tag{11}$$

and

$$u(0,t) = T(0,t) = c(0,t) = 0 \ \forall t \ge 0,$$
$$u(1,t) = T(1,t) = c(y,t) = 1 \ \forall t \ge 0, \tag{12}$$

$\lambda = \frac{v_0}{L}$ is the suction parameter, $M = \frac{\sigma B_0^2 \upsilon}{\rho L^2}$ is the magnetic field parameter, $K = \frac{\upsilon L}{k_1}$ is the permeability parameter, $G_c = \frac{\upsilon g \beta_c (c' - c'_\infty)}{L^3}$ is the solutal Grashof number, $G_r = \frac{\upsilon g \beta_T (T' - T'_\infty)}{L^3}$ is the thermal Grashof number, $P_r = \frac{\upsilon \rho C_p}{K}$ is the Prandtl number, $R = \frac{16(T'_\infty)\sigma *}{3\upsilon \rho C_p K}$ is the thermal radiation parameter, $H = \frac{\upsilon Q}{\rho C_p L^2}$ is the heat source parameter, $S_c = \frac{\upsilon}{D}$ is the Schmidtl number and $\gamma = \frac{\upsilon R}{L^2}$ is the chemical reaction parameter.

where

The equations (8)-(12) constitute the complete mathematical formulation of the problem. It can be easily seen that the problem is an initial-boundary value problem and the required conditions have all been provided, making the problem a closed one; this is unlike many problems that have been posed by several authors which usually do not include the initial conditions. We present the numerical solution of the problem in the next section.

3. Numerical Scheme

3.1 Numerical Formulation

Let $1 \le Q \in \mathbb{Z}^+,\ \Delta y = \frac{1}{Q},\ S_I = \{0, 1, 2, \ldots, Q\},\ S_N = \{0, 1, 2, \ldots\},\ y_i = i\Delta y\ \forall\ i \in S_I$ and Δt be given and $t^n = n\Delta t\ \forall\ n \in S_N$.

Also define the operators:

$\Delta^+ u_i = \frac{u_{i+1} - u_i}{\Delta y},\ \Delta^- u_i = \frac{u_i - u_{i-1}}{\Delta y}$ and $\Delta^\pm u_i = \Delta^+ \Delta^- u_i$.

Adopting the approach in [19, 20, 21, 12], we seek the following $u_i^n \approx u(y_i, t^n),\ T_i^n \approx T(y_i, t^n)$ and $C_i^n \approx C(y_i, t^n)\ \forall\ i, n$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \lambda \Delta^+ u_i^{n+1} + \Delta^\pm u_i^{n+1} - (M + K) u_i^{n+1} + G_r T_i^n + G_c C_i^n + N \qquad (13)$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \lambda \Delta^+ T_i^{n+1} + \left(\frac{1 + R}{P_r} \right) \Delta^\pm T_i^{n+1} + H T_i^{n+1}, \qquad (14)$$

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = \lambda \Delta^+ C_i^{n+1} + \frac{1}{S_c} \Delta^\pm C_i^{n+1} + \gamma C_i^{n+1}, \qquad (15)$$

$$u_0^{n+1} = T_0^{n+1} = C_0^{n+1} = 0\ \forall\ n \in S_N,$$

$$u_Q^{n+1} = T_Q^{n+1} = C_Q^{n+1} = 1\ \forall\ n \in S_N, \qquad (16)$$

$$u_i^0 = u_0(y_i),\ T_i^0 = T_0(y_i),\ C_i^0 = C_0(y_i),\ \forall\ i \in S_I.$$

Observe that the coupling terms have been treated explicitly leading to decoupled systems that can be solved independently, see [12, 22, 23].

3.2 Analysis

The analyses of the scheme formulated above shall focus on the velocity scheme. This is sufficient since the same scheme is applied for both the temperature and concentration equations. Also, without the loss of generality, we ignore the coupling terms in these analyses.

This is valid since these terms are treated in explicit forms. In the following analyses, we let $\alpha = \Delta t / (\Delta y)^2, \beta = (\Delta t)^2 / (\Delta y)^2$.

3.2.1 Stability Analysis
Theorem 1 (Stability Analysis)
The velocity scheme is unconditionally stable.
Proof
Setting $G_T = G_c = N = 0$, define $F = M + K$, ignoring boundary conditions and set $u_i^n = \beta^n e^{jkh}$, $j = \sqrt{-1}, k \in \mathbb{R}$.
Hence, the velocity scheme becomes:

$$
\begin{aligned}
(1+\alpha+\Delta t F)\beta &= 1+\alpha\beta e^{jkh} + \mu\left(e^{jkh} + e^{-jkh} - 2\right)\beta \\
&= 1+\alpha\beta e^{jkh} - 4\mu \sin^2\left(\frac{kh}{2}\right) \\
&\leq 1+\alpha\beta e^{jkh}.
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
(1+\alpha+\Delta t F)|\beta| &\leq \left|1+\alpha\beta e^{jkh}\right| \\
&\leq 1+\alpha|\beta| \qquad (\text{using } \left|e^{jkh}\right| = 1)
\end{aligned}
$$

Hence:

$$(1+\Delta t F)|\beta| \leq 1$$

The fact that $\Delta t F \geq 0$ leads to the result:

$$|\beta| \leq 1$$

This proves that the scheme is unconditionally stable.

3.2.2 Convergence Analysis
Theorem 2 (Convergence Analyses)
The velocity scheme is unconditionally convergent with the order of the truncation error.
Proof:
Let $T_{\max}^n = \max_i \{T_i^n\}$, where T_i^n is the truncation error, then the error equation for the velocity scheme reads:

$$
\begin{aligned}
(1+\alpha+2\mu+\Delta t F)e_i^{n+1} &= e_i^n + (\alpha+\mu)e_{i+1}^{n+1} + \mu e_{i-1}^{n+1} \\
&\leq \left\|e^n\right\| + (\alpha+2\mu)\left\|e^{n+1}\right\| + \Delta t T_{\max}^n.
\end{aligned}
$$

$\Rightarrow$

$$(1+\Delta t F)e_i^{n+1} \leq \left\|e^n\right\| + \Delta t T_{\max}^n,$$

$\Rightarrow$

$$\left\|e^{n+1}\right\| \leq \left\|e^n\right\| + \Delta t T_{\max}^n.$$

By discrete Grownwall's inequality, the following is obtained:

$$
\begin{aligned}
\left\|e^n\right\| &\le \left\|e^0\right\| + n\Delta t T^n_{\max} \\
&= n\Delta t T^n_{\max} \\
&= T_f T^n_{\max}.
\end{aligned}
$$

These theoretical results are numerically verified in the following section.

3.3 Numerical Verification of The Theoretical Results

This section presents numerical results to verify the convergence of the proposed numerical formulation. For this experiment, the following problem, whose exact solution exists, is considered..

$$
\begin{aligned}
&\frac{\partial u}{\partial t} - \lambda\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - (M+K)u + G_r T + G_c c + N \\
&\qquad - 0.1y^2 t - 0.1yte^{1-y} + 1.1y - t + y - 1, \\
&\frac{\partial T}{\partial t} - \lambda\frac{\partial T}{\partial y} = \frac{(1+R)}{P_r}\frac{\partial^2 u}{\partial y^2} + HT + \left(0.9yt + y - t\right)e^{1-y} \\
&\qquad - 1.466666666666667(y-2)e^{1-y}, \\
&\frac{\partial c}{\partial t} - \lambda\frac{\partial c}{\partial y} = \frac{1}{S_c}\frac{\partial^2 u}{\partial y^2} + \gamma c + (1-0.1t)y^2 - 2ty \\
&\qquad + 8.333333333333333t, \\
&u(y,0) = T(y,0) = c(y,0) = 0, \\
&u(0,t) = T(0,y) = c(0,y) = 0, \\
&u(1,t) = T(1,y) = c(1,y) = 0
\end{aligned}
$$

If the following values are set for the parameters:

$$
\begin{aligned}
&\lambda = 1,\ M = 0.1, K = 1,\ G_r = 0.1,\ G_c = 0.1, \\
&\mathrm{P_r} = 0.75,\ R = 0.1,\ H = 0.1,\ S_c = 0.24, \\
&\gamma = 0.1,\ N = 1,
\end{aligned}
$$

then the exact solution is:

$$u(y,t) = yt,\ T(y,t) = yte^{1-y}, c(y,t) = y^2 t.$$

This problem is simulated using the scheme formulated in this paper, and with the following data:

$\Delta t = 0.005$, simulation terminated after $t = 0.1$, on grids with $8 \times 2^k, k = 0, 1, \ldots, 7$, mesh points. The inhomogeneous terms are evaluated implicitly.

The inhomogeneous terms are evaluated implicitly, and the 2-norm error is computed for each equation.

Table 1: The 2-norm error for each field variable is computed. The results are depicted in table 1. It can be seen that for all the numerical solution of each variable converges to the corresponding exact solution. This verifies the convergence of the numerical scheme.

No of Points	Error in u	EOC of u	Error in T	EOC of T	Error in c	EOC of c
8	4.307e-05	1.8166	4.396e-04	1.04534	1.9007e-04	0.990839
16	1.173e-05	1.8761	2.401e-04	0.87275	1.0327e-04	0.880109
32	3.051e-06	1.9433	1.201e-04	0.99979	5.1773e-05	0.996148
64	8.090e-07	1.9150	6.044e-05	0.99015	2.6044e-05	0.991235

128	2.233e-07	1.8573	3.024e-05	0.99896	1.3031e-05	0.999039
256	6.636e-08	1.7506	1.513e-05	0.99887	6.5195e-06	0.999094
512	2.192e-08	1.5979	7.567e-06	0.99968	3.2260e-06	0.999760
1024	8.181e-09	1.4220	3.784e-06	0.99980	1.6303e-06	0.999854

4. Application

In this section the numerical simulations of the flow fields are presented. The effects of the model parameters on the flow fields are examined. The formulated numerical scheme is implemented in an in-house C++ code developed by the first author. The code adopts the Guass Seidal linear solver. The following default values are used for simulations: λ = 1; M = 0:1; K = 1:0, Gr = 0:1; Gc = 0:1; Pr = 0:75, R = 0:0;H = 0:1; Sc = 0:24; = 0:1, The total simulation time is T = 5 on a grid with 51 grid points. The parameters are varied to investigate their impact on the flow and transport.

4.1 Time Evolution of Flow Fields

The time evolution of distribution of the flow fields are presented in Figure 2. It was observed that increase in time leads to decrease in velocity and concentration profiles while the reverse is the case in temperature profile.

4.2 Variation of Velocity

The influences of the suction parameter and magnetic field parameter (M) on the fluid velocity are depicted in Figure 3. It is observed that increase in the suction parameter increases the velocity distribution while increase in the magnetic field parameter decreases the velocity distribution across the boundary layer. This is expected since the transverse magnetic field gives rise to a resistive type of force called the Lorentz force and it has the tendency of slowing the motion of the fluid.

The distributions of the velocity with varying Thermal Grashof number (Gr) and Solutal Grashof number (Gc) are shown in 4. It is noticed that increases either of these parameters results to an increase in the velocity.

4.3 Variation of Concentration

Figure 5 shows the effect of varying the chemical reaction parameter, suction parameter and Schmidt number (Sc) on the concentration profile. It is observed that an increase in each one of them parameters increases the concentration.

4.4 Variation of Temperature

The effects of the suction parameter and heat generation parameter (H) on the temperature distribution are presented in Figure 6. It was shown that an increase in these parameters lead to increase in the temperature distribution across the boundary layer. Also, the influences of the Prandtl number and the radiation parameter on the temperature distribution are depicted in Figures 7 and 8 respectively. We observe that the Prandtl number increases the temperature while an increase in the radiation parameter decreases the temperature.

4.5 Variation of Skin Friction

The impact of suction parameter and magnetic field parameter on the skin friction are represented in Figure 9. The skin friction increases as suction parameter increases while it decreases as magnetic field parameter increases. Figure 10 demonstrates the influences of the Thermal Grashof number and solutal Grashof number on the skin friction. It is observed that as the Thermal Grashof number and the Solutal Grashof number increase, the skin friction also increases in the region close to the plate while it decreases in the region far from the plate.

4.6 Variation of Nusselt Number

Figure 11 display the variation of the Nusselt number with respect to the suction parameter and heat generation parameter. It is seen that as these parameters increase, the Nusselt number decreases in the region close to the plate but increases in the region far away from the plate. This is also the case for Prandtl number, see Figure 12(a). That is to say that the effects of these parameters is to increase the rate of heat transfer in the region close to the plate, and decreases it in the region far from the plate. However, figure 12(b) shows that the radiation parameter has an opposite effect on the Nusselt number.

4.7 Variation of Sherwood Number

Figure 13 shows the effects of the chemical reaction parameter, suction parameter and Schmidt number on the Sherwood number. It is observed that as the chemical reaction parameter increases, the Sherwood number also increases. This implies that the effect of chemical reaction is to increase the rate of mass transfer in the flow. But as the suction parameter and/or Schmidt number increase, the Sherwood number increases in the area close to the plate while the reverse is the case in the area far from the plate. This implies that suction parameter and Schmidt number increase the rate of mass transfer within the flow.

5. Conclusion

The problem of unsteady free convection incompressible flow in a porous medium with thermal radiation and chemical reaction in an infinite vertical plate was studied. The governing equations are transformed to dimensionless form. A stable and convergent scheme is formulated, analysed, verified and then applied to the model. The simulation results are presented graphically. The following conclusion are drawn from the result obtained: (i) Chemical reaction increases concentration distribution and the rate of mass transfer of the flow and (ii) the suction parameter and Schmidt number increase the rate of mass transfer in the flow.

6. Acknowledgement

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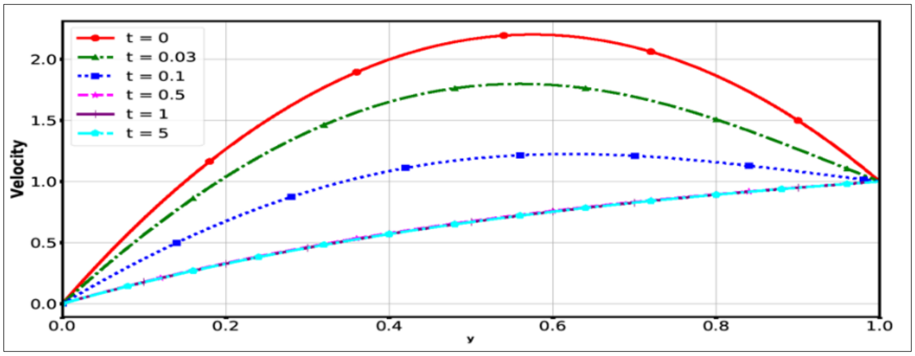

Fig 2a: Time Evolution of Velocity Distribution

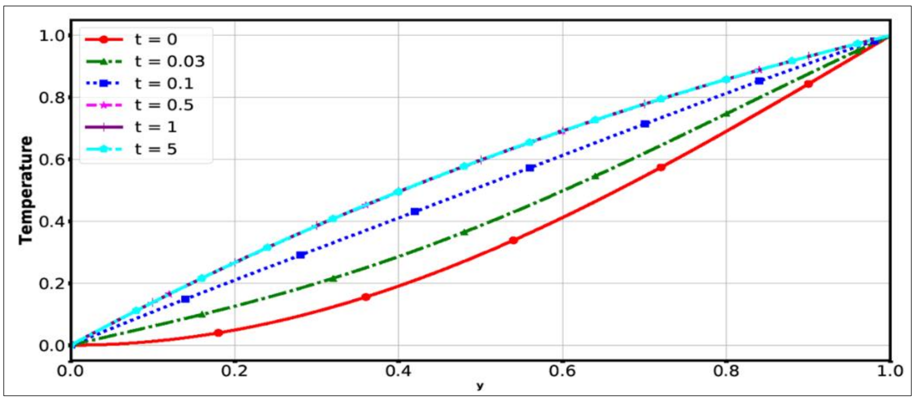

Fig 2b: Time Evolution of Temperature Distribution

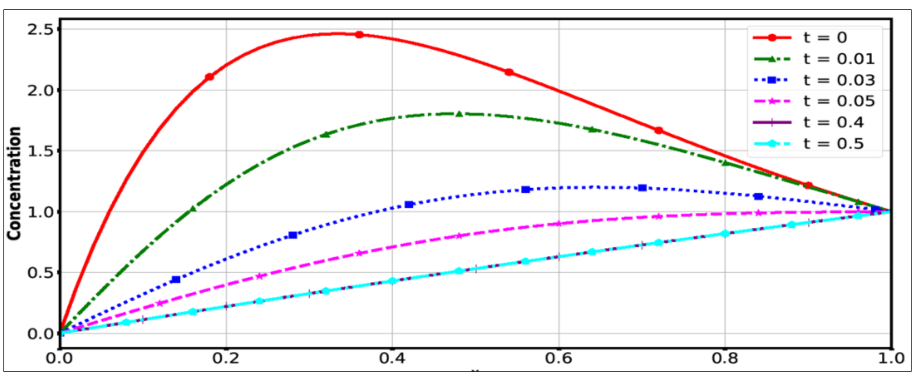

Fig 2c: Time Evolution of Concentration Distribution

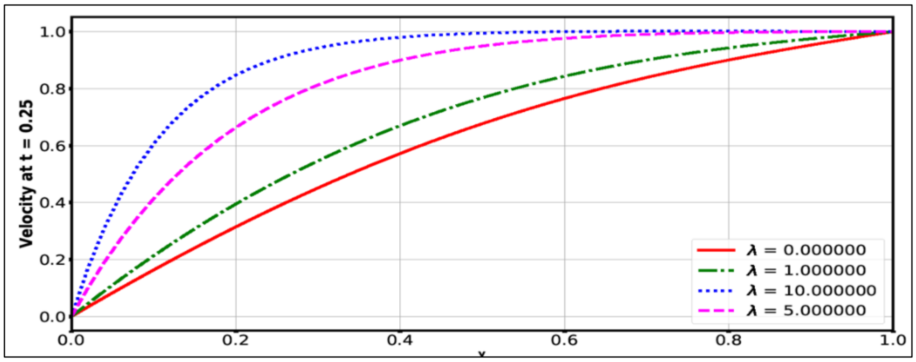

Fig 3a: Effect of suction parameter on the fluid velocity

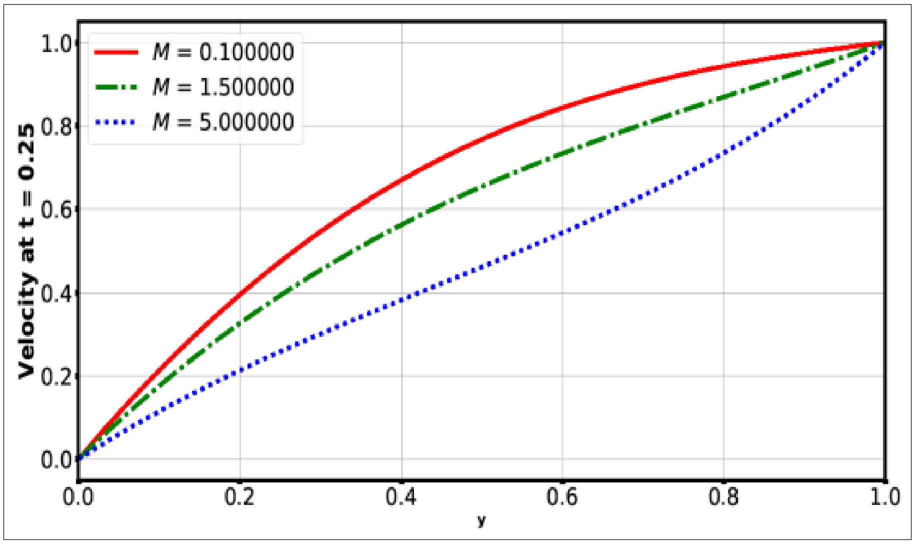

Fig 3b: Effect of Magnetic field parameter on the fluid velocity

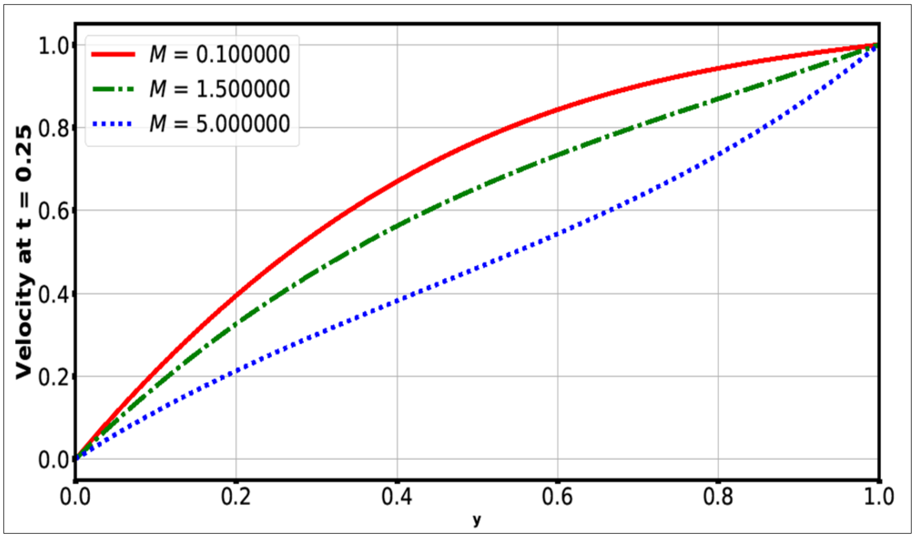

Fig 4a: Effect of thermal Grash of number on the fluid velocity

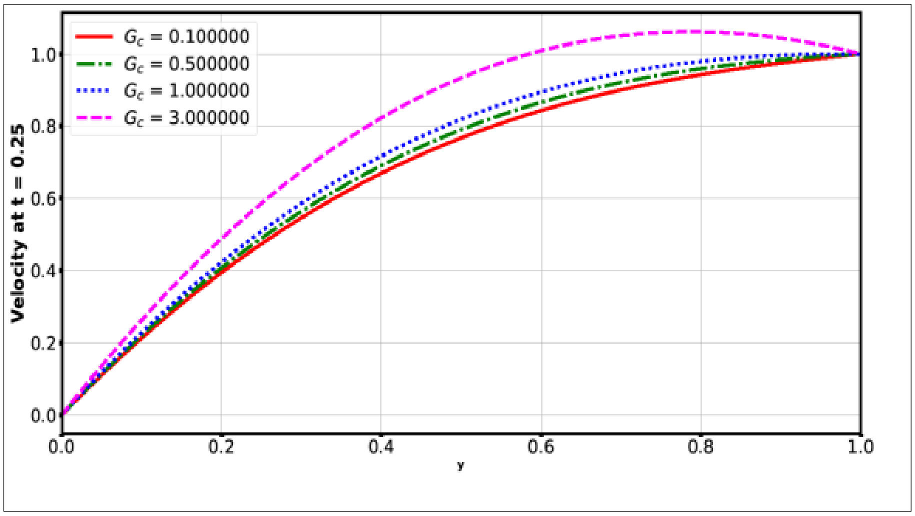

Fig 4b: Effect of solutal Grashof number on the fluid velocity

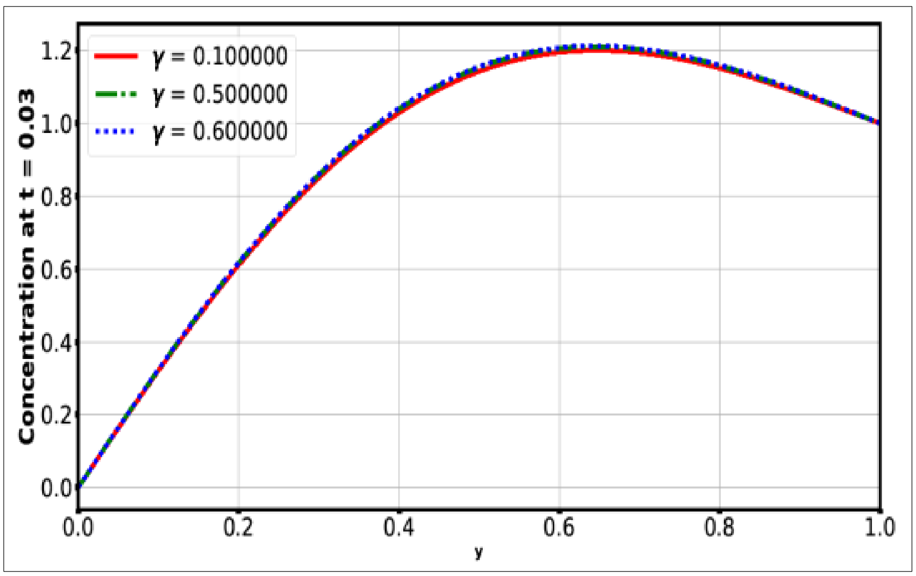

Fig 5a: Effect of reaction parameter on concentration

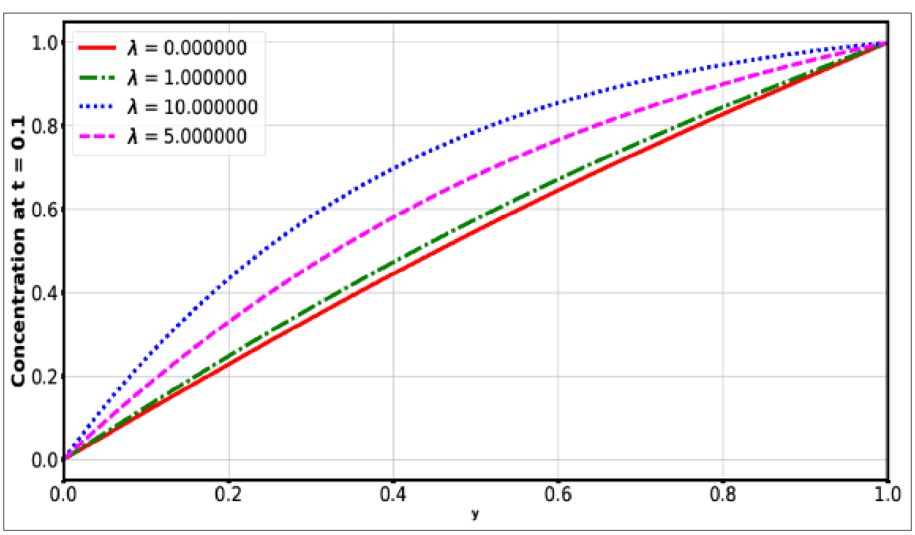

Fig 5b: Effect of suction parameter on concentration

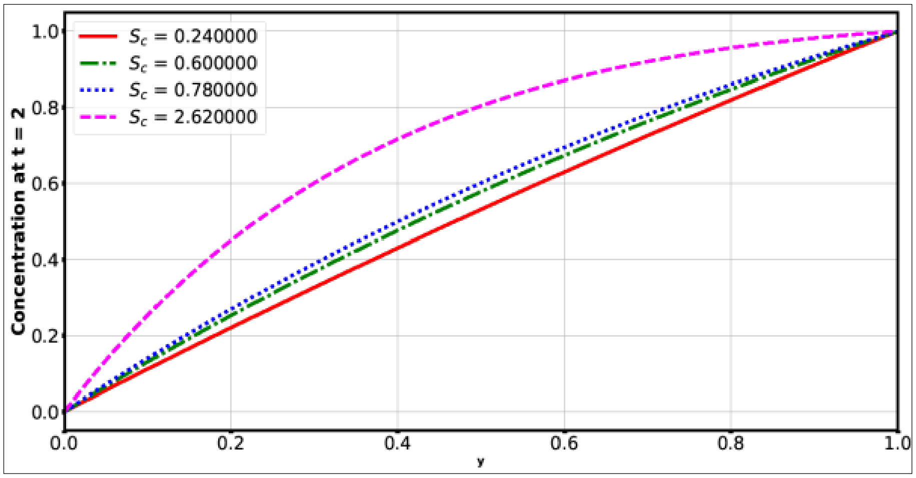

Fig 5c: Effect of Schmidt number on concentration

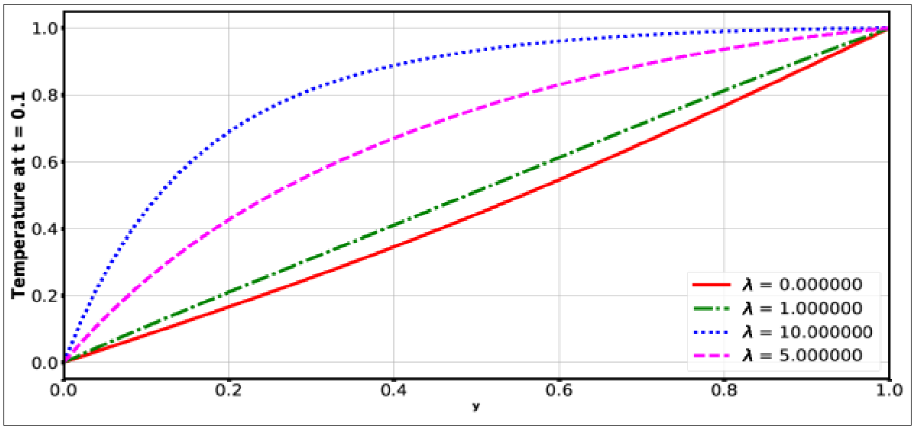

Fig 6a: Effect of suction parameter on temperature

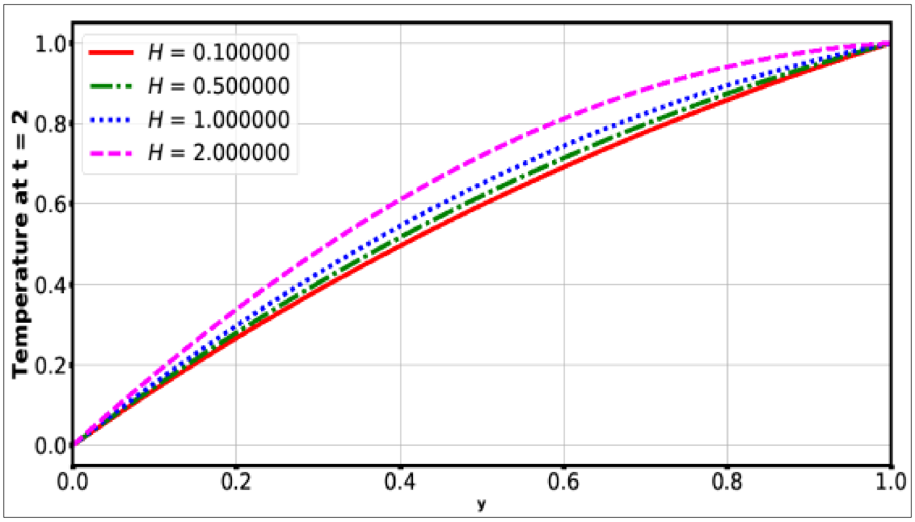

Fig 6b: Effect of heat generation parameter on temperature

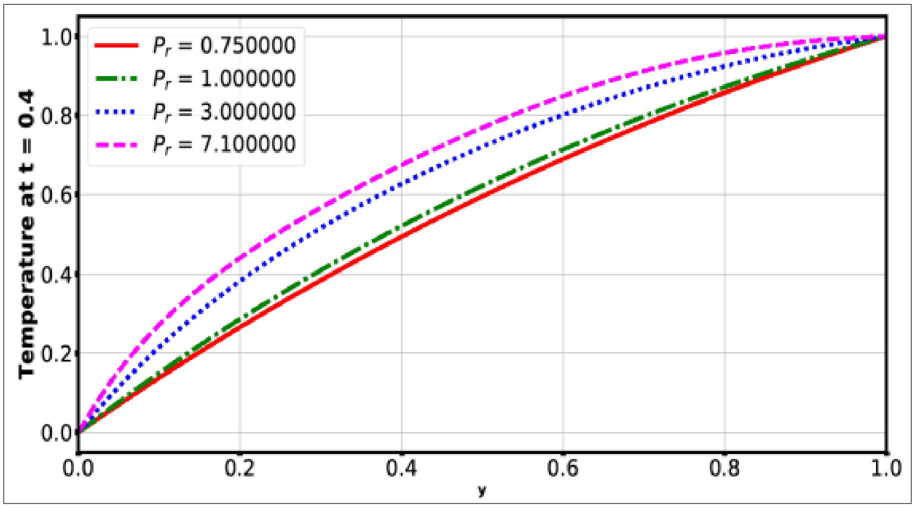

Fig 7: Effect of Prandtl number on temperature

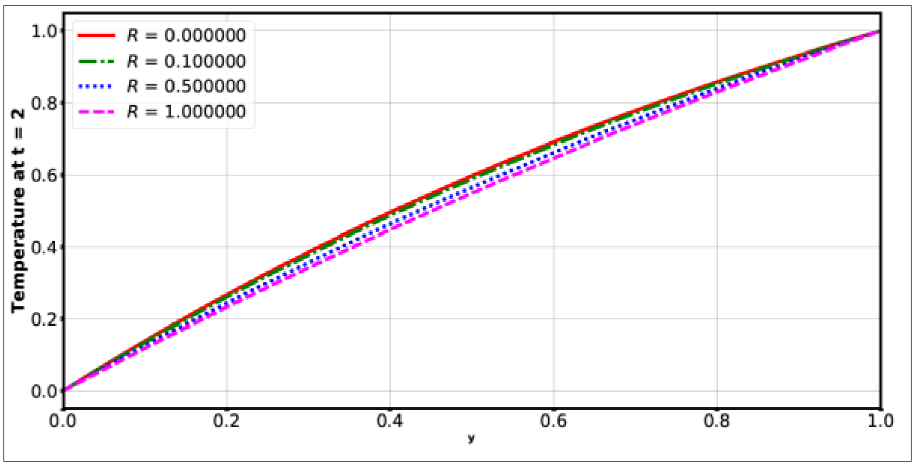

Fig 8: Effect of Radiation parameter on the temperature

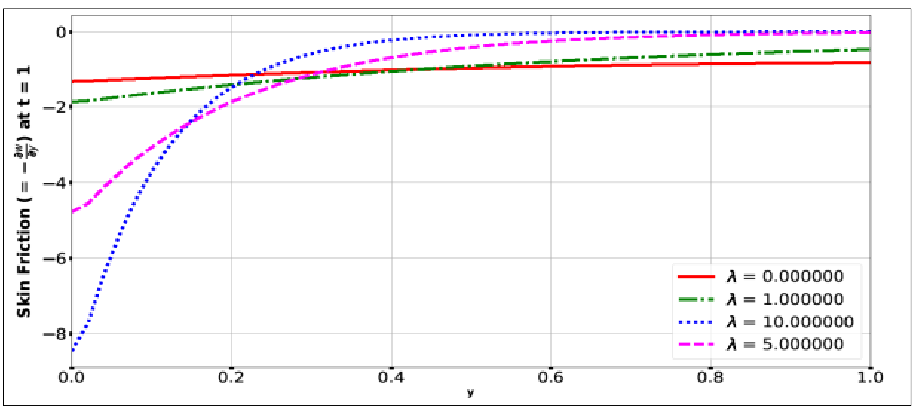

Fig 9a: Effect of the suction parameter on the skin friction

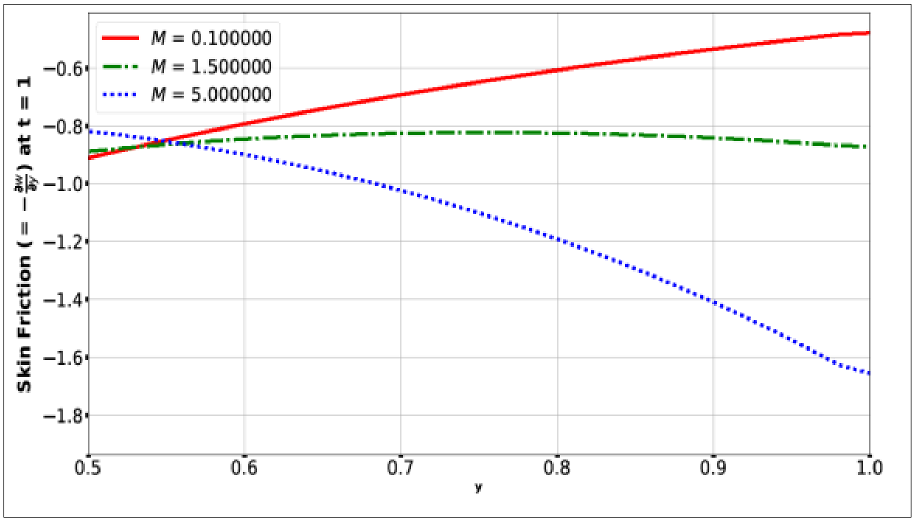

Fig 9b: Effect of the magnetic field parameter on the skin friction

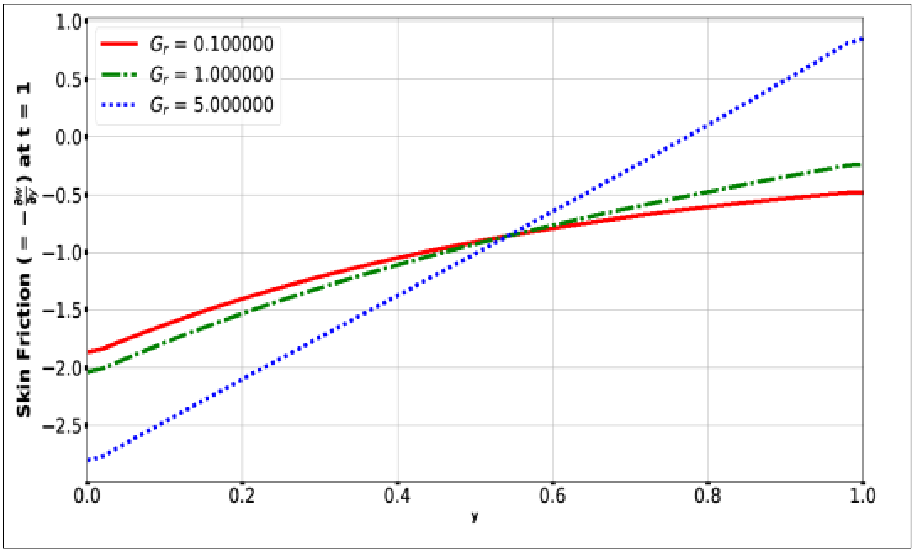

Fig 10a: Effect of the thermal Grashof number on the skin friction

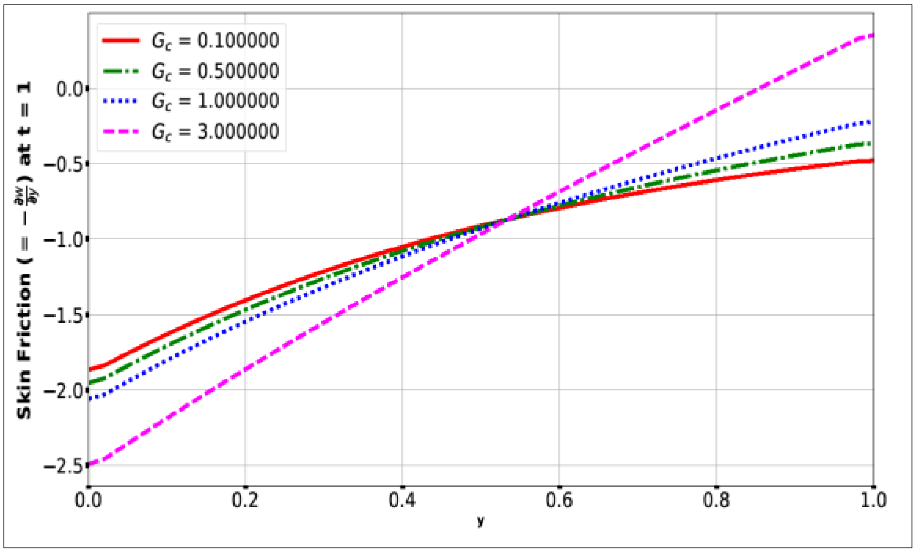

Fig 10b: Effect of the solutal Grash of number on the skin friction

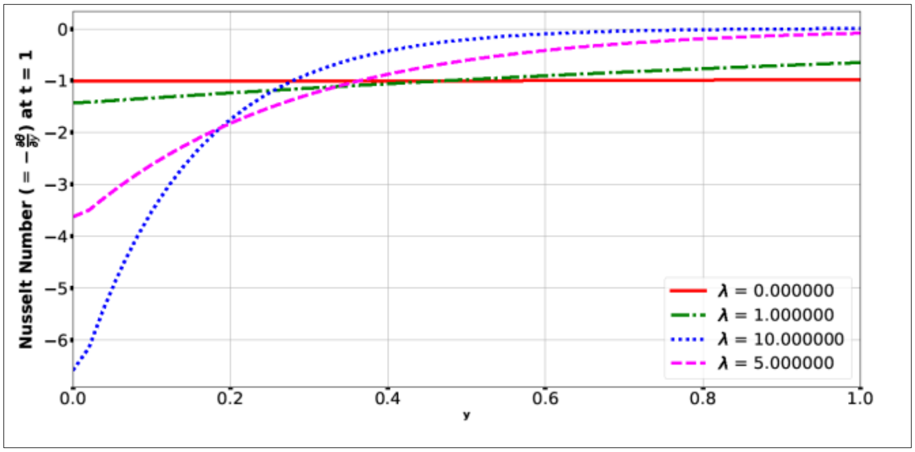

Fig 11a: Effect of suction parameter on the Nusselt number

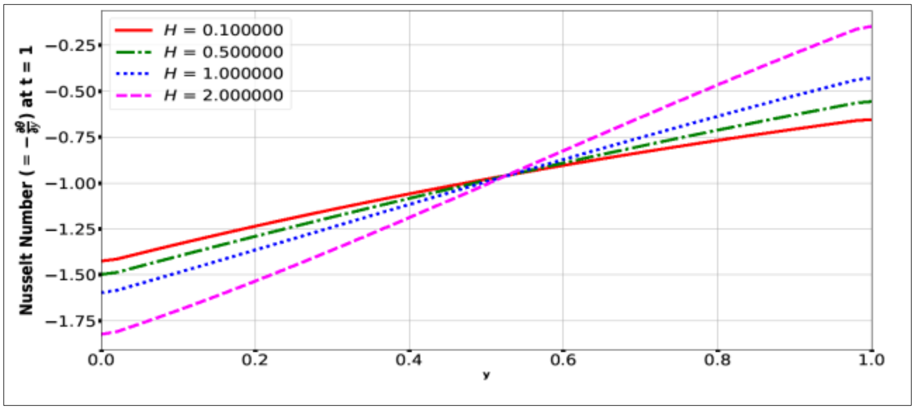

Fig 11b: Effect of heat generation parameter on the Nusselt number

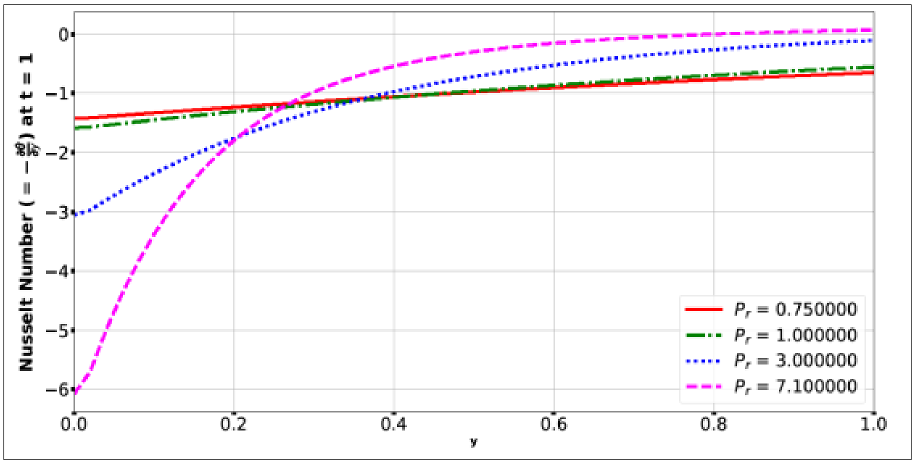

Fig 12a: Effect of Prandtl number on the Nusselt number

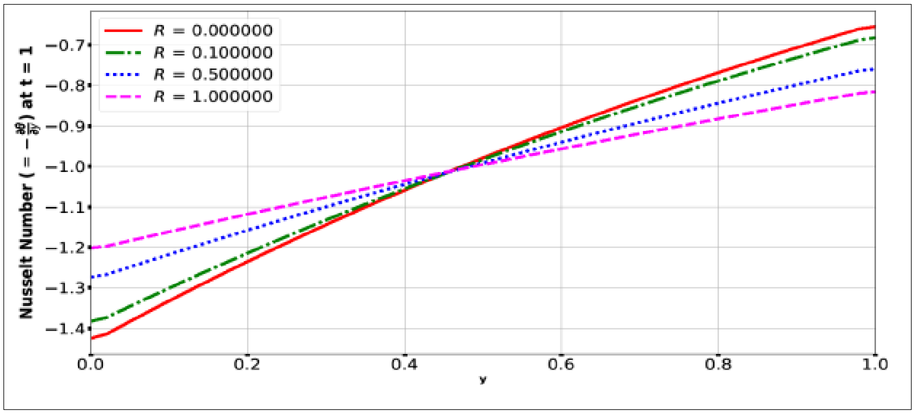

Fig 12b: Effect of radiation parameter on the Nusselt number

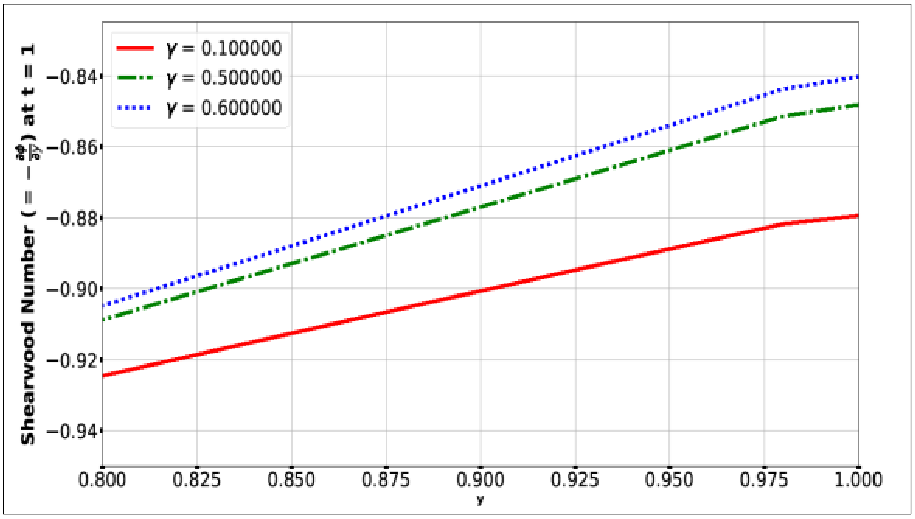

Fig 13a: Effect of reaction parameter on Sherwood number

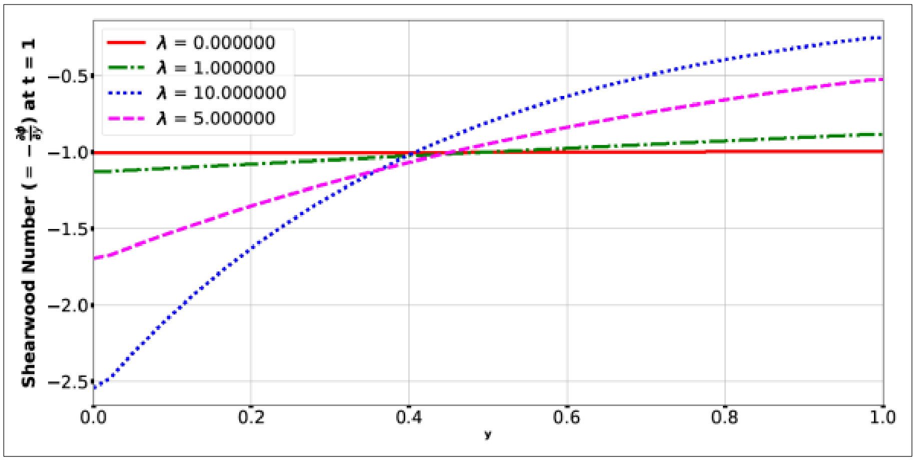

Fig 13b: Effect of suction parameter on Sherwood number

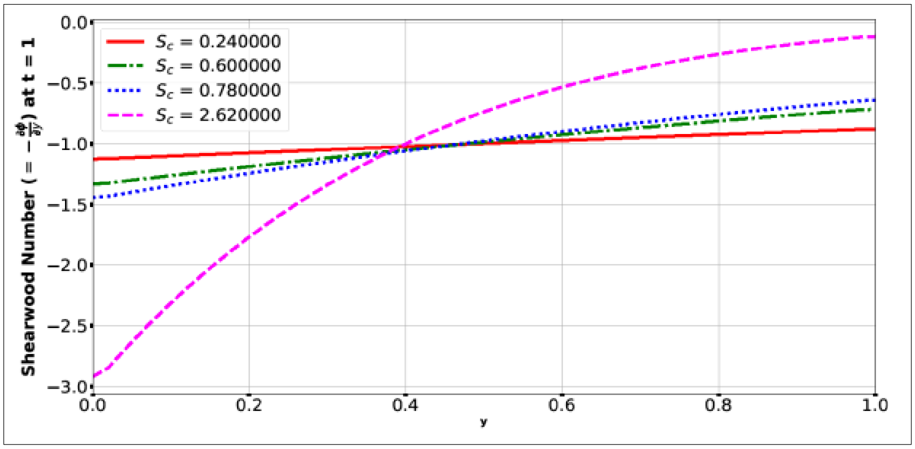

Fig 13c: Effect of Schmidt number on Sherwood number

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