



A mathematical study of porous medium equation under the effect of heat generation and hall current

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Abstract

This study examines the impact of velocity slip and temperature jump on steady fully developed MHD natural convection flow in a vertical micro-porous-channel under the action of Hall current and heat generation. The mathematical model responsible for the present physical situation is presented in non-dimensional form under relevant boundary conditions. The analytical solutions have been obtained for momentum and energy equations. A parametric study of physical parameters is conducted and a representative set of numerical results for the primary velocity, secondary velocity, volume flow rate due to primary flow and volume flow rate due to secondary flow are discussed graphically. It is observed that magnetic field plays a significant role in flow formation inside the micro-channel. Numerical computation reveals that increase in Hartmann number reduces the volume flow rate due to primary flow for symmetric and asymmetric wall heating conditions while impact is just converse on the volume flow rate due to secondary flow for symmetric and asymmetric wall heating conditions.

Keywords: Hall current, MHD, velocity slip, temperature jump, heat generation, micro-channel

1. Introduction

The flows through porous media in tubes or channels have numerous applications in biomedical engineering, for illustration, in the dialysis of blood in simulated kidneys, blood flow in oxygenators, etc. Also the other applications noted, filters design, the transpiration cooling, control in boundary layers and the diffusion in gaseous state. The long emaciated cylinder has the flow of laminar boundary layers and is a special reflection for engineers, mathematicians and physical sciences researchers outstanding to its significant functions in the study of aero-dynamical properties of the motion of air-craft, protracted vehicles and fastest trains, etc., the mechanized of wires, fibres and tumbler in polymer manufacturing. Suction effect on porous walls get rids of the slowdown fluid particles beginning the boundary layer earlier than they grounds separation. So pressure bridges decrease due to the absence of separation. Maximum lift coefficient will increase with decreasing suction. However, the injection of coolant over the perforated wall is a powerful method of cooling the combustion chamber in rockets and jet automobile. Sobhan and Garimella ^[1] discussed the comparative analysis of studies on heat transfer and fluid flow in micro-channels. Yu and Ameel ^[2] considered the slip-flow heat transfer in rectangular micro-channels. Palm ^[3] presented the heat transfer in micro-channels. Chen and Weng ^[4] studied the natural convection in a vertical micro-channel. Aydin and Avci ^[5] considered the analysis of laminar heat transfer in micro-Poiseuille flow. Veera Krishna *et al.* ^[6] presented the investigations of Soret Joule and Hall effects on MHD rotating mixed convective flow past an infinite vertical porous plate. Avci and Aydin ^[7] studied the mixed convection in a vertical parallel plate micro-channel. Venkateswarlu *et al.* ^[8] reported the influence of Hall current and heat source on MHD flow of a rotating fluid in a parallel porous plate channel. Veera Krishna *et al.* ^[9] discussed the Hall effects on MHD peristaltic flow of Jeffrey fluid through porous medium in a vertical stratum. Jha *et al.* ^[10] studied the fully developed magnetohydrodynamics natural convection flow in a vertical micro-porous-channel with Hall effects. Venkateswarlu *et al.* ^[11] presented the influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of heat absorption and chemical reaction. Rundora and Makinde ^[12] considered the effect of suction/injection on unsteady reactive variable viscosity non-Newtonian fluid flow in a channel filled with porous medium and convective boundary conditions. Prabhakar Reddy ^[13] discussed the Hall Effect on MHD transient flow past an impulsively started infinite horizontal porous plate in a rotating system.

The fluid flow and heat transfer through a porous medium have been extensively studied in the past because of its applicability to nuclear waste disposal, solid matrix heat exchanger, thermal insulation and other practical applications. Jha and Aina ^[14] presented the mathematical modeling and exact solution of steady fully developed mixed convection flow in a vertical micro-porous-annulus. Venkateswarlu *et al.* ^[15] discussed the thermodynamic analysis of Hall current and Soret number on hydromagnetic couette flow in a rotating system with a convective boundary condition. Ghara *et al.* ^[16] discussed the Hall Effect on oscillating flow due to eccentrically rotating porous disk and a fluid at infinity. Venkateswarlu *et al.* ^[17] studied the influence of thermal radiation and heat generation on steady hydromagnetic flow in a vertical micro-porous-channel in presence of suction/injection. Akinpelu *et al.* ^[18] discussed the second grade Casson fluid flow with variable viscosity and thermal conductivity through a porous medium. Jha *et al.* ^[19] presented the MHD natural convection in a vertical parallel plate microchannel. Hayat and Abbas ^[20] studied the effects of Hall current and heat transfer on the flow in a porous medium with slip condition. Venkateswarlu *et al.* ^[21] examined Soret and Dufour Effects on radiative hydromagnetic

flow of a chemically reacting fluid over an exponentially accelerated inclined porous plate in presence of heat absorption and viscous dissipation. Gireesha *et al.* [22] presented the entropy generation and heat transport analysis of Casson fluid flow with viscous and Joule heating in an inclined porous microchannel. Kumar *et al.* [23] considered the numerical investigation of heat transfer and fluid flow characteristics in circular wavy microchannel with tangentially branched secondary channels. Venkateswarlu *et al.* [24] studied the influence of heat generation and viscous dissipation on hydromagnetic fully developed natural convection flow in a vertical micro-channel.

The objective of the present article is to study the impact of velocity slip and temperature jump on steady hydromagnetic flow in a parallel plate micro-porous-channel in the presence of Hall current and heat generation, which has not been accounted for in the existing literature. This study benefits the design of micro-heat exchangers.

2. Formation of the problem

The motion of steady hydromagnetic natural convective flow of heat generating, viscous, incompressible and electrically conducting fluid in a parallel plate micro-porous-channel under the effect of transverse magnetic field is considered. Choose a Cartesian coordinate system such that x -axis is parallel to the gravitational acceleration g , y -axis is normal to the channel walls and z -axis is perpendicular to the xy -plane as shown in Fig 1. The micro-channel width is a . A magnetic field of uniform strength B_0 is assumed to be applied in the direction perpendicular to the direction of flow. It is assumed that the magnetic Reynolds number is very small, which corresponds to negligibly induced magnetic field compared to the externally applied one. The micro-porous-channel plates are heated with one plate maintained at a temperature T_1 while the other plate at a temperature T_2 where $T_1 > T_2$. A constant suction/injection velocity is taken into account. Following Jha *et al.* [25, 26], the governing equations in dimensional form can be written as:

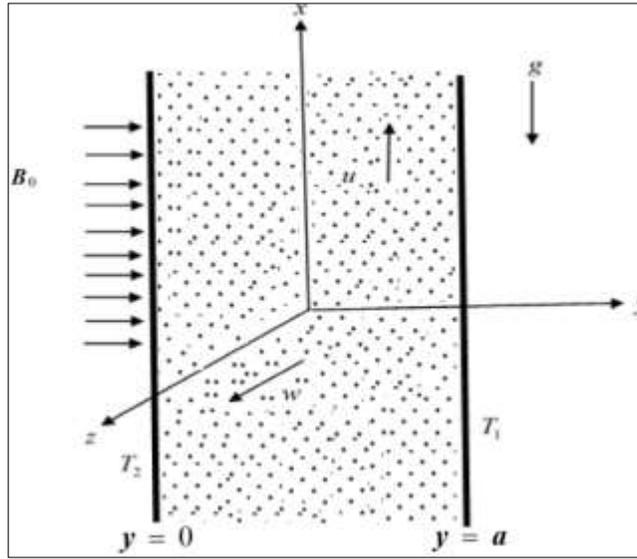


Fig 1: Flow configuration and coordinate system.

Continuity equation:

$$\frac{dv}{dy} = 0 \tag{1}$$

Momentum equations:

$$V_0 \frac{du}{dy} = \nu \frac{d^2u}{dy^2} - \frac{\sigma B_0^2}{\rho} \left[\frac{u + mw}{1 + m^2} \right] + g\beta(T - T_0) - \frac{\nu}{K_1} u \tag{2}$$

$$V_0 \frac{dw}{dy} = \nu \frac{d^2w}{dy^2} + \frac{\sigma B_0^2}{\rho} \left[\frac{mu - w}{1 + m^2} \right] - \frac{\nu}{K_1} w \tag{3}$$

Energy equation:

$$V_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} + \frac{Q_0}{\rho c_p} (T - T_0) \tag{4}$$

Here u – fluid velocity in x – direction, v – fluid velocity in y – direction, w – fluid velocity in z – direction, B_0 – uniform magnetic field, ρ – fluid density, ν – kinematic viscosity of the fluid, σ – fluid electrical conductivity, g – gravitational acceleration, β – thermal expansion coefficient, T – fluid temperature, T_0 – reference temperature, V_0 – constant suction velocity, $m = \omega_e \tau_e$ – Hall current parameter, ω_e – cyclotron frequency, τ_e – electron collision time, Q_0 – dimensional heat generation parameter, c_p – specific heat at constant pressure, K_1 – dimensional permeability parameter and α – thermal diffusivity of the fluid respectively.

We should in prior inform the reader that our model is not the same as that by Jha et al. [25, 26], in which the heat generation parameter and permeability parameter were not taken into account. Interpretation of energy equation was different in both papers. The corresponding boundary conditions which describe velocity slip and temperature jump at the fluid wall interface in dimensional form can be written as:

$$\left. \begin{aligned} u &= \frac{(2 - F_v)}{F_v} \lambda \frac{du}{dy}, w = \frac{(2 - F_v)}{F_v} \lambda \frac{dw}{dy}, T = T_2 + \frac{(2 - F_t)}{F_t} \frac{2\gamma}{(\gamma + 1)} \frac{\lambda}{Pr} \frac{dT}{dy} & \text{at } y = 0 \\ u &= -\frac{(2 - F_v)}{F_v} \lambda \frac{du}{dy}, w = -\frac{(2 - F_v)}{F_v} \lambda \frac{dw}{dy}, T = T_1 - \frac{(2 - F_t)}{F_t} \frac{2\gamma}{(\gamma + 1)} \frac{\lambda}{Pr} \frac{dT}{dy} & \text{at } y = a \end{aligned} \right\} \tag{5}$$

Here F_v – tangential momentum accommodation coefficient, λ – molecular mean free path, T_1 – hot wall temperature, T_2 – cold wall temperature, F_t – tangential thermal accommodation coefficient, $\gamma = c_p/c_v$ – ratio of specific heats, c_v – specific heat at constant volume, Pr – Prandtl number, and a – distance between two plates respectively.

The following non-dimensional quantities are introduced

$$\left. \begin{aligned} \eta &= \frac{y}{a}, \theta = \frac{T - T_0}{T_1 - T_0}, U = \frac{uv}{g\beta(T_1 - T_0)a^2}, W = \frac{wv}{g\beta(T_1 - T_0)a^2}, Pr = \frac{\nu}{\alpha}, \\ \xi &= \frac{T_2 - T_0}{T_1 - T_0}, \beta_v = \frac{2 - F_v}{F_v}, \beta_t = \frac{2 - F_t}{F_t} \frac{2\gamma}{(\gamma + 1)} \frac{1}{Pr}, Kn = \frac{\lambda}{a}, ln = \frac{\beta_t}{\beta_v} \end{aligned} \right\} \tag{6}$$

Equations (2), (3) and (4) can be transformed to the following non-dimensional form

$$\frac{d^2U}{d\eta^2} - S \frac{dU}{d\eta} + \theta - M \left[\frac{U + mW}{1 + m^2} \right] - \frac{1}{K} U = 0 \tag{7}$$

$$\frac{d^2W}{d\eta^2} - S \frac{dW}{d\eta} + M \left[\frac{mU - W}{1 + m^2} \right] - \frac{1}{K} W = 0 \tag{8}$$

$$\frac{d^2\theta}{d\eta^2} - S Pr \frac{d\theta}{d\eta} + H Pr \theta = 0 \tag{9}$$

Here $S = \frac{V_0 a}{\nu}$ is the suction parameter, $M = \frac{\sigma B_0^2 a^2}{\rho \nu}$ is the magnetic parameter, $m = \omega_e \tau_e$ is the Hall current parameter, $K = \frac{K_1}{a^2}$ is the permeability parameter and $H = \frac{Q_0 a^2}{\rho c_p \nu}$ is the heat generation parameter respectively.

The boundary conditions in non-dimensional can be written as

$$\left. \begin{aligned} U &= \beta_v Kn \frac{dU}{d\eta}, W = \beta_v Kn \frac{dW}{d\eta}, \theta = \xi + \beta_v Kn \ln \frac{d\theta}{d\eta} \quad \text{at } \eta = 0 \\ U &= -\beta_v Kn \frac{dU}{d\eta}, W = -\beta_v Kn \frac{dW}{d\eta}, \theta = 1 - \beta_v Kn \ln \frac{d\theta}{d\eta} \quad \text{at } \eta = 1 \end{aligned} \right\} \quad (10)$$

By defining the complex velocity $\psi = U + iW$, equations (2) and (3) can be written as

$$\frac{d^2\psi}{d\eta^2} - S \frac{d\psi}{d\eta} + \theta - \left[\frac{M}{1+im} + \frac{1}{K} \right] \psi = 0 \quad (11)$$

The boundary conditions in complex form can be expressed as

$$\left. \begin{aligned} \psi &= \beta_v Kn \frac{d\psi}{d\eta}, \theta = \xi + \beta_v Kn \ln \frac{d\theta}{d\eta} \quad \text{at } \eta = 0 \\ \psi &= -\beta_v Kn \frac{d\psi}{d\eta}, \theta = 1 - \beta_v Kn \ln \frac{d\theta}{d\eta} \quad \text{at } \eta = 1 \end{aligned} \right\} \quad (12)$$

Referring to the values of F_v and F_t given in Goniak Duffa [27], Eckert and Drake [28] the value of β_v is near unity and the value of β_t ranges from near 1 to more than 100 for actual wall surface conditions, also β_t is near 1.667 for several engineering applications, corresponding to $F_v = 1$, $F_t = 1$, $\gamma = 1.4$ and $Pr = 0.71$.

Given the micro-channel slip velocity, dimensional volume flow rate δ is obtained as

$$\delta = v \int_0^a [u(y) + iv(y)] dy \quad (13)$$

In non-dimensional form the volume flow rate Q is defined as

$$Q = \frac{\delta}{g\beta(T_1 - T_0)a^3} \quad (14)$$

Using the non-dimensional quantities in equation (6) and the equation (13) into equation (14), we obtain

$$Q = \int_0^1 [U(\eta) + iW(\eta)] d\eta \quad (15)$$

3. Solution of the problem

The system of ordinary differential equations is solved analytically with corresponding boundary conditions. Equation (9), subject to the boundary conditions (12), has the following analytical solution:

$$\theta(\eta) = a_4 \exp(m_1 \eta) + a_5 \exp(m_2 \eta) \quad (16)$$

Substituting the equation (16) into the momentum equation (11) and solving it by using the boundary conditions (12), we obtain

$$\psi(\eta) = a_{15} \exp(m_3 \eta) + a_{16} \exp(m_4 \eta) - a_7 \exp(m_1 \eta) - a_8 \exp(m_2 \eta) \quad (17)$$

$$U(\eta) = \text{Real} [a_{15} \exp(m_3 \eta) + a_{16} \exp(m_4 \eta) - a_7 \exp(m_1 \eta) - a_8 \exp(m_2 \eta)] \quad (18)$$

$$W(\eta) = \text{Imag} \left[a_{15} \exp(m_3 \eta) + a_{16} \exp(m_4 \eta) - a_7 \exp(m_1 \eta) - a_8 \exp(m_2 \eta) \right] \quad (19)$$

4. Results and discussion

We have investigated the influence of velocity slip and temperature jump on MHD flow in a vertical micro-porous-channel in presence of heat generation and Hall current. The flow governed by the following non-dimensional parameters namely, Hartmann number M , Hall current parameter m , permeability parameter K , heat generation parameter H , rarefaction parameter $\beta_v Kn$ and fluid wall interaction parameter ln for three distinct values of wall ambient temperature difference ratio ξ on the primary velocity U , secondary velocity W displayed with the aid of line graphs (Figs. 2–13) and discussed consequently. The parametric study has been performed with fixed value of $S = 0.5$, $M = 0.5$, $m = 0.5$, $K = 0.5$, $H = 0.5$, $Pr = 0.71$, $\beta_v Kn = 0.05$ and $ln = 1.667$. In this study $\xi = -1.0$ corresponds to one plate is heating and one plate is cooling, $\xi = 0.0$ corresponds to one plate is heating and one plate is not heating and $\xi = 1.0$ corresponds to both plates are heating. That is $\xi = -1.0$ and $\xi = 0.0$ corresponds to asymmetric heating and $\xi = 1.0$ corresponds to symmetric heating.

Figs. 3 and 3 express the delineating aspect of primary velocity U and secondary velocity W with respect to the Hartmann number M and the wall ambient temperature difference ratio ξ . It is observed that, increasing values of the Hartmann number has a tendency to slow down the primary velocity in the micro-channel for $\xi = 0$ and $\xi = 1$ whereas the reverse trend is recognized in case of secondary velocity. The secondary velocity profiles are decreases near the micro-channel left wall and increases near the micro-channel right wall for $\xi = -1$ but primary velocity profiles increases at both left and right walls for $\xi = 1$. This is due to the fact that the application of magnetic field generates a resistive force related to the drag force that acts in the opposite direction of the fluid motion, consequently primary velocity decreases and secondary velocity increases with an increase in Hartmann number for $\xi = 0$ and $\xi = 1$.

Figs.4 and 5 illustrate the variation of primary velocity U and secondary velocity W with Hall current parameter m under wall ambient temperature difference ratio ξ . It is interesting to observe that in both primary and secondary flow directions, an increase in the Hall current parameter yields a noticeable increase in the fluid velocity for $\xi = 0$ and $\xi = 1$. However the increase is more prevalent along the secondary flow direction. The primary velocity attains their steady state and the secondary velocity decreases with an increase in Hall current parameter for $\xi = -1$. This is due to the physical fact that with the increase in Hall current parameter, the effective conductivity of the fluid decreases, thereby weakening the resistive force induced by the applied magnetic field and consequently increasing the fluid velocity.

Figs. 6 and 7 depict that, the effects of the permeability parameter K as well as the wall ambient temperature difference ratio ξ on the primary velocity U and the secondary velocity W . It is seen that for $\xi = 0$ and $\xi = 1$, escalating value of permeability parameter leads to induce more flow and hence the fluid primary velocity and secondary velocity accelerates. The primary velocity and secondary velocity decreases with an increase of permeability parameter for $\xi = -1$.

Figs. 8 and 9 illustrate the variation of primary velocity U and secondary velocity W for heat generation parameter H and wall ambient temperature difference ratio ξ . It is clear that by enhancing the heat generation parameter the primary and secondary velocity profiles enhances for $\xi = 0$ and $\xi = 1$. It is observed that, velocity profiles are decreases in both directions for $\xi = -1$ with an increase in the heat generation parameter.

Figs. 10 and 11 depict the influence of rarefaction parameter $\beta_v Kn$ as well as wall ambient temperature difference ratio ξ on the fluid primary velocity U and secondary velocity W respectively. It is noticed that for $\xi = 0$ and $\xi = 1$, increasing the value of rarefaction parameter leads to an enhancement in the fluid primary velocity and secondary velocity. These effects are more pronounced in the case of symmetric heating. It is interesting to mention that in both directions, the strength of the velocity is inversely proportional to the rarefaction parameter for $\xi = -1$.

Figs. 12 and 13 display the combined effect of fluid wall interaction parameter ln and wall ambient temperature difference ratio ξ on the primary velocity U and secondary velocity W respectively. It is noticed that, there is an enhancement in the primary velocity near the micro-channel left wall and a reduction near the micro-channel right wall with an increase in fluid wall interaction parameter for $\xi = -1$ and $\xi = 0$ whereas the secondary velocity decreases for $\xi = -1$ and increases for $\xi = 0$ with an increase in the fluid wall interaction parameter. It is observed that, in both directions velocity increases within the channel with increasing values of fluid wall interaction parameter ln for symmetric heating.

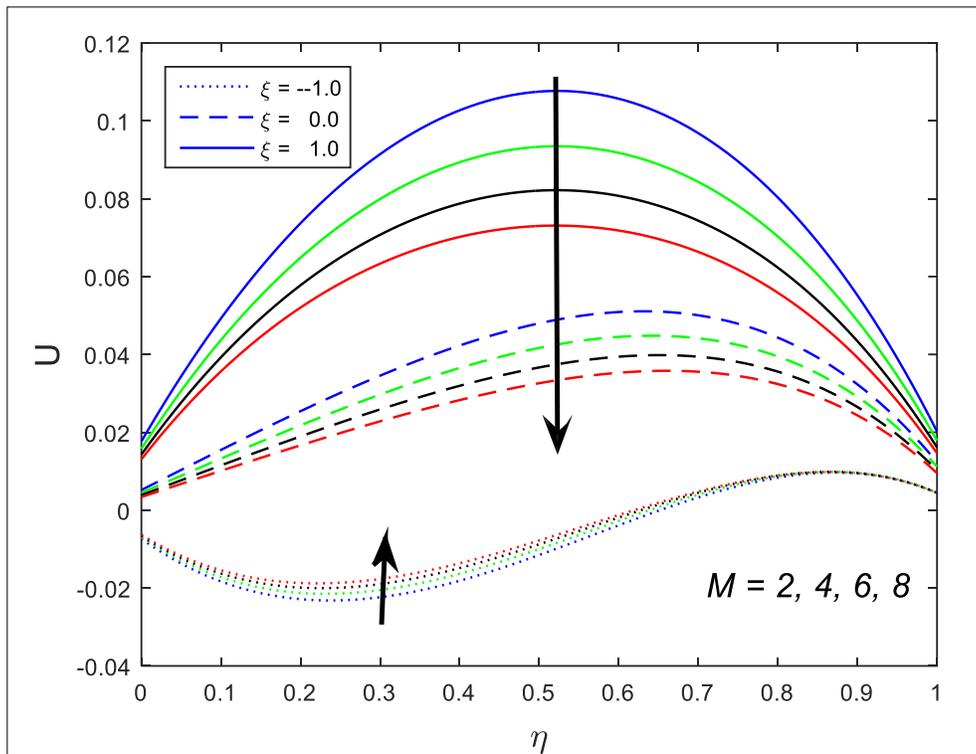


Fig 2: Representation of primary velocity against Hartmann parameter.

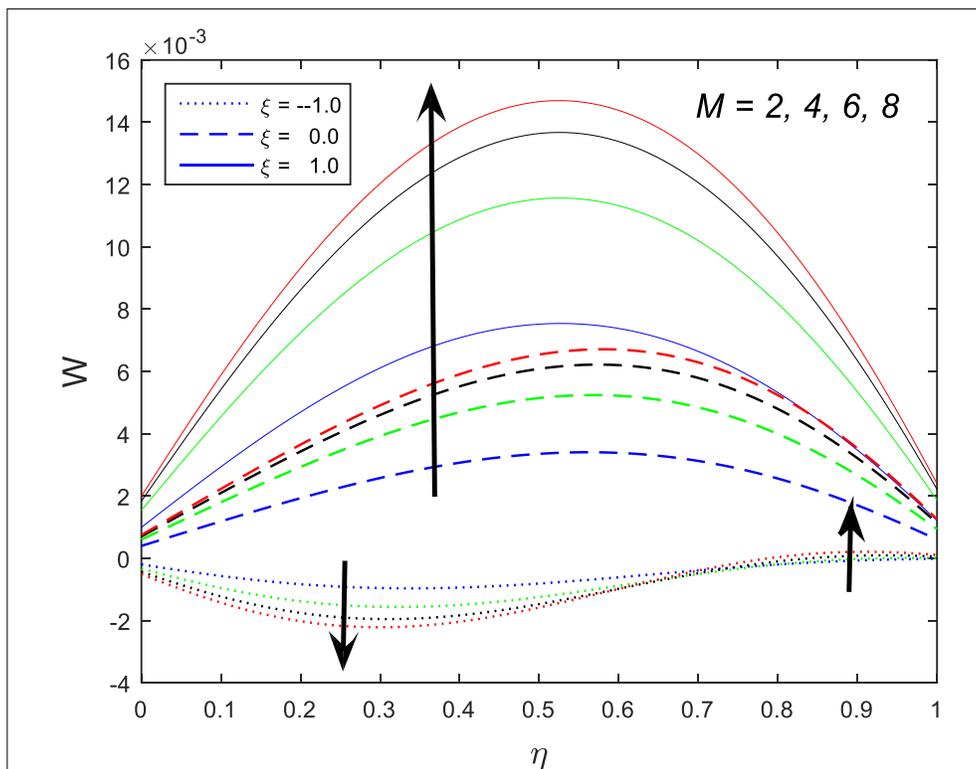


Fig 3: Representation of secondary velocity against Hartmann parameter.

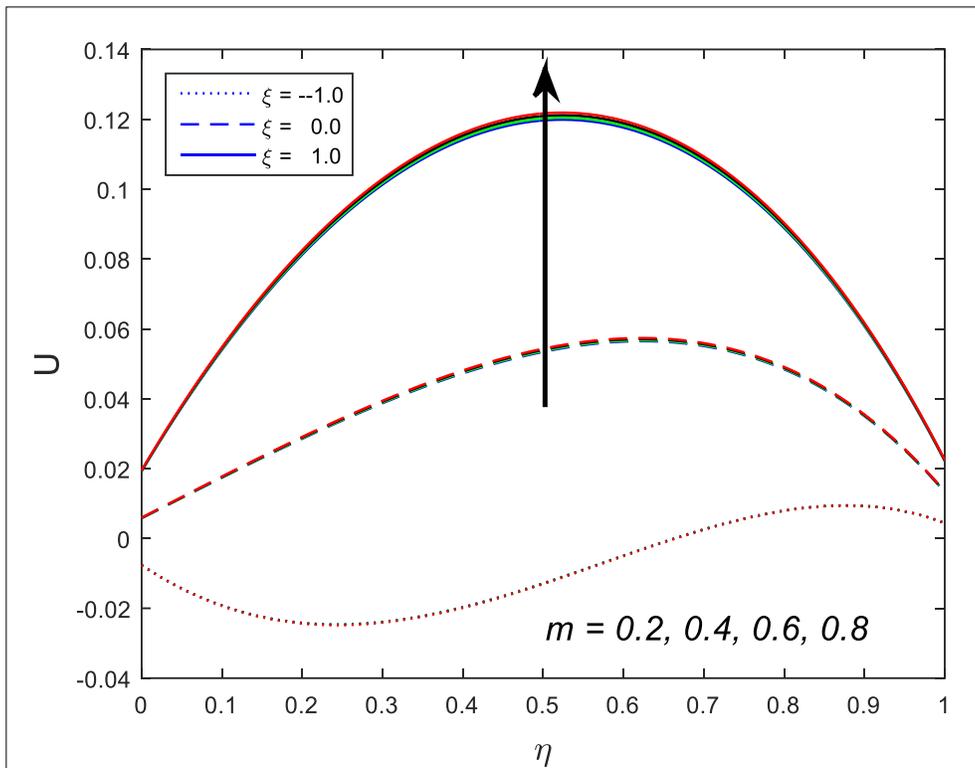


Fig 4: Representation of primary velocity against Hall current parameter.

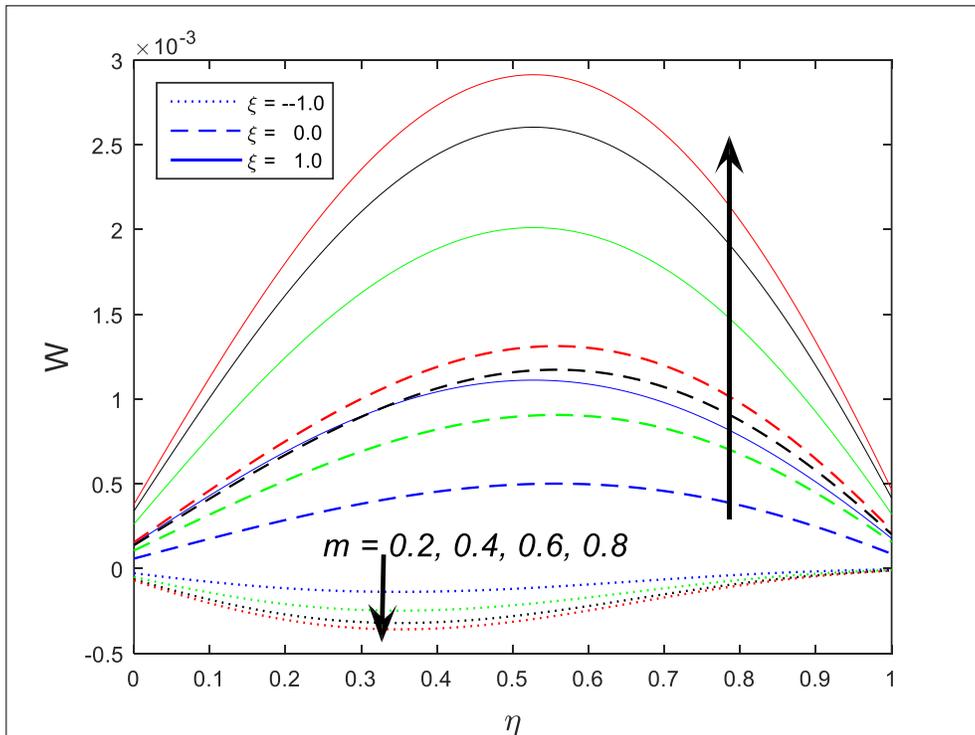


Fig 5: Representation of secondary velocity against Hall current parameter.

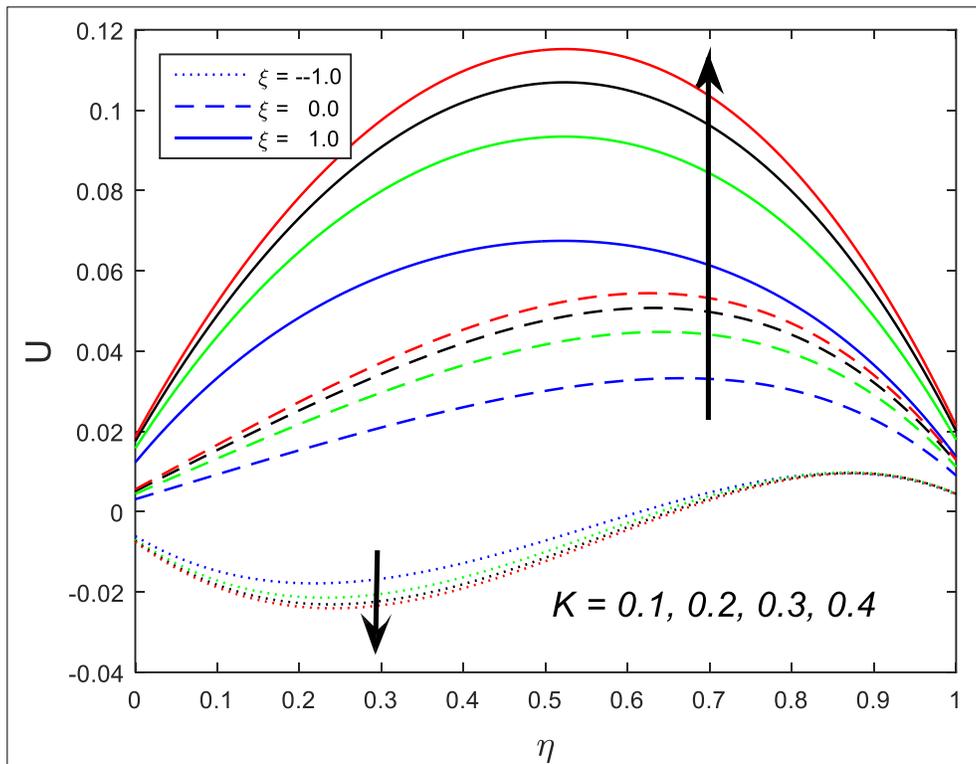


Fig 6: Representation of primary velocity against permeability parameter.

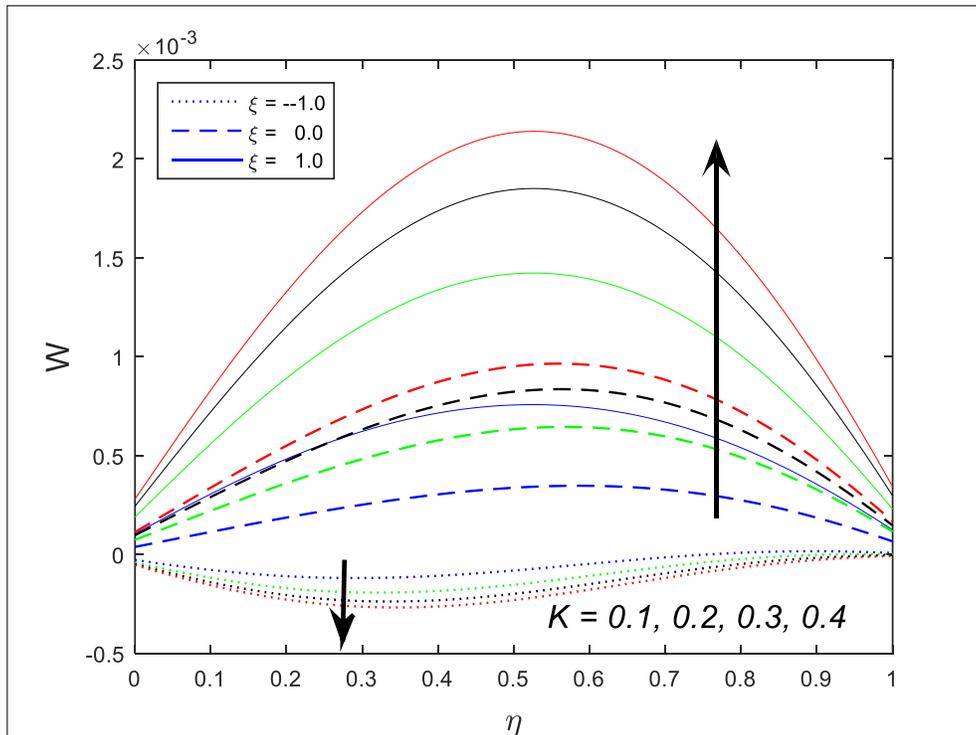


Fig 7: Representation of secondary velocity against permeability parameter.

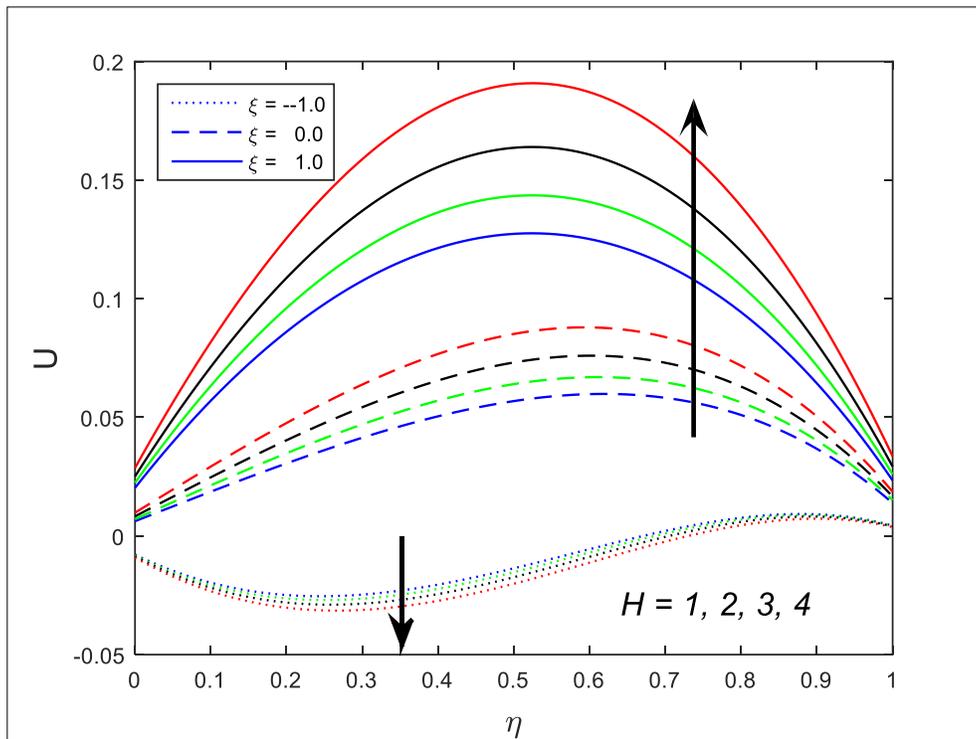


Fig 8: Representation of primary velocity against heat generation parameter.

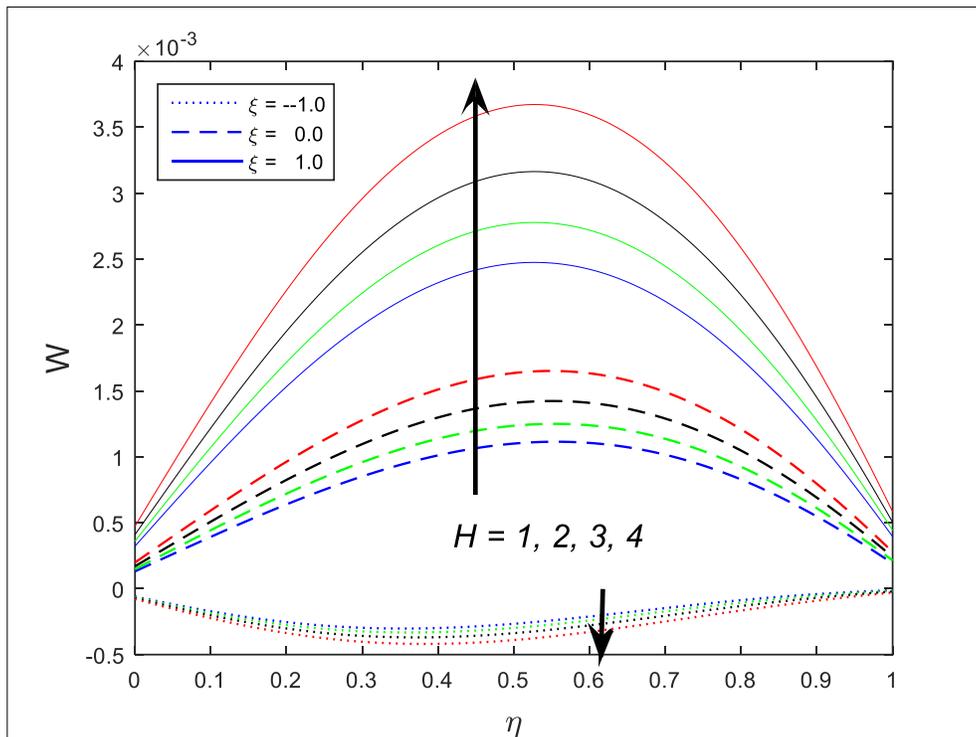


Fig 9: Representation of secondary velocity against heat generation parameter.

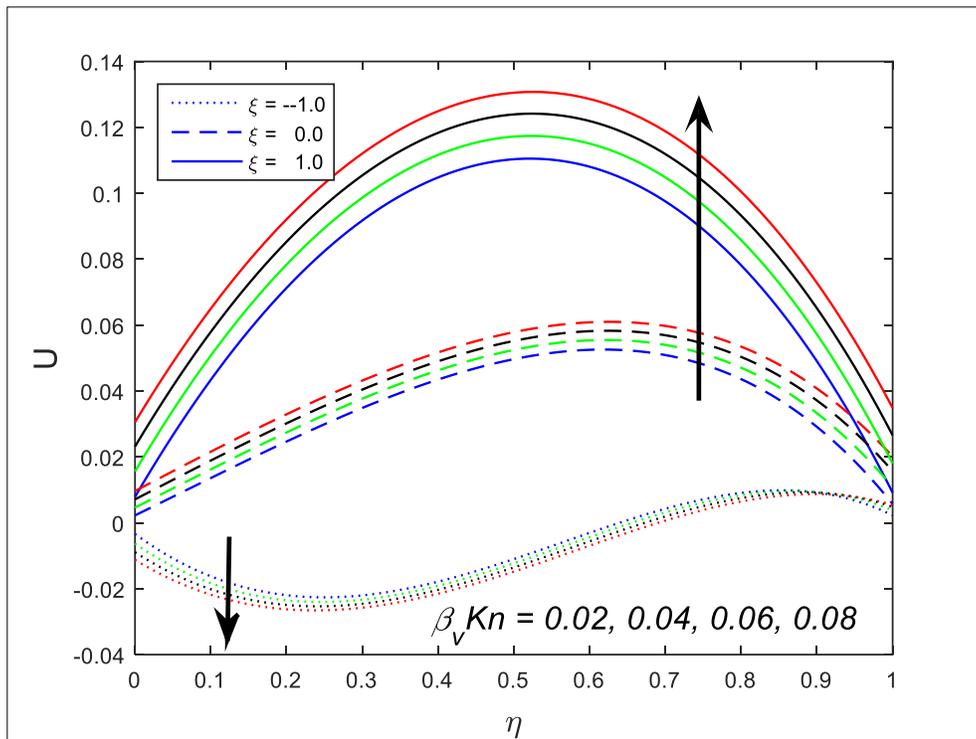


Fig 10: Representation of primary velocity rarefaction parameter.

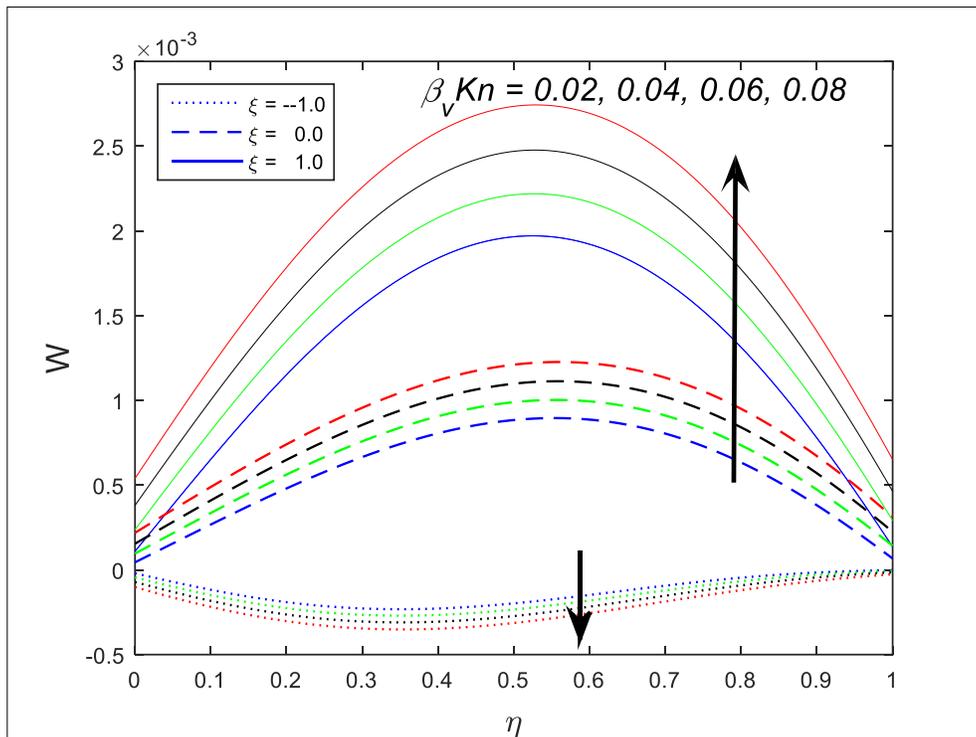


Fig 11: Representation of secondary velocity against rarefaction parameter.

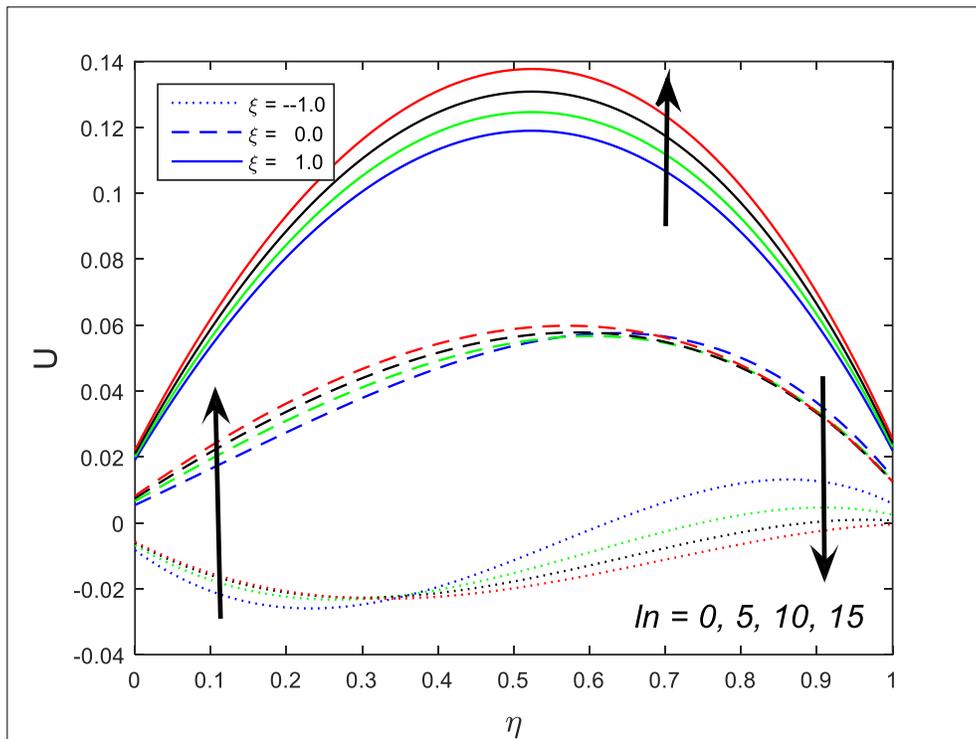


Fig 12: Representation of primary velocity against fluid wall interaction parameter.

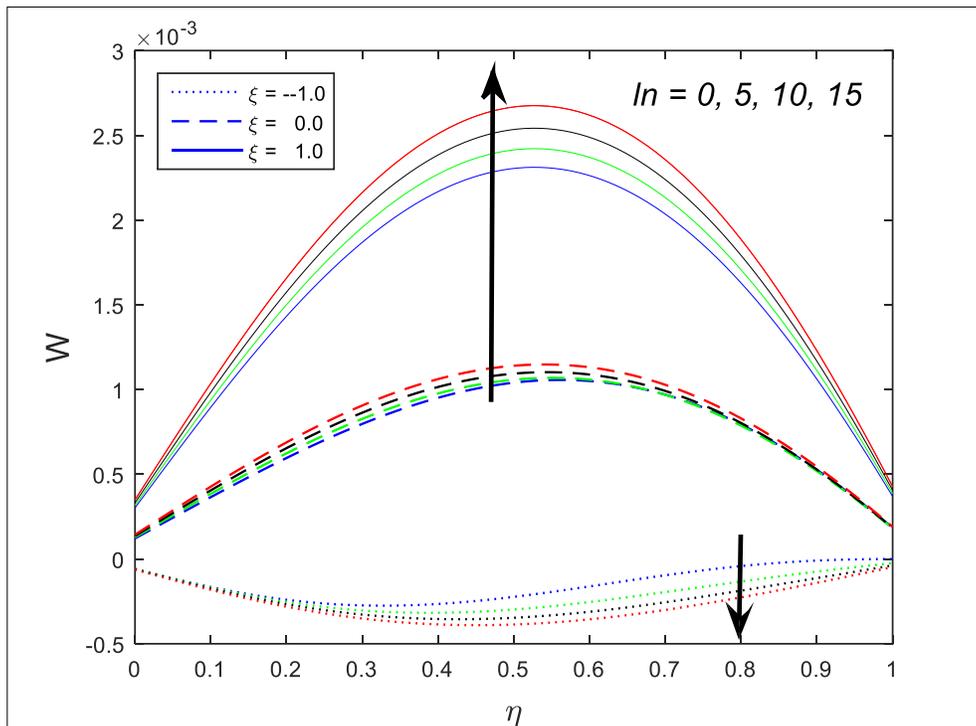


Fig 13: Representation of secondary velocity against fluid wall interaction parameter.

5. Conclusions

Main observations of present research are listed below

- The greater values of Hall current parameter, permeability parameter, heat generation parameter and rarefaction parameter increase the velocity in both directions for $\xi = 0$ and $\xi = 1$.
- Fluid wall interaction parameter increases the primary velocity near the micro-channel left wall and decreases near the micro-channel right wall $\xi = -1$ and $\xi = 0$.

- Primary and secondary velocity profiles are increases for rising values of the wall ambient temperature difference ratio.

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