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Velocity and acceleration in paraboloidal coordinates

JF Omonile[1], JO Adeyemi[2], SXK Howusu[3]
[1] Lecturer, Kogi state university, Ayangba, Kogi state, Nigeria
[2] Assistant Lecturer, Kogi state university, Ayangba, Kogi state, Nigeria
[3] Professor, Kogi state university, Ayangba, Kogi state, Nigeria

Abstract
The expression for the velocity and acceleration in prolate spheroidal coordinates is now well known. In this paper we proceed to derive expression for the instataneous velocity and acceleration in paraboloidal coordinates for application in Newtonian's Mechanics, Eeinstein's special law of Motion and Schrödinger Law of Quantum Mechanics.

Keywords: Vernal keratoconjunctivitis, tacrolimus, immunomodulators, olopatadine

Introduction

We had started the establishe velocities and accelerations of a curvilinear coordinates beyond well-known cartesian,cylindrical and spherical coordinates [1]. In our previous paper, we had established the velocity and acceleration in Prolate coordinates and Parabolic coordinates for application in Mechanics [2,]. In this paper, we continue to derive the expression for velocity and acceleration in araboloidzal coordinates for application in Mechanics. The paraboloidazl coordinates (U, V, Φ) are defined interms of the casrtesian coordinates (x,y,z) by [4].

$$X = UV\cos\Phi \qquad (1)$$

$$Y = UV\sin\Phi \qquad (2)$$

$$Z = \frac{1}{2}(U^2 + V^2) \qquad (3)$$

Where

$$U \geq 0;\ V \geq 0;\ 0 \leq \Phi \leq 2\pi \qquad (4)$$

Consequently, by definition, the Paraboloidal metrical coefficient are given by:

$$h_u = (U^2 + V^2)^{\frac{1}{2}} \qquad (5)$$

$$h_v = (U^2 + V^2)^{\frac{1}{2}} \qquad (6)$$

$$h_{\phi} = UV \qquad (7)$$

These metrical coefficients define the units vectors, line element, volume elemrnts,gradient, divergence, curl and laplacian operations in paraboloidal coordinates according to the theory of orthogonal curvilinear coordinates [5,6,7]. These quantities are necessary and sufficient for the derivation of the field of all paraboloidal distribution of mass, charge, current. Now for the derivation of the equation of motion for test particles. In these fields, we shall derive the expression for instantaneous velocity and acceleration in paraboloidal coordinates.

Mathemathecal analysis

The Cartesian unit vectors are related to the Paraboloidal coordinates unit vectors as:

$$\hat{U} = \frac{v\cos\phi}{(u^2+v^2)^{\frac{1}{2}}}\hat{\imath} + \frac{v\sin\phi}{(u^2+v^2)^{\frac{1}{2}}}\hat{\jmath} + \frac{u}{(u^2)+v^2)^{\frac{1}{2}}}\hat{k} \qquad (8)$$

$$\hat{V} = \frac{u\cos\phi}{(u^2+v^2)^{\frac{1}{2}}}\hat{\imath} + \frac{u\sin\phi}{(u^2+v^2)^{\frac{1}{2}}}\hat{\jmath} - \frac{u}{(u^2+v^2)}\hat{k} \qquad (9)$$

and

$$\widehat{\Phi} = \frac{uv\cos\phi}{uv}\,\hat{\imath} + \frac{uv\sin\phi}{uv}\,\hat{\jmath} \qquad (10)$$

Inversion of (8) –(9) are given as:

$$\hat{\imath} = \frac{v\cos\phi}{(u^2+v^2)^{\frac{1}{2}}}\,\hat{u} + \frac{u\cos\phi}{(u^2+v^2)^{\frac{1}{2}}}\,\hat{v} - \sin\phi\,\widehat{\Phi} \qquad (11)$$

$$\hat{\jmath} = \frac{v\sin\phi}{(u^2+v^2)^{\frac{1}{2}}}\,\hat{u} + \frac{u\sin\phi}{(u^2+v^2)^{\frac{1}{2}}}\,\hat{v} + \cos\phi\,\widehat{\Phi} \qquad (12)$$

And

$$\hat{k} = \frac{u}{(u^2+v^2)^{\frac{1}{2}}}\,\hat{u} - \frac{v}{(u^2+v^2)^{\frac{1}{2}}}\,\hat{v} \qquad (13)$$

Hence denoting one time differentiating by a dot, it follows from (8),(9) and (10) and some manipulations that:

$$\dot{\widehat{U}} = \left[\frac{-v}{u^2+v^2}\,\dot{u} + \frac{u}{u^2+v^2}\,\dot{v}\right]\widehat{V} + \frac{v}{(u^2+v^2)^{\frac{1}{2}}}\,\dot{\phi}\,\widehat{\Phi} \qquad (14)$$

Similarly, it follows from (9), (8), and (10) that:

$$\dot{\widehat{V}} = \left[\frac{v}{u^2+v^2}\,\dot{u} - \frac{u}{u^2+v^2}\,\dot{v}\right]\widehat{U} + \frac{u}{(u^2+v^2)^{\frac{1}{2}}}\,\dot{\phi}\,\widehat{\Phi} \qquad (15)$$

And
Consequently from (10), (11), and (12):

$$\dot{\widehat{\Phi}} = \frac{-1}{(u^2+v^2)^{\frac{1}{2}}}\left[v\,\hat{u} + u\hat{v}\right]\dot{\phi} \qquad (16)$$

Now it follows from definition of instantaneous position $\underline{r}$ as

$$\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad (17)$$

And (8)-(10) and (11)- (13) that the instantaneous position vector may be expressed entirely in terms of paraboloidal coordinates as:

$$\underline{r} = \frac{u}{2(u^2+v^2)^{-\frac{1}{2}}}\,\widehat{U} + \frac{v}{2(u^2+v^2)^{-\frac{1}{2}}}\,\widehat{V} \qquad (18)$$

It now follows defination of instantaneous velocity vector $\underline{u}$ as:

$$\underline{u} = \underline{\dot{r}} \qquad (19)$$

And (18), (14)-(16) that the instantaneous velocity vector may be expressed enyirely in terms of paraboloidal coordinates as :

$$\underline{u} = u_u\,\widehat{U} + u_v\,\widehat{V} + u_\phi\,\widehat{\Phi} \qquad (20)$$

Where,

$$\underline{U}_U = \frac{1}{(u^2+v^2)^{-\frac{1}{2}}}\,\dot{U} \qquad (21)$$

$$\underline{U}_V = \frac{1}{u^2+v^2)^{-\frac{1}{2}}}\,\dot{V} \qquad (22)$$

And

$\underline{U}_{\phi}$ = uv$\dot{\Phi}$ (23)

Similarly, it follows from definition of instataneous acceleration, g, as;

$\underline{a} = \underline{u}$ (24)

And (8)-(24) and (14)-(16) that the instataneousl acceleration may be expressed in terms of paraboloidal coordinates as;

$\underline{a} = a_u\hat{U} + a_v\hat{V} + a_{\phi}\hat{\Phi}$ (25)

Where

$a_u = (u^2 + v^2)^{\frac{1}{2}}\left[\ddot{U} + \frac{2v}{u^2+v^2}\,\dot{u}\dot{v} + \frac{u}{u^2+v^2}\,\dot{U}^2 - \frac{u}{u^2+v^2}\,\dot{V}^2 - \dot{}\frac{uv}{u^2+v^2}\,\dot{V}\,\dot{\Phi}^2\right]$ (26)

$a_v = (u^2 + v^2)^{\frac{1}{2}}\left[\ddot{V} + \frac{2U}{u^2+v^2}\,\dot{u}\dot{v} + \frac{u}{u^2+v^2}\,\dot{V}^2 - \frac{v}{u^2+v^2}\,\dot{U}^2 - \dot{}\frac{uv}{u^2+v^2}\,\dot{U}\,\dot{\Phi}^2\right]$ (27)

$a_{\phi} = \left[uv\ddot{\Phi} + 2u\dot{V}\ \dot{\Phi} + 2v\dot{U}\,\dot{\Phi}^2\right]$ (28)

This is the completion of the paraboloidal coordinates system.

Results and discussion

In thpaper we derived the component of velocity and acceleration in paraboloidal coordinates as (20)-(24) and (25)-(28) respectively. These results obtained in this paper are necessary and sufficient for expresing all mechanical quantities (linear momentum, Kinetic energy and Hamiltonian) in terms of Paraboloidal coordinates.

Conclusion

The velocity and acceleration equation (21), (22), (23), (24), (25), (26), (27) and (28) obtained in this paper paves a way for expressing all dynamical Laws of motion [Newton's Law, Lagrange's Law. Hamilltonian's Law, Einstein's Special relativities Law of Motion and schrödinger's Law of Quantum Mechanics] entirely in terms of paraboloidal coordinates.

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