

The present and the future of some velocity in space-time

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Abstract

In this paper we introduced the present and the future of some velocity $[\omega \in \mathbb{R}]^+$, as hypercones in Space-Time, valid both in the classical mechanics (the Newtonian mechanics) and in the relativistic mechanics if ω does not exceed the speed of light in vacuum. The consequences are multiple including the need to redefine philosophical notions about the present, past and future, the expansion and the size of the universe available to an observer in the Space-Time.

Keywords: space-time, events, past hypercone, present hypercone, future hypercone, Minkowski space

1. Introduction

Both in the classical mechanics (the Newtonian mechanics) and in the relativistic mechanics, the present at time t_0 for an observer, i.e. the events occurring at time t_0 , is considered to be the hiperplan of all events having the equation $t = t_0$.

However, it is more natural to consider that the events affecting the observer at the time t_0 are part of his present at the time t_0 ,

For example, now we can see the North Star, even if this star was extinguished two hundred years ago. The disappearance of the North Star two hundred years ago, does not affect us at this time, nor in the next hundred years. If the northern star is at a distance $d=323$ light-years from Terra, for an observer located on Terra at the moment t , what counts is only the state of the star 323 years ago, i.e. the state of the star at time $t - 323 = t - \frac{d}{c}$, where time t , distance d and speed of the light in vacuum c , uses adequate unities.

The legatus Augusti pro praetore, the military governor of a province of the Roman Empire, get orders from Rome through messengers. If d is the distance to Rome and the messenger travels at velocity $\omega \in \mathbb{R}^+$, then the message received at time t by gouvernor, contains the Emperor's orders given at time $t - \frac{d}{\omega}$, (uses adequate unities).

The events in Rome at time $t - \frac{d}{\omega}$ affects the governor at the moment t , so this events are part of the present at the moment t , for the legatus - governor of that province.

Such considerations suggest new definitions for the present and the future for an observer.

2. Space-Time

In the physical space we consider an observer O with a ruler and a clock. The observer O can set a Cartesian reference frame which allows him to associate to each point of the physical space, its coordinates $(x, y, z) \in \mathbb{R}^3$. So, the mathematical model of physical space is \mathbb{R}^3 , the observer O is placed in the origin of reference frame, its coordinates in the physical space are $(0,0,0)$.

The time t measured by the observer O in a point of physical space is given by a clock placed at that point, synchronized with the clock of the observer O .

All the synchronized clocks placed in every point of physical space show the same time and go with the same rate with the clock of the observer O .

An *event* is a physical phenomenon that takes place at a space position (x, y, z) and at a certain time t . A *signal* can be of any kind like: a sound, a ray of light, a letter sent by post, and so on. Note that an event can generate or receive multiple signals traveling at different speeds. For example an explosion can generate signals, as light and sound.

We represent the event that takes place at a space position (x, y, z) and a time t , as a point $(x, y, z, t) \in \mathbb{R}^4$

Viceversa, any point $(x, y, z, t) \in \mathbb{R}^4$, corresponds to an event (eventually nothing is happening) occurring at the point (x, y, z) , at time t .

So, the set of all events characterized by their coordinates (x, y, z, t) is the Space-Time, mathematically modeled as \mathbb{R}^4 .

Remark 1: The observer O stand "at rest" at the point $(0,0,0)$ in the physical space, but in the Space-Time his position is at the point $(0,0,0,t)$ at the time t given by his clock. So, it is immobile in physical space but not in Space-Time. He standing in the physical space "at rest", travels along the axis of time to his future.

Remark 2: As we said in the introduction, for the observer O in the physical space, the present at the moment t_0 given by its clock, so far, is considered to be the hiperplan of all events with temporary coordination t_0 , i.e. the set of events $\{(x, y, z, t_0) / (x, y, z) \in \mathbb{R}^3\}$

3. New definition for the present past and future of an observer O in Space-Time

In Space-Time, the observer O is placed at the point $(0,0,0, t_0)$, at the time t_0 given by his clock.

Definition 1: The *present* of observer O at time t_0 and velocity $\omega \in \mathbb{R}^+$, is the hypercone

$$P_{t_0, \omega} = \left\{ \left(x, y, z, t_0 - \frac{\|(x,y,z)\|}{\omega} \right) \in R^4 \mid (x, y, z) \in R^3, \|(x, y, z)\| = \sqrt{x^2 + y^2 + z^2} \right\}$$

The implicit equation of this hypercone is $\omega^2(t - t_0)^2 - (x^2 + y^2 + z^2) = 0, t_0 \geq t$.

Definition 2. The *instant present* ("here and now") at t_0 for the observer, is the event $O(0, 0, 0, t_0)$, included in the *present* of observer O , at time t_0 and velocity $\omega \in R^+$

Definition 3. The *total present* at time t_0 is $P_{t_0} = \bigcup_{\omega \in R^+} P_{t_0, \omega}$

Definition 4. The observer's *past*, at time t_0 is the set of events $\{(0,0,0,t)/t < t_0\}$.

So, everything that an observer O sees at the time t_0 , is part of his present, at time t_0 , of velocity of light c i.e. part of the light hypercone. All that the observer hears at the moment t_0 , is part of the hypercone of sound velocity s , in the observer's environment, namely present of velocity s . (Here we refer to the events that can generate those effects).

An event may belong, for example, to the present at a moment t_0 and the speed v , but also to the present at time t_1 and the velocity w other than v , such as an explosion that generates light and sound, the sound reaching the observer later than the luminous signal.

Definition 5. The *influential future* of observer O , at time t_0 and velocity $\omega \in R^+$, is the hypercone:

$$IF_{t_0, \omega} = \left\{ \left(x, y, z, t_0 + \frac{\|(x,y,z)\|}{\omega} \right) \in R^4 \mid (x, y, z) \in R^3, \|(x, y, z)\| = \sqrt{x^2 + y^2 + z^2} > 0 \right\}$$

Minus its vertex, i.e. minus the point $(0, 0, 0, t_0)$.

The implicit equation of this hypercone is $\omega^2(t - t_0)^2 - (x^2 + y^2 + z^2) = 0, t_0 < t$.

Definition 6. The *influential future* of observer O at time t_0 is the

$$\text{Set of events } IF_{t_0} = \bigcup_{\omega \in R^+} IF_{t_0, \omega}$$

So, all events that can receive the signals of speed ω , emitted at time t_0 by the observer $O(0, 0, 0, t_0)$, constitute the *yielding future of speed ω at time t_0* . All events that can receive signals from the event $O(0, 0, 0, t_0)$ constitute the *yielding future* of observer O at time t_0 .

If $\omega = c$, we have the light hypercone of yielding future at time t_0 without $(0, 0, 0, t_0)$. If $\omega = s$ where s is the speed of sound, we have the hypercone of *yielding future* of sound speed at time t_0 without $(0, 0, 0, t_0)$.

Definition 7. If there is a maximum finite velocity of signals in the Space-Time, say the speed of light c , then we can define the *total future of observer O at time t_0* as the exterior of the light hypercone $P_{t_0, c}$ - the present of observer O at time t_0 and velocity c .

In this case, the total future of observer O at time t_0 contains:
 -the events potentially influenced by the event "here and now" $(0, 0, 0, t_0)$ - namely influential future IF_{t_0} ;
 -The *noninfluential* events outside the light hypercones $P_{t_0, c}$ and outside the hypercone $IF_{t_0, c}$ - the "space like events" or "elsewhere" defined in Minkowski space;
 - the events $(0, 0, 0, t)$ with $t_0 < t$.

If there are infinite velocity signals, then *the total future of observer O at time t_0* , will be the hypersemispace with equation $t > t_0$.

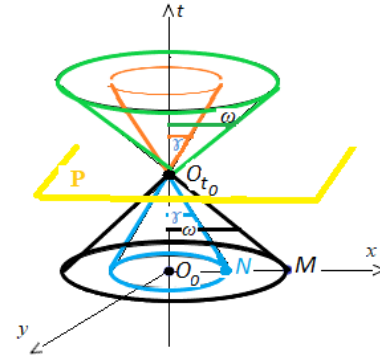


Fig 1: The diagram 3D –shows the Space-Time diagram in three dimensions (x, y, t) , where v and ω are real positive numbers, $v < \omega$.

The blue cone represents the present of the observer O , at time t_0 and velocity $v, N(vt_0, 0, 0)$.

The black cone represents the present of the observer O , at time t_0 and velocity $\omega, M(\omega t_0, 0, 0)$

The orange cone without its vertex, represents the influential future of the observer O , at time t_0 and velocity v .

The green cone without its vertex represents the influential future of the observer O , at time t_0 and velocity ω .

The yellow plane P , is the present of the observer O , at time t_0 and infinite velocity (if there is such velocity).

Observation 1. It is noticed that the interior of a hypercone of a certain speed, contains the lower speed hypercones. In the classical Minkowski space, the events located inside the lower part of the light hypercone are considered to be part of the past. For us, those events that are not on the axis of time, are part of the present of lower speed

Remark 3. The previously definitions are valid both in the classical mechanics (the Newtonian mechanics) and in the relativistic mechanics.

Sometimes it is easier to reason by means of diagrams 2D as in the next Figure.

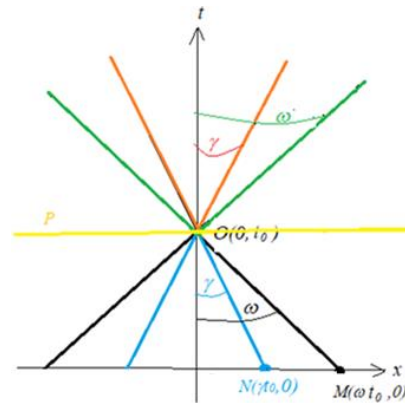


Fig 2: shows the Space-Time diagram in two dimensions (x, t) .

v and ω are real positive numbers, $v < \omega$.

The blue line segments represent the present of the observer O , at time t_0 and velocity v .

The black line segments represent the present of the observer O , at time t_0 and velocity ω .

The orange lines without its vertex represents the influential future of the observer O , at time t_0 and velocity γ .

The green lines without its vertex represent the influential future of the observer O , at time t_0 and velocity ω .

The yellow line P , is the present of the observer, at time t_0 and infinite velocity (if there is such velocity).

The Figure 3-shows the Space-Time diagram in two dimensions in the Mincowski space (a) and the diagram corresponding to some definitions, introduced in this paper, in the hypothesis that the speed of light in vacuum is the maximum velocity of the signals in Space-Time (b).

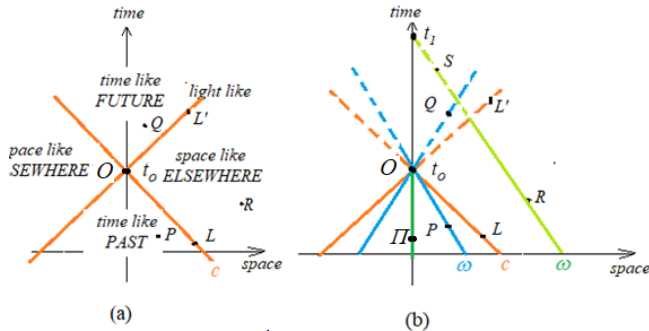


Fig 3

In (a), The event P is in the past, the event Q in the future, the events L, L' in the light hypercone and the event P belong to “Elsewhere”, for the observer O at time t_0 .

In (b), The event P belongs to the present of the observer O , at time t_0 and velocity ω , the events L belong to the present of the observer O , at time t_0 and velocity c , the event Q belongs to the influential future of the observer O , at time t_0 and velocity ω , the event L' belongs to the influential future of the observer O , at time t_0 and velocity c , the event R belongs to the noninfluential future of the observer O , at time t_0 and to the present of the observer O , at time t_1 and velocity ω , the event S belongs to the influential future of the observer O , at time t_0 and to the present of the observer O , at time t_1 and velocity ω , the events $\square\square\square\square\square\square(t), t_0 \geq t$ constitute the *observer's past at time t_0* .

Remark 4. An event E can generate more signals at different speeds, and so it may belong, for example, to both $P_{t_0,\omega}$ (blue line segments) and $P_{t_0,\gamma}$ (red line segments) as in Figure 4.

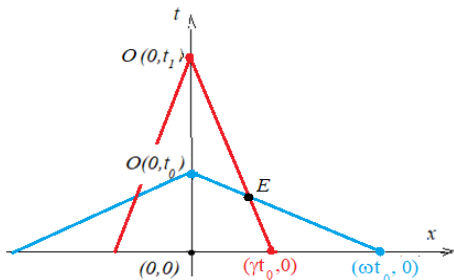


Fig 4

Figure 4-is a Space-Time diagram in two dimensions, in which: the blue line segments represents the present of the observer O ,

At time t_0 and velocity ω , the red line segments represents the present of the observer O , at time t_1 and velocity γ . The event E belong to the present of the observer O , at time t_0 and velocity ω , but also to the present of the observer O , at time t_1 and velocity γ . For example, an explosion that takes place at 3400 m of the terrestrial observer, generates light and sound. If the observer sees the light generated by the explosion at time t_0 he will hear the sound at time $t_1 \approx t_0 + 10$ seconds. So the event belongs to the present $P_{t_0,c}$ and to the present $P_{t_1,s}$ where c is the speed of light and s is the speed of sound.

Both in classical and relativistic mechanics, two fixed observers in the physical space at the time t_0 , have the same present given by the equation. $t = t_0$

According to the new definitions introduced in this paper, the links between two fixed observers in the physical space, become more complex. We will exemplify this in Fig. 5, in the case of the present for two observers.

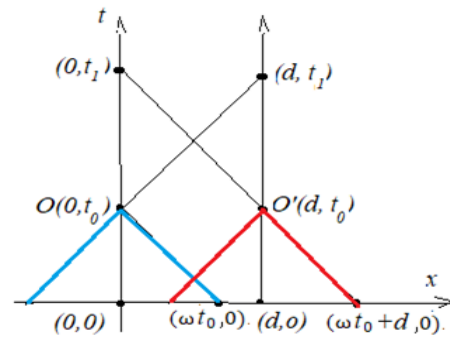


Fig 5

Figure 5-shows the Space-Time diagram in two dimensions (x, t) , and two observers fixed in the physical space, located at a distance d from each other, using the same reference frame (having the synchronized clocks and the same ruler).

The blue line segments represent the present of the observer O , at time t_0 and velocity ω .

The red line segments represent the present of the observer O , at time t_0 and velocity ω .

The present one is obtained from each other through a translation, but the meaning changes.

For example, we can observe that part of the present of the observer O' , at time t_0 And velocity belongs to the future of observer O , or to the present of observer O at time t_1 and velocity ω . The present of the observers O , and O' at time t_0 and velocity ω , may not have the commune events if $d > 2\omega t_0$.

4. The geometry of Space-Time

If we refer at the *present of the observer O at time t_0 and velocity $\omega \in R^+$* (denoted by $P_{t_0,\omega}$) then, the events of this *present* for the observer O , takes place simultaneously at time t_0 . So the observer O , naturally has to use a pseudo metric that produces a “distance” equal to zero, between those events of $P_{t_0,\omega}$.

Because the events of $P_{t_0,\omega}$ satisfy the equation $\omega^2(t - t_0)^2 - (x^2 + y^2 + z^2) = 0, t_0 \geq t$,

It is natural to use the pseudo-Euclidean metric: $(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2) \in R^4$, $\Delta s_{12}^2 = \omega^2(t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$,

invariant under a group of linear Lorentz transformations: $x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{\omega^2}}}$, $y'=y$, $z'=z$, $t' = \frac{t-\frac{vx}{\omega^2}}{\sqrt{1-\frac{v^2}{\omega^2}}}$, $v \in (-\omega, \omega)$ between two inertial

reference frame, whose axes Ox , Ox' coincide, the other axes remain parallel two by two and the second frame is moving along the common axis with the velocity v .

For each value $\omega \in R^+$, the Space-Time becomes a Minkowski space. When ω is speed of light we have the classical situation from Special relativity.

So, instead of using a single pseudo-Euclidean metric we must use a lot of such metrics depending on a parameter ω and a lot of Lorentz groups parameterized by the velocity ω .

As we know, the Special relativity is based on the postulates:

1. The laws of physics are the same for all observers in uniform motion, relative to one another (principle of relativity).
2. The speed of light in a vacuum is the same for all observers, regardless of their relative motion or of the motion of the light source.

Remark 5. If the maximum velocity of the signals for the observer O , is $\omega \in R^+$, then the postulate 2 can be replaced by:

2'. the maximum velocity of the signals for the observer O is $\omega \in R^+$, and the law of composition of two signals speeds $v, w \in (-\omega, \omega)$, with the same direction is $v * w = \frac{v+w}{1+\frac{vw}{\omega^2}}$

In this case, results similar to those obtained by Einstein in Special Relativity can be deduced, including the famous formula of energy, which in this case becomes: $E = m\omega^2$.

Remark 6. If in the Space- time for an Observer there are signals of velocity no matter how big, then, we leave the postulate 2, accepting that the law of composition of two signals speeds $v, w \in R^+$, with the same direction is usual addition of real numbers $v+w$, and we come to the Newtonian mechanics.

The above considerations, generally refer to an observer in the universe accessible to him. These considerations are only a beginning in the study of the properties of the introduced notions. The open issue consists in the compatibility of these structures using possibly new mathematical concepts in accordance with the experimental reality of physical space.

Consequences.

1. If we admit the Big- Bang theory and the fact that the maximum velocity of any signal is the speed of light in vacuum c , then, the diameter of available Universe for an observer O at time t_0 since Big-Bang, will be $2ct_0$. If the maximum velocity of any signal is $\omega \in R^+$, then the diameter of available Universe for an observer O at time t_0 since Big-Bang, will be $2\omega t_0$.
2. If the maximum signal speed is c (or ω), for an observer, the available Universe expands, because its diameter $2ct_0$ (or $2\omega t_0$) increases with time. t_0 since Big-Bang.
3. If in the Space- time for an Observer there are signals of velocity no matter how big, the theory of Big-Bang fall and the Universe is infinite in physical space and in time.

5. Conclusions

In any non-trivial universe model, the information describing an event can travel at different speeds. For an observer, the

perception of the present of the past and of the future, depends on the speed of the information describing the events that affect it at a given time.

The present and the future of some velocity $\omega \in R^+$, introduced as hypercones in Space-Time, both valid in the classical mechanics (the Newtonian mechanics) and in the relativistic mechanics if ω does not exceed the speed of light in vacuum, gives the possibility of a new interpretation of the events based on the speed of the signals emitted by them, in relation to an observer in the Space-Time universe.

At the same time, the introduced notions can lead to a new geometry of the universe, in accordance with the reality of the physical space perceived by an observer.

Models based on present or future of different speeds, could be used in the study of other systems using different speed signals such as encountered in chemistry biology and so on.

There are situations in many areas such as biology, chemistry, society and so on, when the maximum signal speed is less than the speed of light. In these situations, when the maximum signal speed ω is less than the speed of light, the law of composition of two signals speeds $v, w \in (-\omega, \omega)$ with the same direction, could be given by $*w = \frac{v+w}{1+\frac{vw}{\omega^2}}$

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