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Modeling and application of three-parameter Nakagami-m distribution to Nigeria telecommunication

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Abstract
The research paper examined the quality of service of the Nigeria telecommunication service of the major telecommunication providers in Nigeria. The Call Set-Up Success Rate (CSSR) and Dropped Call Rate (DCR) were analyzed using three parameters Nakagami-m distribution. The parameter estimates of the Nakagami-m distribution were provided and were tested if they fit into Nakagami-m distribution. The research work recommended that the National Communications Commission (NCC) which is the Apex body that allocates spectrums should put up effective control mechanisms for radio frequency users to improve infrastructure and Quality of Service. This paper has also attempted to provide mathematical guide for the accurate estimation of bandwidth requirements for organizations to improve the quality of service provided.

Keywords: nakagami-m distribution, quality of service, parameter estimates, CSSR, DCR

1. Introduction
1.1 General Introduction on Probability and Probability Distribution
Probability is the measure of the likelihood that an event will occur. Also, Probability is quantified as a number between 0 and 1, where, in general, 0 tells of impossibility and 1 signifies certainty. The higher the probability of an event, the more credible it is that the event will occur. The idea of probability has been given an axiomatic mathematical formalization in probability theory, which is used widely in such areas of study as Mathematics, Statistics, Finance, Gambling, Science (in particular Physics), Artificial Intelligence/Machine Learning, Computer Science, Game Theory, and Philosophy.
There have been at least two successful attempts to standardized Probability, namely the Kolmogorov formulation and the Cox formulation. In Kolmogorov's formulation, sets are interpreted as events and probability itself as a measure on a class of sets. In Cox's theorem, probability is taken as a primitive (that is, not further analysed) and the emphasis is on constructing a consistent assignment of probability values to propositions. In both cases, the laws of probability are the same, except for technical details. There are other methods for quantifying uncertainty, such as the Dempster–Shafer theory or possibility theory, but those are essentially different and not compatible with the laws of probability as usually understood.
Probability distribution is a mathematical function that furnishes the probabilities of occurrence of different possible outcomes in an experiment. A probability distribution is defined in terms of an underlying sample space, which is the set of all possible outcomes of the random phenomenon being observed. The sample space may be the set of real numbers or a higher-dimensional vector space, or it may be a list of non-numerical values; for example, the sample space of a coin flip would be {heads, tails}.
Probability distributions are generally divided into two classes. A discrete probability distribution (applicable to the scenarios where the set of possible outcomes is discrete, such as a coin toss or a roll of dice) can be encoded by a discrete list of the probabilities of the outcomes, known as a probability mass function. On the other hand, a continuous probability distribution (applicable to the scenarios where the set of possible outcomes can take on values in a continuous range (e.g. real numbers), such as the temperature on a given day) is typically described by probability density functions (with the probability of any individual outcome actually being 0). The probability distributions can have either univariate or multivariate random variables.

1.2 History of Nakagami Distribution
The Nakagami distribution (NKD) is relatively new; compared to other statistical distribution. It was proposed in 1960, it has been used to model attenuation of wireless signals traversing multiple paths and to study the impact of fading of radio signals, data regarding communicational engineering, and so forth. [1] The distribution may also be employed to model failure times of a variety of products (and electrical components) such as ball bearing, vacuum tubes, and electrical insulation. It is also widely considered in biomedical fields, such as to model the time to the occurrence of tumours and appearance of lung cancer. It has the applications in medical imaging studies to model the ultrasounds especially in Echo (heart efficiency test). [2], [3] and [4] used the Nakagami distribution to model ultrasound data in medical imaging studies. Similarly, [5] and [6] had shown the utility of the NKD to deal with the formation of high frequency seismogram envelopes. This distribution is extensively used in reliability theory and reliability engineering and to model the constant hazard rate portion because of its memory less property.

This distribution was introduced by M. Nakagami in 1960 [1]. In communications theory, Nakagami distribution (NKD) is mostly used for modelling the fading of radio signals. It has two parameters one is shape parameter (m-parameter/fading parameter) and other is scale (w) parameter. It is used to model scattered signals that reach a receiver from different paths. Depending on the thickness of the scatter, the signal will display diverse fading properties. NKD can be reduced to Rayleigh distribution, but gives more control over the extent of the fading. The NKD has also been applied successfully in many other fields as well. In order to use NKD to model a given set of data, we will have to estimate its parameters from the given data. Shape parameter is important in the sense as its knowledge is required by the receiver for optimal reception of signals in Nakagami fading.

The probability density function (pdf) of the Nakagami distribution is given as mentioned in below:

$$f(x|m,\Omega) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1}\exp\left(-\frac{m}{\Omega}x^2\right) \tag{1}$$

2. Research Methodology

In wireless telecommunications and all waves' related equipments, the received signal is subjected to fading due to two physical mechanisms. On one hand, the multipath components cause rapid and deep fading in displacements of few wavelengths (small-scale area). This is the well-known short-term fading or fast fading, which has been extensively analyzed in the literature [7, 17]. This fast fading signal has been modeled statistically using the Rice, Rayleigh, Nakagami-m, and Weibull distributions [18, 20].

The Nakagami-m distribution, whose pdf is defined in equation (1) above, is frequently employed to model the fast fading since it fits better than the other distributions in many measurement campaigns [1]. On the other hand, the received signal fluctuates slowly around a mean in displacements of hundreds of wavelengths (large-scale area). This variation is known as long-term fading or shadowing. This shadowing is due to the temporal blockage of the direct component between the transmitter and receiver terminals. The shadowing is commonly modeled statistically by a lognormal distribution.

2.1 Estimation of Two Parameters of Nakagami-M Distribution

The probability density function (p.d.f.) of the Nakagami-m distribution with two parameters $\{m,\Omega\}$ is given as [1, 21]:

$$\mathrm{f}(\mathrm{x}/\mathrm{m},\Omega) = \frac{2\mathrm{m}^{\mathrm{m}}}{\Gamma(m)\Omega^m}x^{2m-1}\exp\left(-\frac{m}{\Omega}x^2\right) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{2m-1}\exp\left(-\frac{m}{\Omega}x^2\right) \quad \begin{matrix} 0 \le \mathrm{x} \le \infty \\ 0.5 < m \le \infty \\ 0 \le \Omega \le \infty \end{matrix} \tag{1}$$

And its cumulative distribution function is given as:

$$F(\mathrm{x}/\mathrm{m},\Omega) = \frac{\gamma(\mathrm{m},\frac{m}{\Omega}x^2)}{\Gamma(m)} \tag{2}$$

Where m is the shape parameter and Ω is the speed parameter.

Its measures are given as:

$$\text{Mean; } E(x) = s + \left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} \tag{3}$$

$$\text{Variance; } Var\ (x) = \Omega\left[1 - \frac{1}{m}\left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right] \tag{4}$$

$$\text{Skewness; } Skewness = \frac{\Gamma(m+3/2)}{\mathrm{m}^{3/2}\Gamma(m)} \tag{5}$$

$$\text{Kurtosis; } Kurtosis = \frac{(m+1)}{m} \tag{6}$$

Its parameters are estimated as:

$$m = \frac{E^2(X^2)}{Var\ (X^2)} \tag{7}$$

$$\text{And } \Omega = \mathrm{E}\ (X^2) \tag{8}$$

2.2 Estimation of Three Parameters of Nakagami-M Distribution

The probability density function (p.d.f) of the extended Nakagami distribution with three parameters $\{\mathrm{m},\Omega,\mathrm{s}\}$ as defined by [22] and

Also known as Nakagami-Akintunde distribution is given as:

$$f(x/m,\Omega,s) = \frac{2m^m}{\Gamma(m)\Omega^m}(x-s)^{2m-1}\, exp\left(-\frac{m}{\Omega}(x-s)^2\right)$$

$$= \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m (x-s)^{2m-1} \exp\left(-\frac{m}{\Omega}(x-s)^2\right) \quad \begin{matrix} 0 \le x \le \infty \\ 0.5 < m \le \infty \\ 0 \le \Omega \le \infty \\ x \le s \le \infty \end{matrix} \tag{9}$$

And its cumulative distribution function is given as:

$$F(x/m,\Omega,s) = \frac{\gamma(m,\frac{m}{\Omega}(x-s)^2)}{\Gamma(m)} \tag{10}$$

Where m is the shape parameter, Ω is the speed parameter and s is the location parameter.

The measures of the three parameter distribution are given as:

Mean; $E(X) = s + \left(\frac{\Omega}{m}\right)^{1/2} \frac{\Gamma(m+1/2)}{\Gamma(m)}$ (11)

Variance; $Var\ (x) = \Omega\left[1 - \frac{1}{m}\left(\frac{\Gamma(m+1/2)}{\Gamma(m)}\right)^2\right]$ (12)

Skewness; $Skewness == \frac{\left[s^3 + 3s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 3s\Omega + \left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)}\right]}{\left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right]^{3/2}}$ (13)

Kurtosis; $Kurtosis = \frac{\left[s^4 + 4s^2\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + 6s^2\Omega + 4s\left(\frac{\Omega}{m}\right)^{3/2}\frac{\Gamma(m+3/2)}{\Gamma(m)} + \frac{\Omega^2(m+1)}{m}\right]}{\left[s^2 + 2s\left(\frac{\Omega}{m}\right)^{1/2}\frac{\Gamma(m+1/2)}{\Gamma(m)} + \Omega\right]^2}$ (14)

The parameters are estimated as:

$$m = \frac{E^2((X-s)^2)}{Var\ ((X-s)^2)} \tag{15}$$

$$\Omega = \mathrm{E}\ ((X-s)^2) \tag{16}$$

And $s = \frac{\sum x}{n}$ (17)

3. Analyses and Results

In this research work, analyses are done on the application of three-parameter Nakagami-m distribution to telecommunication industry in Nigeria. Thus, the following term are briefly defined as:

Telecommunication: The science and technology of communication or messages over a distance especially using electric, electronic or electromagnetic impulses. Furthermore, it is the technology of sending signals, images, and messages over a long distance electronically for example radio, telephone, television, and satellite and so on.

Quality of Service: This is abbreviated as QOS, it is the quality of service rendered at a particular time.

Key Performance Indicators: This is abbreviated as KPIS, and it is a qualitative measure used to evaluate the success of an organization employee in meeting objectives for performance.

Call Setup Success Rate: This is abbreviated as CSSR, and it is the number of the unblocked calls attempts, divide by the total of all attempts.

Dropped Call Rate: This is abbreviated as DCR, and it is the number of dropped calls divide by the total of all attempts.

SDCCCR: This is defined by the probability of failure of accessing a standalone dedicated control channel (SDCCH) during call setup.

SDCCH: This is used in GSM system to provide a reliable connection for signalling and SMS.

TCCCR: This is the probability of failure of accessing a traffic channel during call setup.

Traffic Control Channels: This is abbreviated as TCCH and it is responsible for transferring information between mobiles and BTS.

The Call Set-Up Success Rate (CSSR) and Dropped Call Rate (DCR) for the two major leading telecommuniicaion industries (MTN Nigeria and Globalcom Nigeria) are explicitly analyzed using the three-parameter Nakagami-m distribution. The results are as shown in Table below:

Table 1: Observed and Expected Valeus/pdf values of MTN and GLO CSSR and DCR

Date	MTN CSSR	GLO CSSR	MTN DCR	GLO DCR	Expected MTN CSSR	Expected GLO CSSR	MTN DCR pdf	GLO DCR pdf
Jan 2017	99.07	98.10	0.57	0.61	87.367050	51.722050	0.436794	0.436794
Feb 2017	98.98	98.15	0.58	0.60	82.318960	58.345500	0.431301	0.431301
Mar 2017	98.98	98.16	0.58	0.56	82.318960	59.491050	0.435995	0.435995
Apr 2017	98.92	98.44	0.26	0.70	74.638970	62.850570	0.600557	0.600557
May 2017	99.24	98.51	0.28	0.79	74.638970	55.001510	0.734994	0.935167
Jun 2017	99.26	98.63	0.29	0.74	71.393340	35.359210	0.765678	0.732048
Jul 2017	99.34	98.60	0.66	0.76	55.532130	40.868370	0.808769	0.808769
Aug 2017	99.33	98.40	0.70	0.52	57.735900	65.878100	0.928618	0.482876
Sept 2017	98.82	98.71	0.72	0.56	55.532133	49.334860	0.876900	0.435995
OcT 2017	98.81	98.21	0.66	0.53	53.274570	64.242370	0.467293	0.467293
Nov 2017	98.93	98.11	0.61	0.52	76.138920	53.160690	0.482876	0.482876
Dec 2017	98.95	98.02	0.55	0.52	78.881250	38.449060	0.482876	0.482876
Jan 2018	99.04	98.10	0.52	0.48	86.593300	51.722050	0.569644	0.569644
Feb 2018	98.86	98.02	0.52	0.52	63.992520	38.449060	0.482876	0.482876
Mar 2018	99.21	98.25	0.45	0.55	78.881250	66.801300	0.443815	0.443815
Apr 2018	99.17	98.59	0.46	0.53	83.277130	42.626930	0.467293	0.467293
May 2018	99.29	98.30	0.50	0.53	65.949500	68.358900	0.467293	0.467296
Jun 2018	99.24	98.40	0.53	0.50	74.638970	65.878100	0.521495	0.521495

The three parameters of the Nakagami-m distribution, namely shape parameter (m), speed parameter (Ω) and location parameter (s), were calculated obtained and fixed. Their values were obtained as follows:

1. For MTN CSSR; m=0.5, Ω=0.18124 and s=99.08
2. For GLO CSSR, m=0.5, Ω=0.22432 and s=98.3167
3. For MTN DCR, m=0.5, Ω=0.13656 and s=0.5244
4. For GLO DCR, m=0.5, Ω=0.5844 and s=0.09624

The pictorial representations of the observed and fitted three-parameter Nakagami-m distribution for the CSSR and DCR are as shown below.

Having done the fitting and pictorial representation, it was then tested, examined and found that all of MTN CSSR, Glo CSSR, MTN DCR and Glo DCR fit into Nakagami-m distribution. Thus, the three-parameter Nakagami-m distribution is recommended for use in Nigeria telecommunication industry [23].

4. Conclusion

In this research work, the probability density function (pdf) of the new extended three-parameter Nakagami-m distribution developed has been presented with all its parameters estimation and properties.

Knowing all these, after fixing the parameters, we discover and know the class and quality of signal sent and received. Hence, we are able to determine the signal that is expected.

Conclusively, this study affirm and revalidates that Nakagami distribution is massively applicable in the sending and in receiving telecommunication signals. We can easily figure out where error is and provide quantitative and qualitative measure in correcting and preventing them.

5. References

1. Nakagami M. The m-distribution-a general formula of intensity distribution of rapid fading in Statistical Methods in Radio Wave Propagation: Proceedings of a Symposium Held at the University of California, Los Angeles, June 18-20, W.C. Hoffman, Ed, Pergamon Press, Oxford, UK, 1960, 1960:336.
2. Sarkar S. "Adequacy of Nakagami-m distribution function to derive GIUH," Journal of Hydrologic Engineering. 2009; 14(10):1070-1079.
3. Sarkar S. "Performance investigation of Nakagami-m distribution to derive flood hydrograph by genetic algorithm optimization approach," Journal of Hydrologic Engineering. 2010; 15(8):658-666.
4. Tsui P. "Use of Nakagami distribution and logarithmic compression in ultrasonic tissue characterization," Journal of Medical and Biological Engineering. 2006; 26(2):69.
5. Carcole E, Sato H. "Statistics of the fluctuations of the amplitude of coda waves of local earthquakes," Proc. Seismological Society of Japan, 2009 Fall Meeting, C31-13, Kyoto, Japan, 2009.

6. Nakahara H, Carcolé E. "Maximum likelihood method for estimating coda Q and the Nakagami-m parameter," Bulletin of the Seismological Society of America. 2010; 100(6):3174-3182.
7. Haowei Bai, Mohammed Atiquzzaman. "Error Modeling Schemes for Fading Channels in Wireless Communications: a Survey", IEEE Communication Surveys, Fourth Quarter, 2003, 5(2).
8. Suzuki H. A statistical model for urban radio propagation. IEEE Transactions on Communications. 1977; 25(7):673-680.
9. Cheng YC, Robertazzi TG. "Critical Connectivity Phenomena in Multi-Hop Radio Models", IEEE Transactions on Communication. 2011; 37(7):770-777. International Journal of Computer Applications (0975 – 8887) Volume 26– No.2, July 2011 12. 49
10. Gupta P, Kumar PR. "Critical Power for Asymptotic Connectivity in Wireless Networks", in proceedings of IEEE conference on Design & Control, 1998, 676-681.
11. Liaoruo Wang. "Connectivity in Cooperative Wireless Ad Hoc Networks," proc. of ACM MobiHoc, 2008.
12. Bettstetter C, Hartmann C. "Connectivity of Wireless Multi-Hop Networks in a Shadow Fading Environment", Wireless Networks. 2005; 11(5):571-579.
13. Hekmat R, Van Mieghem P. "Connectivity in Wireless Adhoc Networks with a Lognormal Radio Model", Mobile Networks and Applications. 2006; (11):351-360.
14. Orriss J, Barton SK. "A Statistical Model for Connectivity between Mobiles and Base Stations: The Extension to Suzuki", COST 273 TD. 2002; (02)121.
15. Orriss J, Barton SK. "A Statistical Model for Connectivity between Mobiles and Base Stations: from Suzuki to Rice and beyond", COST 273 TD, 2003, (03)89.
16. Haenggi M. "A Geometry-inclusive Fading Model for Random Wireless Networks", in proceedings of IEEE ISIT, 2006, 1329-1333. 51
17. Dousse O, Baccelli F, Thiran P "Impact of Interferences on Connectivity in Adhoc networks," IEEE/ACM Transactions on Networking. 2005; 13(2):425-436.
18. Abu-Dayya AA, Beaulieu NC. Micro-and macro-diversity NCFSK (DPSK) on shadowed Nakagami-fading channels IEEE Transactions on Communications. 1994; 42(9):2693-2702.
19. Akintunde Oyetunde A, Adelagun ROA. Modeling Air Pollution by the use of Weibull Distribution for Sustainable Management. International Conference Proceeding of Sustainable Development and Environmental Protection by Institute for Environmental Research and Development, Bells University of Technology Ota, 2012, 134-139.
20. Ahmad SP, Ahmad K. Bayesian analysis of Weibull distribution using R software Australian Journal of Basic and Applied Sciences. 2013; 7(9):156-164.
21. Pandey BN, Dwividi N, Pulastya B. Comparison between Bayesian and maximum likelihood estimation of the scale parameter in Weibull distribution with known shape under linex loss function Journal of Scientific Research, 2011; 55:163-172.
22. Al-Athari FM. Parameter estimation for the double-pareto distribution Journal of Mathematics and Statistics. 2011; 7(4):289-294.
23. Akintunde Oyetunde A. On the introduction of location parameter to Nakagami-m distribution. International Journal of Statistics and Applied Mathematics. 2018; 3(4):147-154. ISSN: 2456-1452 Maths. 2018; 3(4):147-154 © 2018 Stats & Maths www.mathsjournal.com
24. Akintunde Oyetunde A. Modeling Call Set-up Success Rate to Nigeria Telecommunication Using Three-Parameter Nakagami-m Distribution. International Journal of Advanced Scientific Research. 2019; 4(1):34-38. March 2019
25. Santi P, Blough DM, Vainstein F. "A Probabilistic Analysis for the Radio Rang Assignment Problem in Ad-hoc Networks", in proceedings of ACM MobiHoc, 2001, 212-220.
26. Santi P, Blough DM. "The Critical Transmitting Range for Connectivity in Sparse Wireless Adhoc Networks", IEEE Transactions on Mobile Computing. 2003; 2(1):25-39.