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Mathematical modelling on two phase hepatic systolic blood flow through arteries due to malaria

Ram Naresh Yadav
Assistant Professor, Department of Mathematics, Goswami Tulsidas Govt. PG College Karwi, Chitrakoot, Uttar Pradesh, India
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Abstract
The aim of this study is to analyse the mathematical model of the two-phase systolic blood flow in the human hepatic artery during malaria. Wherein, red blood cells and blood plasma make up the two phases. In this study we applied Navier-stoke equation and equation of continuity for cylindrical co-ordinate system and all required equations presented in tensorial form. We used a Newtonian model based on the artery's stress and strain rate, and adopted both an analytical and numerical solution technique. Clinical data for both blood pressure and haemoglobin have been gathered in order to conduct a graphical analysis of the blood pressure drop vs haematocrit. It is clear how the haematocrit affects how much blood pressure drops.

Keywords: Systolic pressure, haematocrit, circulatory system, non-newtonian power law model, hepatic blood flow

1. Introduction

1.1 Function of Hepatic Artery

The common hepatic artery is one of the final branches of the celiac artery. One important blood channel that gives the liver oxygenated blood is the hepatic artery. Together with the hepatic vein and the portal vein, it is one of the three principal blood vessels that supply the liver. The common hepatic artery, which splits off from the celiac artery, is usually where the hepatic artery originates. One of the branches of the abdominal aorta is the celiac artery. The true hepatic artery, which sends blood to the liver, and the gastroduodenal artery, which supplies blood to the stomach, and the duodenum, the first segment of the small intestine, split off from the common hepatic artery. The primary function of the hepatic artery is to deliver oxygen-rich blood to the liver. This oxygen is crucial for the metabolic processes carried out by the liver cells (hepatocytes) [1].

1.2 Constitution of Blood

Blood circulates throughout the body and is a complicated, necessary fluid that serves a number of vital purposes for preserving homeostasis. In addition to liquid components known as plasma, blood also consists of many cell components. Erythrocytes or red blood cells: Supply the body's tissues with oxygen from the lungs and return carbon dioxide from the tissues to the lungs for expiration. Leukocytes, or white blood cells, protect the body from infections and foreign substances in order to support the immune system and Platelets, or thrombocytes, form clots to halt bleeding, which are a vital part of their function in blood clotting and wound healing [2].

1.3 Description of Disease

Plasmodium parasites are the source of malaria, an infectious disease that can be fatal. Humans contract it from female *Anopheles* mosquitoes that are infected. *Anopheles* mosquitoes are common in tropical and subtropical locations, where malaria is still a serious worldwide health concern. Many Plasmodium parasite species are known to cause malaria in humans. Though *Plasmodium vivax*, *P. ovale*, *P. malariae*, and *P. knowlesi* are also present, *Plasmodium falciparum* is the most prevalent and lethal species [3, 4].

2. Real Modal

2.1 Frame of reference

In this model we employing Navier-Stoke's equation and equation of continuity and have choose orthogonal curvilinear generalized three-dimensional co-ordinate system denoted by E^3 called three dimensional Euclidean space of the moving blood. It is imperative that the biophysical laws be true since, in Mishra's opinion, they fully hold true in any coordinate system [5]. All quantities related to blood flow written in tensorial form which is comparatively more realistic. Let P be any point in space with co-ordinate x^i with respect to axes Ox^i, O as origin where i = 1, 2, 3. At time t, $v^k = v^k(x^i, t)$ be velocity of blood, $p = p(x^i, t)$ thermodynamical pressure

and ρ = ρ (x^i, t) density. Since blood vessels are cylindrical the governing equations have to transform into cylindrical co-ordinates system. The following are the boundary conditions:

1. The velocity of blood flow on the axis of artery at r =0 will be maximum and finite, say v_0.
2. The velocity of blood flow on the wall of the blood vessels at r=R, Where, R is the radius of traverse. Section of artery, will be zero. This condition is well known as no-slip condition.

3. Basic Bio-Fluid Equations for Two Phase Blood Flow

As per Sherman I.W. and Sherman V.G., blood is classified as a mixed fluid. Blood typically goes through two stages. A semi-permeable membrane with a density higher than that of plasma envelops the first phase, which is plasma, and the second phase is blood cells. In the plasma, these blood cells are evenly spaced. Therefore, blood can be thought of as a uniform mixing of the two phases [6].

3.1 Equation of Continuity for Two Phase Blood Flow

The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X, this X is replaced by H/100, where H is the Haematocrit the volume percentage of blood cells. Then the volume portion covered by the plasma will be 1−X. If the mass ratio of blood cells to plasma is r then clearly.

$$r = \frac{X\rho_c}{(1-X)\rho_p} \tag{1}$$

Where ρ_c and ρ_p are densities of blood cells and blood plasma respectively. Usually, this mass ratio is not a constant; even then this may be supposed to constant in present context. The both phase of blood, i.e. blood cells and plasma move with the common velocity. Campbell and Pitcher have presented a model for this situation. According to this model, we consider the two phases of blood separately. Hence equation of continuity for two phases according to the principle of conservation of mass defined by J.N Kapoor and Gupta R.C. as follows:

$$\frac{\partial(X\rho_c)}{\partial t} + \left(X\rho_c v^i\right)_{,i} = 0 \text{ and } \frac{\partial(1-X)\rho_p}{\partial t} + \left((1-X)\,\rho_p v^i\right)_{,i} = 0 \tag{2, 3}$$

$(X\rho_c v^i)_{,i}$ Represents the covariant derivative of $(X\rho_c v^i)$with respect to x^i, and $\left((1-X)\rho_p v^i\right)_{,i}$ represents the covariant derivative of $\left((1-X)\rho_p v^i\right)$with respect to x^i, where v represents the common velocity of blood cells and plasma in two phases.

If ρ_m be uniform density of blood then

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \tag{4}$$ [7]

Where $\rho_m = X\rho_c + (1-X)\rho_p$ (5)

By combining equations (2) and (3) and using equation (5), we derive the following expression:

$$\frac{\partial\rho_m}{\partial t} + \left(\rho_m v^i\right)_{,i} = 0 \tag{6}$$

3.2 Equation of motion for two phase blood flow

The two phases of blood, or plasma and blood cells, are constantly in an equilibrium state, so Ruch, T.C. and H.D. [8] claim that the hydrodynamical pressure p between them can be assumed to be uniform. By assuming that blood cells have a viscosity coefficient η_c and using the conservation of momentum principle, we may obtain the following equation of motion for two phases of blood flow:

$$X\rho_c \frac{\partial v^i}{\partial t} + \left(X\rho_c v^j\right)v^i_{,j} = -Xp_{,j}g^{ij} + X\eta_c\left(g^{jk}v^i_{,k}\right)_{,j} \tag{7}$$

And $$(1-X)\,\rho_p \frac{\partial v^i}{\partial t} + \left\{(1-X)\rho_p v^j\right\}v^i_{,j} = -(1-X)\,p_{,j}g^{ij} + (1-X)\eta_p\left(g^{jk}v^i_{,k}\right)_{,j} \tag{8}$$

Now, by combining equations (7) and (8) together and making use of relation (5), one can obtain the equation of motion for blood flow.

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j) v^i_{,j} = -p_{,j} g^{ij} + \eta_m \left(g^{jk} v^i_{,k}\right)_{,j} \quad (9)$$

Where: $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient and $\rho_m = X\rho_C + (1-X)\rho_p$ is the density of blood as mixture of red blood cells and plasma.

4. Mathematical Modelling

We consider the two-layer blood flow to be Newtonian. The first layer is that of plasma while second one is core layer. Let the viscosity of plasma layer be η_p and that of the core layer η_c is the viscosity of the blood cells and X is portion of blood cells in unit volume.

The tensorial representation of the equation of continuity and equation of motion respectively as follows:

$$\frac{1}{\sqrt{g}} (\sqrt{g} v^i)_{,i} = 0 \quad (10)$$

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j) v^i{}_{,j} = -p_{,j} g^{ij} + \eta_m \left(g^{jk} v^i_{,k}\right)_{,j} \quad (11)$$

Since the blood vessels are cylindrical, the above governing equations have to transform into cylindrical co-ordinates. Let $x^1 = r\,,\quad x^2 = \theta\,,\quad x^3 = z$

Matrix of corresponding metric tensor in cylindrical coordinate system is as follows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } [g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas Christoffel's symbols of 2nd kind for cylindrical coordinate system are as follows:

$\left\{ {1 \atop 2\ 2} \right\} = -r\,,\ \left\{ {2 \atop 2\ 1} \right\} = \left\{ {2 \atop 1\ 2} \right\} = \frac{1}{r}$ Except of these all are zero.

Physical components of velocity of blood flow will be related as

$\sqrt{g_{11}} v^1 = v_r$ or, $v_r = v^1$

$\sqrt{g_{22}} v^2 = v_\theta$ or, $v_\theta = r v^2$ and $\sqrt{g_{33}} v^3 = v_z or,\quad v_z = v^3$

The direction of blood flow within the blood vessel is symmetrical with respect to its axis. Therefore, the variables hence $v_\theta = 0$, v_z, v_r and p are independent of the variable θ.

Now $v_r = 0, v_\theta = 0$ and $v_z = v$

Since blood flow is steady, then

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

The equation of continuity and equation of motion are transformed and take on the following form based on the previously provided information:

The equation of continuity in a cylindrical coordinate system is expressed as follows:

4.1 Equation of continuity for blood flow

$\frac{1}{\sqrt{g}} \left(\sqrt{g} v^i\right)_{,i} = 0$, i= 1, 2, 3

$$\frac{1}{\sqrt{g}} \left(\sqrt{g} v^1\right)_{,1} + \frac{1}{\sqrt{g}} \left(\sqrt{g} v^2\right)_{,2} + \frac{1}{\sqrt{g}} \left(\sqrt{g} v^3\right)_{,3} = 0$$

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^1}\left(\sqrt{g}v^1\right)+\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^2}\left(\sqrt{g}v^2\right)+\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^3}\left(\sqrt{g}v^3\right)=0$$

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial(v_\theta)}{\partial\theta}+\frac{\partial v_z}{\partial z}=0$$

$$\frac{\partial v_z}{\partial z}=0$$

Or, $v_z=v(r)$ (12)

4.2 Equation of motion for blood flow

Radial Momentum Equation

$$\rho_m\frac{\partial v^i}{\partial t}+\rho_m v^j v^i_{,j}=-\tau^{ij}_{,j}$$

$$\rho_m\frac{\partial v^1}{\partial t}+\rho_m v^1 v^1_{,1}+\rho_m v^2 v^1_{,2}+\rho_m v^3 v^1_{,3}=\{\tau^{11}_{,1}+\tau^{12}_{,2}+\tau^{13}_{,3}\}$$

Where: $\tau^{,11}_{,1}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)$, $\tau^{,12}_{,2}=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^1}\left(\sqrt{g}\tau^{,11}\right)+\begin{Bmatrix}1\\2\end{Bmatrix}\tau^{,22}$

$$\tau^{,22}=\eta_m\frac{1}{r^2}\left[\left(\frac{\partial(\frac{v_\theta}{r})}{\partial\theta}\right)+\frac{1}{r}v_r\right],\quad \tau^{,13}_{,3}=\eta_m\frac{\partial^2 v_r}{\partial z^2}$$

$$\rho_m\left[\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right]=-\frac{\partial p}{\partial r}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{\partial v_r}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}-\frac{v_r}{r^2}+\frac{\partial^2 v_r}{\partial z^2}\right] \quad (13)$$

Tangential Momentum Equation

$$\rho_m\frac{\partial v^2}{\partial t}+\rho_m v^2 v^1_{,1}+\rho_m v^2 v^2_{,2}+\rho_m v^3 v^2_{,3}=\{\tau^{,21}_{,1}+\tau^{,22}_{,2}+\tau^{,23}_{,3}\}$$

$$\tau^{,21}_{,1}=\eta_m\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{1}{r}\frac{\partial v_\theta}{\partial r}\right\}+\eta_m\frac{1}{r}\frac{1}{r^2}\left(\frac{\partial v_r}{\partial\theta}-r\frac{v_\theta}{r}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left(r\frac{1}{r^2}\left[\frac{\partial(\frac{v_\theta}{r})}{\partial\theta}+\frac{1}{r}v_r\right]\right)+\eta_m\frac{1}{r}\frac{1}{r}\frac{\partial v_\theta}{\partial r}\text{ and }\tau^{,23}_{,3}=\eta_m\frac{1}{r}\frac{\partial}{\partial z}\left(r\frac{\partial v_\theta}{\partial z}\right)=\eta_m\frac{\partial^2 v_\theta}{\partial z^2}$$

$$\rho_m\left[\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}-\frac{v_\theta v_r}{r}+v_z\frac{\partial v_\theta}{\partial z}\right]=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial v_\theta}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}-\frac{v_\theta}{r^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right] \quad (14)$$

Axial Momentum Equation:

$$\rho_m\frac{\partial v^3}{\partial t}+\rho_m v^1 v^3_{,1}+\rho_m v^2 v^3_{,2}+\rho_m v^3 v^3_{,3}=\{\tau^{31}_{,1}+\tau^{32}_{,2}+\tau^{33}_{,3}\}$$

Where: $\tau^{31}_{,1}=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r})$, $\tau^{32}_{,2}=\eta_m\frac{1}{r}\frac{\partial}{\partial\theta}\left(r\frac{1}{r^2}\frac{\partial v_z}{\partial\theta}\right)=\eta_m\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}$

$$\tau^{33}_{,3}=\eta_m\frac{1}{r}\frac{\partial}{\partial z}\left(r\frac{\partial v_z}{\partial z}\right)=\eta_m\frac{\partial^2 v_z}{\partial z^2}$$

$$\rho_m\left[\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}+v_z\frac{\partial v_r}{\partial z}\right]=-\frac{\partial p}{\partial z}+\eta_m\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial v_z}{\partial r}\right\}+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right] \quad (15)$$

Radial component $\frac{\partial p}{\partial r}=0,$

Or, $p=p(z)$ (16)

Tangential component 0=0

And axial component

$$-\frac{\partial p}{\partial z}+\frac{\eta_m}{r}\frac{\partial}{\partial r}\left[r\left\{\frac{\partial v_z}{\partial r}\right\}\right]=0 \quad (17)$$

$-\frac{\partial p}{\partial z}=P$, If the pressure gradient within a blood vessel located at a distance from the heart remains consistent, then equation (17) can be modified.

$$-P+\frac{\eta_m}{r}\frac{\partial}{\partial r}\left[r\left\{\frac{\partial v_z}{\partial r}\right\}\right]=0 \quad (18)$$

Integrate equation (18), then we obtain

$$r\frac{dv}{dr}=-\frac{Pr^2}{2\eta_m}+A \quad (19)$$

By imposing the boundary condition at r = 0 and $v = v_0$. (a constant) on equation (19), the value of A is determined to be 0. Consequently, equation (19) takes on the following form:

$$r\frac{dv}{dr}=-\frac{Pr^2}{2\eta_m} \quad (20)$$

Again, integrate equation (20), and then we get

$$v=-\frac{Pr^2}{4\eta_m}+B \quad (21)$$

By implementing the no-slip boundary condition, which states that the velocity (v) is zero at the radial distance r = R in equation (21), we obtain

$$B=\frac{Pr^2}{4\eta_m} \quad (22)$$

Using (22) in equation (21), we get

$$v=\frac{P}{4\eta_m}(R^2-r^2) \quad (23)$$

Where R is radius of blood vessel

5. Results and Discussion

Haematocrit vs blood pressure is taken from medical college Jhansi
Diagnosis- Dr. Anurag

Table 1: Patient's Systolic Blood Pressure, Hemoglobin, and Hematocrit Levels

S. No.	Date	Systolic B.P (In mm hg)	Hemoglobin (gm/dl)	Hematocrit (H)	Systolic B.P (In Pascal)
1.	08/09/2018	160	11.6	34.8	21331.2
2.	10/09/2018	155	10.8	32.4	20664.6
3.	12/09/2018	140	10.1	30.3	18664.8
4.	14/09/2018	145	9.6	28.8	19331.4
5.	16/09/2018	138	9.2	27.6	18398.16

The aggregate blood flow, or flux, across the transverse cross-section of the arteries is as follows:

$$Q=\int_0^R v.2\pi r dr=\int_0^R \frac{P}{4\eta_m}(R^2-r^2)2\pi r dr$$

Or, $Q=\frac{P}{4\eta_m}.\left[R^2\frac{r^2}{2}-\frac{r^4}{4}\right]_0^R$,

Or, $Q = \frac{\pi R^4 P}{8\eta_m}$

$where\ P = -\frac{dp}{dz}$

Or, $\int_{p_i}^{p_f} \mathrm{dp} = -\mathrm{P} \int_{z_i}^{z_f} \mathrm{dz}$

Or, $(p_f - p_i) = -\mathrm{P}\,(z_f - z_i)$

Or, $(p_f - p_i) = -\frac{8\eta_m}{Q\pi R^4}(z_f - z_i)$ (21)

Where $(p_f - p_i)$ blood pressure drops and $(z_f - z_i)$ = length of hepatic artery
Average Systolic pressure =19678.032 Pascal
H = Average haematocrit = 30.78
According to Glenn Elert (2010)
η_m = Viscosity of mixture = 0.035 pa. s. [9]
According to Gustafson, Daniel R. (1980)
η_p = Viscosity of plasma =0.0015 pa. s. [10]
Length of common hepatic arteries = 0.0347 m
Radius of hepatic artery R=2.5$\times 10^{-3}$ m [11]

We know that

$$\eta_m = \eta_c X + \eta_p (1 - X)$$

$$\eta_m = \eta_c \frac{H}{100} + \eta_p \left(1 - \frac{H}{100}\right) \quad (22)$$

Where: $X = \frac{H}{100}$

By substituting the values of η_m, η_p and H into the above relation (22), we may solve for η_c

$$0.035 = \eta_c \frac{30.78}{100} + 0.0015\left(1 - \frac{30.78}{100}\right)$$

We get
$\eta_c = 0.110336907$ p.s. =Viscosity of cells

By substituting the values of $\eta_c$and η_p into the aforementioned equation (22), we obtain

$$\eta_m = 0.110336907\ \frac{H}{100} + 0.0015\left(1 - \frac{H}{100}\right)$$

$$\eta_m = 108.836907 \times 10^{-5} H + 0.0015 \quad (23)$$

$p_f - p_i = 1.67 \times 10^{-5} \frac{8 \times 0.0347}{3.13(2.5 \times 10^{-3})^4} \eta_m$

$p_f - p_i = 37796.0355 \eta_m$

$p_f - p_i = 37796.0355(108.836907 \times 10^{-5} H + 0.0015)$

$p_f - p_i = 41.136036\ H + 56.6940532$ (24)

This is a linear relationship between blood pressure drop and haematocrit.

Table 2: Hematocrit levels and blood pressure drop over time

Date	08/09/2018	10/09/2018	12/09/2018	14/09/2018	16/09/2018
Hematocrit (H)	34.8	32.4	30.3	28.8	27.6
Blood pressure drop (ΔP)	1488.23	1389.5	1303.16	1241.41	1192.05

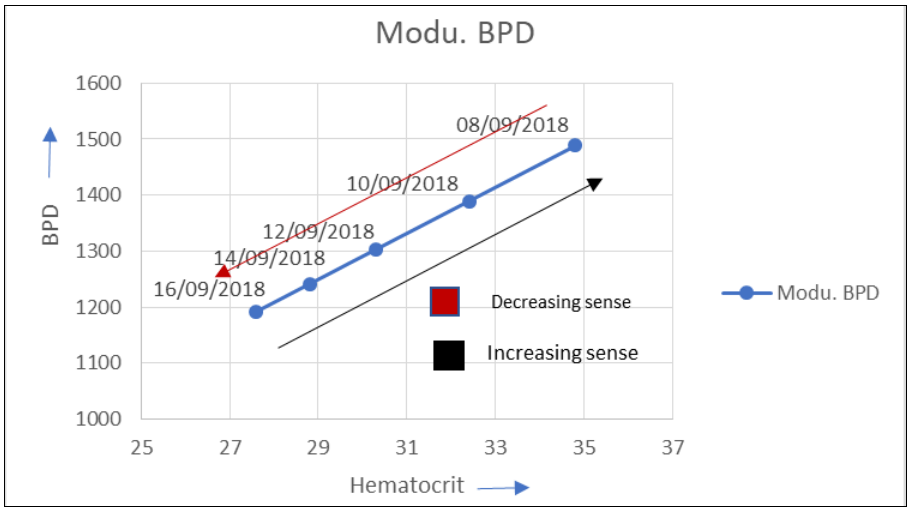

Fig 1: Blood Pressure drop Vs. Haematocrit

6. Conclusion

This graph shows a linear relationship between blood pressure drop and haematocrit described as p_f - p_i= 41.136036 H + 56.6940532, where slope 41.136036 represents the fluctuation of blood pressure drop with respect to haematocrit. Trend of above graph is downward indicating the blood pressure drop decreases with respect to haematocrit then we can advise increasing dose of medicine for malaria patient.

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8. References

1. Yoneyama F, Miyazaki K, Sugawara Y, *et al*. Quantification of hepatic arterial and portal venous blood flow using dynamic contrast-enhanced computed tomography in patients with chronic liver disease. J Gastroenterol. 2016;51(1):72-81.
2. Om Prakash, Upadhyay V, Agrawal AK, Pandey PN. A mathematical model on two-phase hepatic systolic blood flow in hepatic arterioles with special reference to Hepatitis B. Int. J Appl. Res. 2015;(8):318-323.
3. Barber BE, Russel B, Grigg MJ, Zhang R, William T, Amir A, *et al*. Reduced red blood cells deformability in Plasmodium knowlesi malaria. Blood Adv. 2018;2:433-443. DOI: 10.1182/bloodadvances.201701373.
4. Dasana AK, Schwarz US, *et al*. Multiscale modeling of malaria-infected red blood cells. 2018;978-3-319-50257-1-66-1.
5. Mishra RS. Tensors and Riemannian geometry. Pothishala Pvt. Ltd. Allahabad; 1990. DOI: 10.12691/ajmo-3-1-3.
6. Sherman IW, Sherman VG. Biology - A Human Approach. New York, Oxford: Oxford Univ. Press; 1983. p. 278-279. ISBN 0195031768.
7. Campbell IJ, Picher AS. Shock waves in a liquid containing gas bubbles. Proc. Roy Soc. A. 1958;243. DOI: 10.1098/rspa.1958.0018.
8. Ruch TC, Patton HD (Eds.). Physiology and Bio-Physics. W.B.S., 1973, 2-3. ISBN10:0721678181.
9. Elert G. Viscosity, the physics book. Hypertextbook.com. 2010; pp.09-14. Available from: https://physics.info/.
10. Denial RG. Physics: Health and Human body. Wadsworth Publishing Company; 1980. p. 518. ISBN10:0534007562.
11. Singh JP, Agrawal AK, Upadhyay V. Mathematical Modeling on Two Phase Hepatic Mean Blood Flow through in Liver Lobule with Special Reference to Hepatitis A. IJIRSET. 1999;5(1):2016. DOI: 10.15680/IJIRSET.2015.0501076.
12. Kapoor JN. Mathematical models in biology & Medicine. New Delhi: E.W.P.; 1992. p. 346. ISBN 81-85336-82-2.
13. Ye T, Phan N, Khoo TB, Lim CT. Stretching and relaxation of malaria-infected red blood cells. Biophys J. 2013;105(5):1103-1109.
14. Balasubramanian S. Simulation of hepatic blood flow with oxygen transport. Proc. Inst. Mech. Eng. H. 2013;227(8):904-913.